

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.4-f-x-
 $\wedge m-d+e-x^n-q-a+b-x^n+c-x^{2-n-p}$

Nasser M. Abbasi

June 29, 2021

Compiled on June 29, 2021 at 11:37am

Contents

1	Introduction	9
1.1	Listing of CAS systems tested	9
1.2	Results	10
1.3	Performance	14
1.4	list of integrals that has no closed form antiderivative	15
1.5	list of integrals solved by CAS but has no known antiderivative	16
1.6	list of integrals solved by CAS but failed verification	16
1.7	Timing	17
1.8	Verification	17
1.9	Important notes about some of the results	17
1.9.1	Important note about Maxima results	17
1.9.2	Important note about FriCAS and Giac/XCAS results	18
1.9.3	Important note about finding leaf size of antiderivative	18
1.9.4	Important note about Mupad results	19
1.10	Design of the test system	20
2	detailed summary tables of results	21
2.1	List of integrals sorted by grade for each CAS	21
2.1.1	Rubi	21
2.1.2	Mathematica	21
2.1.3	Maple	22

2.1.4	Maxima	22
2.1.5	FriCAS	22
2.1.6	Sympy	23
2.1.7	Giac	23
2.1.8	Mupad	23
2.2	Detailed conclusion table per each integral for all CAS systems	24
2.3	Detailed conclusion table specific for Rubi results	55
3	Listing of integrals	63
3.1	$\int (d + ex^3)^5 (a + bx^3 + cx^6) dx$	63
3.2	$\int (d + ex^3)^4 (a + bx^3 + cx^6) dx$	67
3.3	$\int (d + ex^3)^3 (a + bx^3 + cx^6) dx$	70
3.4	$\int (d + ex^3)^2 (a + bx^3 + cx^6) dx$	73
3.5	$\int (d + ex^3) (a + bx^3 + cx^6) dx$	76
3.6	$\int \frac{a+bx^3+cx^6}{d+ex^3} dx$	79
3.7	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2} dx$	85
3.8	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^3} dx$	91
3.9	$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx$	97
3.10	$\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6} dx$	104
3.11	$\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} dx$	110
3.12	$\int \frac{d+ex^3}{x(a+bx^3+cx^6)} dx$	115
3.13	$\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)} dx$	121
3.14	$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6} dx$	129
3.15	$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} dx$	141
3.16	$\int \frac{x(d+ex^3)}{a+bx^3+cx^6} dx$	152
3.17	$\int \frac{d+ex^3}{a+bx^3+cx^6} dx$	166
3.18	$\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)} dx$	181
3.19	$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)} dx$	192
3.20	$\int \frac{x^8(1-x^3)}{1-x^3+x^6} dx$	204
3.21	$\int \frac{x^5(1-x^3)}{1-x^3+x^6} dx$	208

3.22	$\int \frac{x^2(1-x^3)}{1-x^3+x^6} dx$	211
3.23	$\int \frac{1-x^3}{x(1-x^3+x^6)} dx$	215
3.24	$\int \frac{1-x^3}{x^4(1-x^3+x^6)} dx$	219
3.25	$\int \frac{x^6(1-x^3)}{1-x^3+x^6} dx$	223
3.26	$\int \frac{x^4(1-x^3)}{1-x^3+x^6} dx$	230
3.27	$\int \frac{x^3(1-x^3)}{1-x^3+x^6} dx$	238
3.28	$\int \frac{x(1-x^3)}{1-x^3+x^6} dx$	245
3.29	$\int \frac{1-x^3}{1-x^3+x^6} dx$	252
3.30	$\int \frac{1-x^3}{x^2(1-x^3+x^6)} dx$	258
3.31	$\int \frac{1-x^3}{x^3(1-x^3+x^6)} dx$	265
3.32	$\int \frac{x^2(-2+x^3)}{1-x^3+x^6} dx$	272
3.33	$\int \frac{1+x^3}{x(1-x^3+x^6)} dx$	276
3.34	$\int \frac{1+x^3}{x-x^4+x^7} dx$	280
3.35	$\int (d+ex^3)^{5/2} (a+bx^3+cx^6) dx$	285
3.36	$\int (d+ex^3)^{3/2} (a+bx^3+cx^6) dx$	291
3.37	$\int \sqrt{d+ex^3} (a+bx^3+cx^6) dx$	296
3.38	$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}} dx$	301
3.39	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}} dx$	306
3.40	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}} dx$	311
3.41	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}} dx$	316
3.42	$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}} dx$	321
3.43	$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8} dx$	326
3.44	$\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8} dx$	353
3.45	$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8} dx$	359
3.46	$\int \frac{x(d+ex^4)}{a+bx^4+cx^8} dx$	383

3.47	$\int \frac{d+ex^4}{a+bx^4+cx^8} dx$	390
3.48	$\int \frac{d+ex^4}{x(a+bx^4+cx^8)} dx$	417
3.49	$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)} dx$	426
3.50	$\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)} dx$	449
3.51	$\int \frac{d+ex^4}{x^4(a+bx^4+cx^8)} dx$	463
3.52	$\int \frac{x^4(1-x^4)}{1-x^4+x^8} dx$	498
3.53	$\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx$	503
3.54	$\int \frac{x^2(1-x^4)}{1-x^4+x^8} dx$	507
3.55	$\int \frac{x(1-x^4)}{1-x^4+x^8} dx$	513
3.56	$\int \frac{1-x^4}{1-x^4+x^8} dx$	516
3.57	$\int \frac{1-x^4}{x(1-x^4+x^8)} dx$	522
3.58	$\int \frac{1-x^4}{x^2(1-x^4+x^8)} dx$	526
3.59	$\int \frac{1-x^4}{x^3(1-x^4+x^8)} dx$	531
3.60	$\int \frac{1-x^4}{x^4(1-x^4+x^8)} dx$	536
3.61	$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)} dx$	543
3.62	$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)} dx$	550
3.63	$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)} dx$	556
3.64	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)} dx$	562
3.65	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)} dx$	568
3.66	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)} dx$	573
3.67	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)} dx$	578
3.68	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)} dx$	584
3.69	$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)} dx$	590

3.70	$\int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$	597
3.71	$\int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$	604
3.72	$\int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$	611
3.73	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$	618
3.74	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x(d+ex)^2} dx$	624
3.75	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^2(d+ex)^2} dx$	630
3.76	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^3(d+ex)^2} dx$	636
3.77	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^4(d+ex)^2} dx$	643
3.78	$\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^5(d+ex)^2} dx$	651
3.79	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d+ex} dx$	660
3.80	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^3 \sqrt{d+ex} dx$	666
3.81	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^2 \sqrt{d+ex} dx$	672
3.82	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} dx$	678
3.83	$\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex} dx$	686
3.84	$\int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex}}{x} dx$	695
3.85	$\int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex}}{x^2} dx$	705
3.86	$\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p dx$	716
3.87	$\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p dx$	718
3.88	$\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p dx$	722
3.89	$\int (fx)^m (d+ex^n) (a+cx^{2n})^p dx$	726
3.90	$\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx$	730
3.91	$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2} dx$	734
3.92	$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx$	738

3.93	$\int (b + 2cx) (a + bx + cx^2)^{13} dx$	742
3.94	$\int x (b + 2cx^2) (a + bx^2 + cx^4)^{13} dx$	747
3.95	$\int x^2 (b + 2cx^3) (a + bx^3 + cx^6)^{13} dx$	753
3.96	$\int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^{13} dx$	759
3.97	$\int (b + 2cx) (-a + bx + cx^2)^{13} dx$	766
3.98	$\int x (b + 2cx^2) (-a + bx^2 + cx^4)^{13} dx$	771
3.99	$\int x^2 (b + 2cx^3) (-a + bx^3 + cx^6)^{13} dx$	777
3.100	$\int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^{13} dx$	783
3.101	$\int (b + 2cx) (bx + cx^2)^{13} dx$	790
3.102	$\int x (b + 2cx^2) (bx^2 + cx^4)^{13} dx$	793
3.103	$\int x^2 (b + 2cx^3) (bx^3 + cx^6)^{13} dx$	797
3.104	$\int x^{-1+n} (b + 2cx^n) (bx^n + cx^{2n})^{13} dx$	801
3.105	$\int \frac{b+2cx}{a+bx+cx^2} dx$	805
3.106	$\int \frac{x(b+2cx^2)}{a+bx^2+cx^4} dx$	808
3.107	$\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6} dx$	811
3.108	$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}} dx$	814
3.109	$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx$	817
3.110	$\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$	820
3.111	$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8} dx$	824
3.112	$\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8} dx$	828
3.113	$\int \frac{b+2cx}{-a+bx+cx^2} dx$	832
3.114	$\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx$	835
3.115	$\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx$	838
3.116	$\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx$	841
3.117	$\int \frac{b+2cx}{(-a+bx+cx^2)^8} dx$	844
3.118	$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx$	847

3.119	$\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8} dx$	851
3.120	$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8} dx$	855
3.121	$\int \frac{b+2cx}{bx+cx^2} dx$	859
3.122	$\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx$	861
3.123	$\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx$	864
3.124	$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx$	867
3.125	$\int \frac{b+2cx}{(bx+cx^2)^8} dx$	871
3.126	$\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx$	874
3.127	$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx$	877
3.128	$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx$	880
3.129	$\int (b+2cx)(a+bx+cx^2)^p dx$	884
3.130	$\int x(b+2cx^2)(a+bx^2+cx^4)^p dx$	887
3.131	$\int x^2(b+2cx^3)(a+bx^3+cx^6)^p dx$	890
3.132	$\int x^{-1+n}(b+2cx^n)(a+bx^n+cx^{2n})^p dx$	893
3.133	$\int (b+2cx)(-a+bx+cx^2)^p dx$	896
3.134	$\int x(b+2cx^2)(-a+bx^2+cx^4)^p dx$	899
3.135	$\int x^2(b+2cx^3)(-a+bx^3+cx^6)^p dx$	902
3.136	$\int x^{-1+n}(b+2cx^n)(-a+bx^n+cx^{2n})^p dx$	905
3.137	$\int (b+2cx)(bx+cx^2)^p dx$	908
3.138	$\int x(b+2cx^2)(bx^2+cx^4)^p dx$	911
3.139	$\int x^2(b+2cx^3)(bx^3+cx^6)^p dx$	914
3.140	$\int x^{-1+n}(b+2cx^n)(bx^n+cx^{2n})^p dx$	917
3.141	$\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}} dx$	920
3.142	$\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2} dx$	923
3.143	$\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3} dx$	927
3.144	$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}} dx$	932
3.145	$\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$	936

3.146	$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$	940
3.147	$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$	944
3.148	$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$	948
3.149	$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})} dx$	951
3.150	$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})} dx$	956
3.151	$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})} dx$	960
3.152	$\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p dx$	964
3.153	$\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p dx$	970
3.154	$\int (fx)^m (a+bx^n+cx^{2n})^p dx$	974
3.155	$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{d+ex^n} dx$	977
3.156	$\int \frac{(fx)^m (a+bx^n+cx^{2n})^p}{(d+ex^n)^2} dx$	980
4	Listing of Grading functions	983
4.0.1	Mathematica and Rubi grading function	983
4.0.2	Maple grading function	985
4.0.3	Sympy grading function	990
4.0.4	SageMath grading function	993

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric₂F₁ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (156)	% 0.00 (0)
Mathematica	% 94.23 (147)	% 5.77 (9)
Maple	% 87.82 (137)	% 12.18 (19)
Maxima	% 44.23 (69)	% 55.77 (87)
Fricas	% 69.23 (108)	% 30.77 (48)
Sympy	% 48.08 (75)	% 51.92 (81)
Giac	% 70.51 (110)	% 29.49 (46)
Mupad	% 78.21 (122)	% 21.79 (34)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

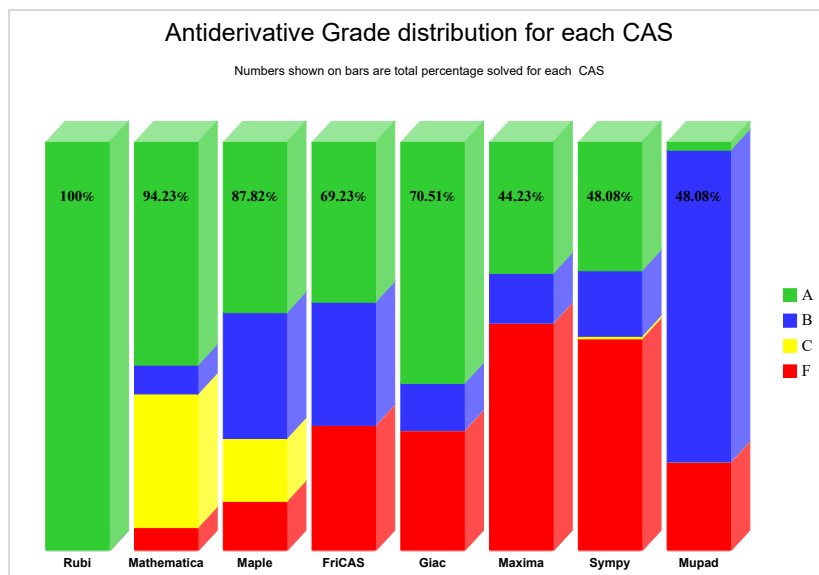
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

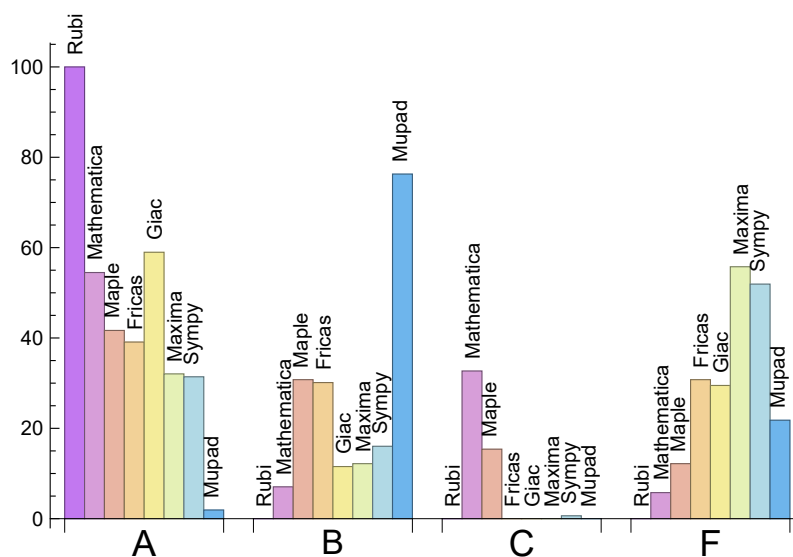
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	54.49	7.05	32.69	5.77
Maple	41.67	30.77	15.38	12.18
Maxima	32.05	12.18	0.00	55.77
Fricas	39.10	30.13	0.00	30.77
Sympy	31.41	16.03	0.64	51.92
Giac	58.97	11.54	0.00	29.49
Mupad	1.92	76.28	0.00	21.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	100.00 %	0.00 %	0.00 %
Maple	19	100.00 %	0.00 %	0.00 %
Maxima	87	66.67 %	4.60 %	28.74 %
Fricas	48	64.58 %	35.42 %	0.00 %
Sympy	81	3.70 %	90.12 %	6.17 %
Giac	46	78.26 %	10.87 %	10.87 %
Mupad	34	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

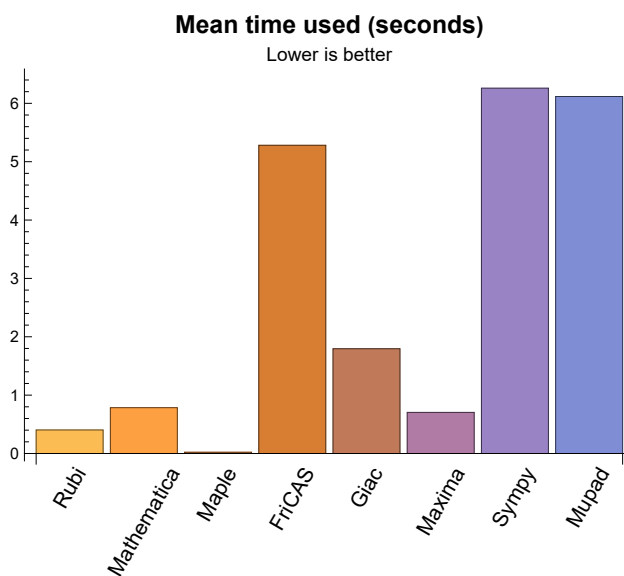
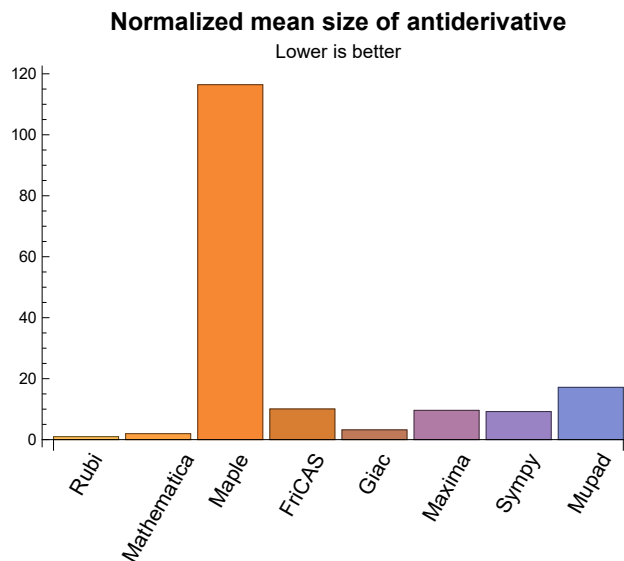
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.40	208.71	0.98	153.50	1.00
Mathematica	0.79	457.80	1.96	80.00	0.96
Maple	0.02	2589.11	116.42	70.00	1.24
Maxima	0.70	214.09	9.61	37.00	1.00
Fricas	5.28	940.94	10.07	216.00	2.49
Sympy	6.26	225.01	9.21	87.00	1.02
Giac	1.80	220.13	3.21	70.00	1.00
Mupad	6.12	3286.17	17.16	295.00	3.15

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{86, 155, 156}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {79, 143, 152, 153, 154}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

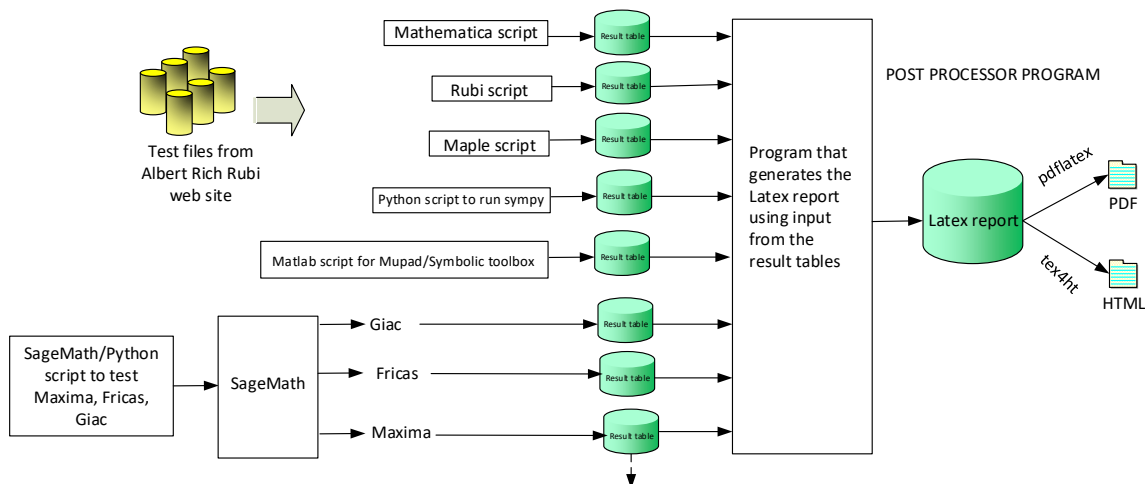
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 96, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 144, 149, 152, 153, 154, 155, 156 }

B grade: { 93, 94, 95, 97, 98, 99, 101, 102, 103, 142, 143 }

C grade: { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 138, 139, 140 }

F grade: { 90, 91, 92, 145, 146, 147, 148, 150, 151 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 53, 55, 57, 59, 64, 65, 66, 67, 74, 86, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 155, 156 }

B grade: { 6, 9, 35, 36, 37, 38, 39, 40, 41, 42, 46, 50, 61, 62, 63, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128 }

C grade: { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 60, 140 }

F grade: { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 32, 33, 53, 57, 86, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 94, 95, 96, 98, 99, 100, 102, 103, 104, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 55, 57, 58, 61, 62, 63, 64, 65, 66, 86, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 8, 16, 17, 25, 26, 27, 28, 29, 30, 31, 45, 46, 47, 50, 54, 56, 59, 60, 71, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade: { }

F grade: { 14, 15, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 49, 51, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123, 124, 137 }

B grade: { 9, 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 110, 111, 117, 118, 119, 125, 126, 127, 129, 133, 138 }

C grade: { 144 }

F grade: { 12, 13, 14, 15, 16, 17, 18, 19, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 112, 116, 120, 128, 130, 131, 132, 134, 135, 136, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 155, 156 }

B grade: { 25, 26, 27, 28, 29, 30, 31, 46, 50, 93, 94, 95, 96, 97, 98, 99, 100, 104 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 49, 51, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.8 Mupad

A grade: { 86, 155, 156 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

C grade: { }

F grade: { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	169	166	182	187	173	158
normalized size	1	1.00	1.01	1.04	1.02	1.12	1.15	1.06	0.97
time (sec)	N/A	0.185	0.047	0.002	0.813	0.687	0.101	0.340	1.599
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	135	147	151	141	130
normalized size	1	1.00	1.00	1.01	1.00	1.09	1.12	1.04	0.96
time (sec)	N/A	0.125	0.037	0.000	0.720	0.903	0.093	0.341	0.056
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	102	112	117	109	102
normalized size	1	1.00	1.01	1.00	0.99	1.09	1.14	1.06	0.99
time (sec)	N/A	0.097	0.029	0.000	0.653	0.803	0.085	0.304	0.043

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	76	75	76	70
normalized size	1	1.00	1.00	0.96	0.95	1.04	1.03	1.04	0.96
time (sec)	N/A	0.062	0.022	0.000	0.747	0.826	0.077	0.332	0.035

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	40	39	43	38
normalized size	1	1.00	1.00	0.88	0.86	0.95	0.93	1.02	0.90
time (sec)	N/A	0.028	0.009	0.001	0.552	0.907	0.066	0.325	0.043

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	313	169	465	175	173	165
normalized size	1	1.00	0.94	1.66	0.90	2.47	0.93	0.92	0.88
time (sec)	N/A	0.211	0.155	0.007	1.523	0.896	0.901	0.372	0.269

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	345	204	697	206	199	187
normalized size	1	1.00	0.93	1.62	0.96	3.27	0.97	0.93	0.88
time (sec)	N/A	0.226	0.192	0.008	1.600	0.972	1.676	0.380	1.801

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	209	362	240	941	246	224	221
normalized size	1	1.00	0.86	1.50	0.99	3.89	1.02	0.93	0.91
time (sec)	N/A	0.262	0.267	0.010	1.694	0.556	5.234	0.401	0.288

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	260	0	430	620	131	3586
normalized size	1	1.00	0.95	1.97	0.00	3.26	4.70	0.99	27.17
time (sec)	N/A	0.218	0.068	0.006	0.000	1.839	55.468	0.999	2.404

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	305	434	95	2624
normalized size	1	1.00	0.96	1.80	0.00	3.14	4.47	0.98	27.05
time (sec)	N/A	0.120	0.070	0.003	0.000	1.360	17.489	1.065	2.950

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	216	287	70	1632
normalized size	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	22.67
time (sec)	N/A	0.073	0.051	0.004	0.000	1.095	6.548	1.206	2.627

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	240	0	76	4149
normalized size	1	1.00	1.03	1.36	0.00	3.08	0.00	0.97	53.19
time (sec)	N/A	0.128	0.035	0.006	0.000	1.427	0.000	1.046	6.765

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	130	191	0	385	0	128	7282
normalized size	1	1.00	1.16	1.71	0.00	3.44	0.00	1.14	65.02
time (sec)	N/A	0.197	0.050	0.008	0.000	2.321	0.000	1.071	9.569

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	0	0	0	13112
normalized size	1	1.00	0.12	0.10	0.00	0.00	0.00	0.00	18.14
time (sec)	N/A	1.813	0.048	0.013	0.000	0.000	0.000	0.000	42.007

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	0	0	0	11453
normalized size	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	15.95
time (sec)	N/A	1.457	0.050	0.004	0.000	0.000	0.000	0.000	30.152

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	13607	0	0	7457
normalized size	1	1.00	0.09	0.08	0.00	21.46	0.00	0.00	11.76
time (sec)	N/A	0.728	0.030	0.005	0.000	93.486	0.000	0.000	24.559

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	14094	0	0	7469
normalized size	1	1.00	0.10	0.07	0.00	22.23	0.00	0.00	11.78
time (sec)	N/A	0.654	0.030	0.007	0.000	27.687	0.000	0.000	18.962

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	85	70	0	0	0	0	11174
normalized size	1	1.00	0.13	0.11	0.00	0.00	0.00	0.00	17.11
time (sec)	N/A	1.175	0.047	0.013	0.000	0.000	0.000	0.000	38.020

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	0	0	0	13466
normalized size	1	1.00	0.14	0.10	0.00	0.00	0.00	0.00	20.56
time (sec)	N/A	1.110	0.047	0.010	0.000	0.000	0.000	0.000	37.903

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.058	0.016	0.003	0.988	0.719	0.137	0.418	0.056

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	26
normalized size	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84
time (sec)	N/A	0.035	0.008	0.003	0.960	0.499	0.121	0.577	0.038

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.040	0.009	0.003	0.951	0.615	0.135	0.568	0.046

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	35	36
normalized size	1	1.00	1.07	0.85	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.055	0.013	0.006	0.973	0.861	0.149	0.592	1.859

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	26
normalized size	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84
time (sec)	N/A	0.045	0.013	0.006	0.960	0.804	0.144	0.449	0.045

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	1036	31	642	332
normalized size	1	1.00	0.11	0.11	0.00	2.48	0.07	1.54	0.79
time (sec)	N/A	0.538	0.011	0.007	0.000	1.018	0.182	0.576	0.652

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	1588	32	817	309
normalized size	1	1.00	0.13	0.12	0.00	4.16	0.08	2.14	0.81
time (sec)	N/A	0.328	0.013	0.004	0.000	1.143	0.185	0.692	2.279

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	1030	24	632	330
normalized size	1	1.00	0.12	0.11	0.00	2.72	0.06	1.67	0.87
time (sec)	N/A	0.259	0.013	0.006	0.000	0.992	0.180	0.626	2.376

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	1583	22	821	281
normalized size	1	1.00	0.13	0.11	0.00	3.85	0.05	2.00	0.68
time (sec)	N/A	0.276	0.013	0.005	0.000	1.220	0.181	0.582	2.264

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	1031	26	637	319
normalized size	1	1.00	0.14	0.11	0.00	2.51	0.06	1.55	0.78
time (sec)	N/A	0.277	0.012	0.004	0.000	1.121	0.179	0.722	2.299

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	47	46	0	1598	31	829	313
normalized size	1	1.00	0.11	0.11	0.00	3.84	0.07	1.99	0.75
time (sec)	N/A	0.275	0.014	0.007	0.000	1.317	0.195	0.708	0.398

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	1062	32	642	332
normalized size	1	1.00	0.11	0.11	0.00	2.54	0.08	1.54	0.79
time (sec)	N/A	0.359	0.012	0.007	0.000	1.167	0.200	0.642	2.399

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	32	32	37	32	34
normalized size	1	1.00	1.03	0.92	0.89	0.89	1.03	0.89	0.94
time (sec)	N/A	0.039	0.010	0.003	0.977	0.962	0.134	0.462	1.845

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	38	34	41	35	36
normalized size	1	1.00	1.41	0.90	0.97	0.87	1.05	0.90	0.92
time (sec)	N/A	0.056	0.014	0.006	0.989	1.050	0.150	0.540	1.847

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	0	34	41	35	36
normalized size	1	1.00	1.41	0.90	0.00	0.87	1.05	0.90	0.92
time (sec)	N/A	0.063	0.010	0.006	0.000	1.050	0.144	0.410	0.039

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	103	1070	0	0	400	0	-1
normalized size	1	1.00	0.26	2.70	0.00	0.00	1.01	0.00	-0.00
time (sec)	N/A	0.416	0.176	0.255	0.000	1.079	9.014	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	101	1010	0	0	257	0	-1
normalized size	1	1.00	0.28	2.84	0.00	0.00	0.72	0.00	-0.00
time (sec)	N/A	0.311	0.150	0.043	0.000	1.193	5.918	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	98	956	0	0	124	0	-1
normalized size	1	1.00	0.31	3.03	0.00	0.00	0.39	0.00	-0.00
time (sec)	N/A	0.246	0.128	0.036	0.000	1.010	3.449	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	98	907	0	0	119	0	-1
normalized size	1	1.00	0.35	3.26	0.00	0.00	0.43	0.00	-0.00
time (sec)	N/A	0.182	0.090	0.033	0.000	1.042	2.904	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	102	934	0	0	119	0	-1
normalized size	1	1.00	0.35	3.23	0.00	0.00	0.41	0.00	-0.00
time (sec)	N/A	0.189	0.105	0.046	0.000	0.999	15.073	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	129	1005	0	0	119	0	-1
normalized size	1	1.00	0.42	3.25	0.00	0.00	0.39	0.00	-0.00
time (sec)	N/A	0.211	0.138	0.051	0.000	1.026	86.879	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	166	1095	0	0	0	0	-1
normalized size	1	1.00	0.48	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.188	0.054	0.000	1.021	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	200	1182	0	0	0	0	-1
normalized size	1	1.00	0.51	3.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	0.238	0.056	0.000	0.805	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	0	0	0	50213
normalized size	1	1.00	0.20	0.15	0.00	0.00	0.00	0.00	115.97
time (sec)	N/A	1.132	0.075	0.006	0.000	0.000	0.000	0.000	9.632

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	216	287	70	3704
normalized size	1	1.00	0.99	1.38	0.00	3.00	3.99	0.97	51.44
time (sec)	N/A	0.072	0.057	0.004	0.000	1.015	18.295	20.738	4.206

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	13521	0	0	29445
normalized size	1	1.00	0.16	0.14	0.00	36.06	0.00	0.00	78.52
time (sec)	N/A	0.456	0.046	0.003	0.000	48.370	0.000	0.000	9.569

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	179	340	0	1535	0	1406	4501
normalized size	1	1.00	0.97	1.85	0.00	8.34	0.00	7.64	24.46
time (sec)	N/A	0.213	0.152	0.020	0.000	1.470	0.000	20.309	7.053

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	13304	0	0	36707
normalized size	1	1.00	0.16	0.13	0.00	35.48	0.00	0.00	97.89
time (sec)	N/A	0.351	0.047	0.002	0.000	9.666	0.000	0.000	8.746

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	240	0	78	8454
normalized size	1	1.00	1.03	1.36	0.00	3.08	0.00	1.00	108.38
time (sec)	N/A	0.126	0.033	0.007	0.000	2.397	0.000	20.628	5.271

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	85	72	0	0	0	0	39028
normalized size	1	1.00	0.22	0.18	0.00	0.00	0.00	0.00	99.56
time (sec)	N/A	0.683	0.064	0.007	0.000	0.000	0.000	0.000	9.459

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	89	365	0	2772	0	3006	15013
normalized size	1	1.00	0.45	1.83	0.00	13.93	0.00	15.11	75.44
time (sec)	N/A	0.311	0.045	0.023	0.000	2.630	0.000	22.519	7.620

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	0	0	0	65350
normalized size	1	1.00	0.22	0.17	0.00	0.00	0.00	0.00	165.86
time (sec)	N/A	0.625	0.071	0.007	0.000	0.000	0.000	0.000	10.224

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	218	170	208	56
normalized size	1	1.00	0.17	0.12	0.00	0.78	0.61	0.75	0.20
time (sec)	N/A	0.300	0.016	0.007	0.000	0.963	0.226	0.452	1.920

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
normalized size	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.042	0.014	0.006	1.507	0.887	0.146	0.628	0.049

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	715	27	253	248
normalized size	1	1.00	0.15	0.13	0.00	2.01	0.08	0.71	0.70
time (sec)	N/A	0.289	0.016	0.008	0.000	1.195	3.164	0.481	1.985

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	41	42	31	20
normalized size	1	1.00	0.88	0.78	0.00	0.82	0.84	0.62	0.40
time (sec)	N/A	0.040	0.016	0.017	0.000	1.055	0.129	0.440	1.889

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	715	26	253	208
normalized size	1	1.00	0.16	0.12	0.00	2.01	0.07	0.71	0.59
time (sec)	N/A	0.216	0.014	0.000	0.000	0.923	3.226	0.536	0.002

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	38	36
normalized size	1	1.00	1.07	0.85	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.053	0.013	0.007	0.965	0.792	0.160	0.453	1.886

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	224	168	210	58
normalized size	1	1.00	0.17	0.14	0.00	0.80	0.60	0.75	0.21
time (sec)	N/A	0.208	0.015	0.010	0.000	0.971	0.234	0.582	1.860

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	70	0	188	76	81	56
normalized size	1	1.00	0.55	0.79	0.00	2.11	0.85	0.91	0.63
time (sec)	N/A	0.090	0.016	0.011	0.000	0.841	0.232	0.528	0.099

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	47	46	0	608	32	258	479
normalized size	1	1.00	0.13	0.12	0.00	1.64	0.09	0.70	1.29
time (sec)	N/A	0.269	0.015	0.009	0.000	1.130	3.175	0.444	0.068

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	283	662	0	1027	0	295	2490
normalized size	1	1.00	1.01	2.36	0.00	3.67	0.00	1.05	8.89
time (sec)	N/A	0.597	0.229	0.014	0.000	80.215	0.000	0.376	6.206

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	218	512	0	798	0	224	2051
normalized size	1	1.00	1.00	2.35	0.00	3.66	0.00	1.03	9.41
time (sec)	N/A	0.395	0.172	0.010	0.000	35.524	0.000	0.368	5.242

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	388	0	596	0	185	1367
normalized size	1	1.00	1.01	2.20	0.00	3.39	0.00	1.05	7.77
time (sec)	N/A	0.285	0.185	0.007	0.000	10.870	0.000	0.401	4.339

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	275	0	405	0	149	966
normalized size	1	1.00	0.89	1.85	0.00	2.72	0.00	1.00	6.48
time (sec)	N/A	0.210	0.119	0.006	0.000	3.568	0.000	0.366	3.668

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	169	0	305	0	127	801
normalized size	1	1.00	0.86	1.36	0.00	2.46	0.00	1.02	6.46
time (sec)	N/A	0.145	0.074	0.006	0.000	1.490	0.000	0.392	3.407

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	168	0	305	0	126	521
normalized size	1	1.00	0.85	1.37	0.00	2.48	0.00	1.02	4.24
time (sec)	N/A	0.107	0.073	0.006	0.000	1.453	0.000	0.341	3.817

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	152	285	0	0	0	164	2399
normalized size	1	1.01	0.96	1.80	0.00	0.00	0.00	1.04	15.18
time (sec)	N/A	0.271	0.186	0.007	0.000	0.000	0.000	0.351	5.400

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	194	412	0	0	0	210	2388
normalized size	1	1.00	1.01	2.13	0.00	0.00	0.00	1.09	12.37
time (sec)	N/A	0.343	0.174	0.011	0.000	0.000	0.000	0.342	20.389

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	252	562	0	0	0	279	3530
normalized size	1	1.00	1.00	2.23	0.00	0.00	0.00	1.11	14.01
time (sec)	N/A	0.428	0.220	0.013	0.000	0.000	0.000	0.348	26.162

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	338	943	0	0	0	565	3503
normalized size	1	1.00	0.99	2.75	0.00	0.00	0.00	1.65	10.21
time (sec)	N/A	0.907	0.356	0.014	0.000	0.000	0.000	0.420	8.039

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	269	765	0	2139	0	476	2495
normalized size	1	1.00	0.98	2.79	0.00	7.81	0.00	1.74	9.11
time (sec)	N/A	0.563	0.287	0.011	0.000	103.931	0.000	0.401	6.003

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	207	580	0	1465	0	412	2037
normalized size	1	1.00	0.84	2.36	0.00	5.96	0.00	1.67	8.28
time (sec)	N/A	0.395	0.227	0.008	0.000	36.698	0.000	0.422	5.112

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	159	389	0	1120	0	331	1585
normalized size	1	1.00	0.82	2.01	0.00	5.77	0.00	1.71	8.17
time (sec)	N/A	0.306	0.225	0.010	0.000	13.249	0.000	0.347	6.091

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	328	0	1059	0	323	1768
normalized size	1	1.00	0.81	1.79	0.00	5.79	0.00	1.77	9.66
time (sec)	N/A	0.236	0.244	0.008	0.000	14.162	0.000	0.366	8.070

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	386	0	1079	0	331	1782
normalized size	1	1.00	0.80	2.04	0.00	5.71	0.00	1.75	9.43
time (sec)	N/A	0.305	0.211	0.009	0.000	7.015	0.000	0.354	8.109

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	249	246	589	0	0	0	391	3510
normalized size	1	1.00	0.99	2.38	0.00	0.00	0.00	1.58	14.15
time (sec)	N/A	0.409	0.253	0.013	0.000	0.000	0.000	0.408	25.284

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	287	791	0	0	0	487	4948
normalized size	1	1.00	0.99	2.72	0.00	0.00	0.00	1.67	17.00
time (sec)	N/A	0.563	0.340	0.014	0.000	0.000	0.000	0.361	31.159

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	993	0	0	0	587	7144
normalized size	1	1.00	0.99	2.67	0.00	0.00	0.00	1.58	19.20
time (sec)	N/A	0.851	0.425	0.016	0.000	0.000	0.000	0.488	45.611

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	11938	0	0	0	0	-1
normalized size	1	1.00	11.12	12.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.172	14.199	0.161	0.000	1.256	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	9182	0	0	0	0	-1
normalized size	1	1.00	9.68	11.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.349	13.735	0.066	0.000	0.864	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	5350	6302	0	0	0	0	-1
normalized size	1	1.00	8.41	9.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.993	13.051	0.052	0.000	1.305	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	693	4361	0	0	0	0	-1
normalized size	1	1.00	1.26	7.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	11.771	0.047	0.000	1.194	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	955	955	1258	3023	0	0	0	0	-1
normalized size	1	1.00	1.32	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.313	10.527	0.047	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	929	929	1372	3553	0	0	0	0	-1
normalized size	1	1.00	1.48	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.725	11.413	0.049	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1287	1287	811	4957	0	0	0	0	-1
normalized size	1	1.00	0.63	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.302	12.532	0.057	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.019	0.195	0.432	0.000	0.912	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	-1
normalized size	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.310	0.089	0.000	0.946	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.159	0.093	0.000	0.806	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.077	0.087	0.000	0.850	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.110	0.109	0.000	0.627	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	0.168	0.112	0.000	0.648	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.449	0.706	0.111	0.000	0.481	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	201	46548	14	1446	1326	216	1203
normalized size	1	1.00	12.56	2909.25	0.88	90.38	82.88	13.50	75.19
time (sec)	N/A	0.060	0.176	0.003	0.430	0.463	0.350	0.434	3.341

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1240	1454	1384	246	1210
normalized size	1	1.00	12.94	2586.22	68.89	80.78	76.89	13.67	67.22
time (sec)	N/A	0.329	0.177	0.003	0.490	0.511	0.338	0.661	3.234

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1240	1454	1394	246	1210
normalized size	1	1.00	12.94	2586.22	68.89	80.78	77.44	13.67	67.22
time (sec)	N/A	0.302	0.182	0.003	0.546	0.643	0.342	0.609	3.183

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	2042	2041	1297	0	1693	1395
normalized size	1	1.00	0.96	88.78	88.74	56.39	0.00	73.61	60.65
time (sec)	N/A	0.056	0.068	0.062	0.857	0.736	0.000	1.002	5.778

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	201	47685	16	1450	1326	218	1208
normalized size	1	1.00	11.17	2649.17	0.89	80.56	73.67	12.11	67.11
time (sec)	N/A	0.069	0.175	0.004	0.444	0.502	0.360	0.473	1.376

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1242	1454	1384	246	1214
normalized size	1	1.00	11.65	2384.40	62.10	72.70	69.20	12.30	60.70
time (sec)	N/A	0.322	0.169	0.002	0.525	0.496	0.359	0.572	3.251

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1242	1454	1394	246	1214
normalized size	1	1.00	11.65	2384.40	62.10	72.70	69.70	12.30	60.70
time (sec)	N/A	0.310	0.167	0.001	0.505	0.543	0.360	0.683	1.282

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	2046	2045	1299	0	1693	1401
normalized size	1	1.00	0.96	81.84	81.80	51.96	0.00	67.72	56.04
time (sec)	N/A	0.060	0.054	0.059	0.827	0.763	0.000	1.089	5.776

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	13	154	175	13	154
normalized size	1	1.00	11.47	10.33	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.014	0.005	0.001	0.435	0.478	0.129	0.402	2.088

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	15	156
normalized size	1	1.00	11.38	9.81	9.75	9.75	11.38	0.94	9.75
time (sec)	N/A	0.054	0.006	0.001	0.433	0.770	0.127	0.383	2.080

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	15	156
normalized size	1	1.00	11.62	9.81	9.75	9.75	11.56	0.94	9.75
time (sec)	N/A	0.056	0.006	0.001	0.439	0.701	0.127	0.508	2.079

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
normalized size	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.033	0.117	0.035	0.478	0.683	0.000	0.433	2.628

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	11	11	10	12	11
normalized size	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00
time (sec)	N/A	0.004	0.003	0.002	0.446	0.752	0.155	0.347	1.962

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.019	0.006	0.002	0.428	0.714	0.280	1.734	1.956

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
normalized size	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.024	0.006	0.001	0.439	0.632	0.413	1.092	0.052

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	23	19	0	19	121
normalized size	1	1.00	1.00	1.26	1.21	1.00	0.00	1.00	6.37
time (sec)	N/A	0.027	0.105	0.025	0.598	0.704	0.000	0.464	2.317

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	350	359	14	358
normalized size	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38
time (sec)	N/A	0.005	0.012	0.000	0.438	0.644	4.791	0.395	3.616

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
normalized size	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.020	0.012	0.000	0.948	0.734	7.662	6.776	12.162

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
normalized size	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.023	0.013	0.002	0.954	0.647	11.757	22.371	18.211

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	416	394	0	21	496
normalized size	1	1.00	0.96	0.96	18.09	17.13	0.00	0.91	21.57
time (sec)	N/A	0.027	0.060	0.064	2.393	0.748	0.000	0.633	23.011

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	13	13	10	14	13
normalized size	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00
time (sec)	N/A	0.005	0.005	0.001	0.435	0.495	0.151	0.390	0.049

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.019	0.007	0.000	0.437	0.555	0.281	1.624	0.049

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
normalized size	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.024	0.007	0.001	0.431	0.615	0.392	1.040	0.059

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	25	21	0	21	199
normalized size	1	1.00	1.00	1.24	1.19	1.00	0.00	1.00	9.48
time (sec)	N/A	0.029	0.117	0.025	0.599	0.676	0.000	0.374	2.676

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	16	354	359	16	358
normalized size	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89
time (sec)	N/A	0.004	0.013	0.002	0.428	0.674	5.060	0.427	5.221

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
normalized size	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.020	0.017	0.000	0.967	0.763	8.089	7.187	11.044

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
normalized size	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.024	0.017	0.001	0.938	0.535	11.792	22.351	16.597

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	419	397	0	23	496
normalized size	1	1.00	0.92	0.96	16.76	15.88	0.00	0.92	19.84
time (sec)	N/A	0.029	0.066	0.067	2.423	0.705	0.000	0.790	22.399

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
normalized size	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.004	0.004	0.002	0.440	0.807	0.122	0.456	0.051

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
normalized size	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.024	0.006	0.006	0.426	0.839	0.191	0.486	0.064

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
normalized size	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.030	0.007	0.006	0.427	0.841	0.202	0.340	1.991

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	47	17	48	17	28
normalized size	1	1.00	1.00	1.20	3.13	1.13	3.20	1.13	1.87
time (sec)	N/A	0.035	0.012	0.022	0.442	0.873	31.234	0.374	2.225

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
normalized size	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.004	0.020	0.016	0.422	0.819	0.879	0.318	4.296

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.021	0.029	0.019	0.520	0.883	1.384	0.450	2.332

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.025	0.037	0.013	0.514	0.811	1.897	0.606	5.065

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	20	107
normalized size	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10
time (sec)	N/A	0.032	0.181	0.052	0.663	0.741	0.000	0.440	2.357

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	28	104	20	39
normalized size	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95
time (sec)	N/A	0.005	0.009	0.003	0.425	0.968	57.114	0.414	2.037

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
normalized size	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.019	0.011	0.004	0.595	0.757	0.000	0.460	2.092

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
normalized size	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.024	0.012	0.006	0.578	0.786	0.000	0.319	2.115

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	39	38	0	27	56
normalized size	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07
time (sec)	N/A	0.028	0.029	0.059	0.722	0.878	0.000	0.845	2.569

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	104	22	42
normalized size	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91
time (sec)	N/A	0.005	0.011	0.001	0.422	0.860	56.660	0.397	2.049

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
normalized size	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.020	0.015	0.003	0.597	0.907	0.000	0.454	2.047

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
normalized size	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.025	0.016	0.006	0.588	0.597	0.000	0.385	2.079

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	43	42	0	29	59
normalized size	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03
time (sec)	N/A	0.028	0.034	0.060	0.708	0.928	0.000	0.820	2.536

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	19	26	46	19	23
normalized size	1	1.00	0.89	1.26	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.004	0.010	0.005	0.428	0.811	0.662	0.410	2.034

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	85	22	31
normalized size	1	1.00	4.04	1.29	1.46	1.29	3.54	0.92	1.29
time (sec)	N/A	0.014	0.073	0.003	0.588	0.763	17.117	0.400	2.074

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	35	31	0	22	31
normalized size	1	1.00	4.04	1.29	1.46	1.29	0.00	0.92	1.29
time (sec)	N/A	0.020	0.074	0.003	0.590	0.731	0.000	0.671	2.069

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	111	155	40	36	0	26	34
normalized size	1	1.00	4.27	5.96	1.54	1.38	0.00	1.00	1.31
time (sec)	N/A	0.079	0.130	0.106	0.766	0.644	0.000	0.834	2.126

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	158	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.305	0.034	0.000	0.759	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	5363	0	0	0	0	0	-1
normalized size	1	1.00	14.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.380	6.519	0.041	0.000	0.597	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	-1
normalized size	1	1.00	25.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.552	7.537	0.061	0.000	1.011	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	34	33	126	0	31
normalized size	1	1.00	1.00	0.77	0.72	0.70	2.68	0.00	0.66
time (sec)	N/A	0.062	0.023	0.002	0.445	0.885	6.321	0.000	2.463

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	0.161	0.092	0.000	0.978	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.161	0.092	0.000	0.653	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.132	0.090	0.000	0.618	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.083	0.089	0.000	0.654	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	218	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	0.675	0.095	0.000	0.689	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.489	0.132	0.091	0.000	0.634	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.125	0.089	0.000	0.715	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	1.136	0.069	0.000	0.733	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.371	0.600	0.052	0.000	0.665	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.302	0.050	0.000	0.556	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.190	0.088	0.000	0.821	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.025	0.269	0.092	0.000	0.850	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	22	0.045
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	22	0.045
5	A	2	1	1.00	20	0.050
6	A	8	8	1.00	22	0.364
7	A	8	8	1.00	22	0.364
8	A	8	8	1.00	22	0.364
9	A	7	6	1.00	25	0.240
10	A	6	6	1.00	25	0.240
11	A	5	5	1.00	25	0.200
12	A	7	6	1.00	25	0.240
13	A	7	6	1.00	25	0.240
14	A	14	8	1.00	25	0.320
15	A	14	8	1.00	25	0.320
16	A	13	7	1.00	23	0.304
17	A	13	7	1.00	22	0.318
18	A	14	8	1.00	25	0.320
19	A	14	8	1.00	25	0.320
20	A	7	6	1.00	23	0.261
21	A	4	4	1.00	23	0.174
22	A	5	5	1.00	23	0.217
23	A	7	6	1.00	23	0.261
24	A	5	4	1.00	23	0.174
25	A	15	9	1.00	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	9	1.00	23	0.391
27	A	14	8	1.00	23	0.348
28	A	13	7	1.00	21	0.333
29	A	13	7	1.00	20	0.350
30	A	14	8	1.00	23	0.348
31	A	15	9	1.00	23	0.391
32	A	5	5	1.00	21	0.238
33	A	7	6	1.00	21	0.286
34	A	8	7	1.00	18	0.389
35	A	6	4	1.00	24	0.167
36	A	5	4	1.00	24	0.167
37	A	4	4	1.00	24	0.167
38	A	3	3	1.00	24	0.125
39	A	3	3	1.00	24	0.125
40	A	3	3	1.00	24	0.125
41	A	4	4	1.00	24	0.167
42	A	5	4	1.00	24	0.167
43	A	8	5	1.00	25	0.200
44	A	5	5	1.00	25	0.200
45	A	7	4	1.00	25	0.160
46	A	4	3	1.00	23	0.130
47	A	7	4	1.00	22	0.182
48	A	7	6	1.00	25	0.240
49	A	8	5	1.00	25	0.200
50	A	5	4	1.00	25	0.160
51	A	8	5	1.00	25	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	20	7	1.00	23	0.304
53	A	5	5	1.00	23	0.217
54	A	21	7	1.00	23	0.304
55	A	4	3	1.00	21	0.143
56	A	19	6	1.00	20	0.300
57	A	7	6	1.00	23	0.261
58	A	20	7	1.00	23	0.304
59	A	11	8	1.00	23	0.348
60	A	21	9	1.00	23	0.391
61	A	7	6	1.00	25	0.240
62	A	7	6	1.00	25	0.240
63	A	7	6	1.00	23	0.261
64	A	7	6	1.00	22	0.273
65	A	7	6	1.00	25	0.240
66	A	7	7	1.00	25	0.280
67	A	7	6	1.01	25	0.240
68	A	7	6	1.00	25	0.240
69	A	7	6	1.00	25	0.240
70	A	7	6	1.00	25	0.240
71	A	7	6	1.00	25	0.240
72	A	7	6	1.00	23	0.261
73	A	7	6	1.00	22	0.273
74	A	7	6	1.00	25	0.240
75	A	8	7	1.00	25	0.280
76	A	7	6	1.00	25	0.240
77	A	7	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
78	A	7	6	1.00	25	0.240
79	A	11	7	1.00	29	0.241
80	A	10	7	1.00	29	0.241
81	A	8	7	1.00	29	0.241
82	A	8	7	1.00	27	0.259
83	A	16	11	1.00	26	0.423
84	A	16	11	1.00	29	0.379
85	A	24	12	1.00	29	0.414
86	A	0	0	0.00	0	0.000
87	A	13	4	1.00	26	0.154
88	A	10	4	1.00	26	0.154
89	A	7	4	1.00	24	0.167
90	A	6	3	1.00	26	0.115
91	A	8	3	1.00	26	0.115
92	A	10	3	1.00	26	0.115
93	A	1	1	1.00	19	0.053
94	A	2	2	1.00	24	0.083
95	A	2	2	1.00	26	0.077
96	A	2	2	1.00	30	0.067
97	A	1	1	1.00	21	0.048
98	A	2	2	1.00	26	0.077
99	A	2	2	1.00	28	0.071
100	A	2	2	1.00	32	0.062
101	A	1	1	1.00	18	0.056
102	A	3	3	1.00	23	0.130
103	A	3	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
104	A	3	3	1.00	29	0.103
105	A	1	1	1.00	19	0.053
106	A	2	2	1.00	24	0.083
107	A	2	2	1.00	26	0.077
108	A	2	2	1.00	30	0.067
109	A	1	1	1.00	19	0.053
110	A	2	2	1.00	24	0.083
111	A	2	2	1.00	26	0.077
112	A	2	2	1.00	30	0.067
113	A	1	1	1.00	21	0.048
114	A	2	2	1.00	26	0.077
115	A	2	2	1.00	28	0.071
116	A	2	2	1.00	32	0.062
117	A	1	1	1.00	21	0.048
118	A	2	2	1.00	26	0.077
119	A	2	2	1.00	28	0.071
120	A	2	2	1.00	32	0.062
121	A	1	1	1.00	18	0.056
122	A	4	3	0.94	23	0.130
123	A	4	3	0.94	25	0.120
124	A	4	3	1.00	29	0.103
125	A	1	1	1.00	18	0.056
126	A	3	3	1.00	23	0.130
127	A	3	3	1.00	25	0.120
128	A	3	3	1.00	29	0.103
129	A	1	1	1.00	19	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	2	2	1.00	24	0.083
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	30	0.067
133	A	1	1	1.00	21	0.048
134	A	2	2	1.00	26	0.077
135	A	2	2	1.00	28	0.071
136	A	2	2	1.00	32	0.062
137	A	1	1	1.00	18	0.056
138	A	1	1	1.00	23	0.043
139	A	1	1	1.00	25	0.040
140	A	2	2	1.00	29	0.069
141	A	4	2	1.00	29	0.069
142	A	5	3	1.00	29	0.103
143	A	6	3	1.00	29	0.103
144	A	3	3	1.00	59	0.051
145	A	5	3	1.00	31	0.097
146	A	5	3	1.00	29	0.103
147	A	5	3	1.00	27	0.111
148	A	5	3	1.00	26	0.115
149	A	8	5	1.00	29	0.172
150	A	5	3	1.00	29	0.103
151	A	5	3	1.00	29	0.103
152	A	10	4	1.00	31	0.129
153	A	7	4	1.00	29	0.138
154	A	2	2	1.00	22	0.091
155	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (d + ex^3)^5 (a + bx^3 + cx^6) dx$

Optimal. Leaf size=163

$$\frac{1}{16}e^3x^{16} (e(ae + 5bd) + 10cd^2) + \frac{5}{13}de^2x^{13} (e(ae + 2bd) + 2cd^2) + \frac{1}{2}d^2ex^{10} (2e(ae + bd) + cd^2) + \frac{1}{7}d^3x^7 (5e(2ae + b$$

```
[Out] a*d^5*x+1/4*d^4*(5*a*e+b*d)*x^4+1/7*d^3*(c*d^2+5*e*(2*a*e+b*d))*x^7+1/2*d^2
*e*(c*d^2+2*e*(a*e+b*d))*x^10+5/13*d*e^2*(2*c*d^2+e*(a*e+2*b*d))*x^13+1/16*
e^3*(10*c*d^2+e*(a*e+5*b*d))*x^16+1/19*e^4*(b*e+5*c*d)*x^19+1/22*c*e^5*x^22
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{16}e^3x^{16} (e(ae + 5bd) + 10cd^2) + \frac{5}{13}de^2x^{13} (e(ae + 2bd) + 2cd^2) + \frac{1}{2}d^2ex^{10} (2e(ae + bd) + cd^2) + \frac{1}{7}d^3x^7 (5e(2ae + b$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1407

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
```

, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]

Rubi steps

$$\int (d + ex^3)^5 (a + bx^3 + cx^6) dx = \int (ad^5 + d^4(bd + 5ae)x^3 + d^3(cd^2 + 5e(bd + 2ae))x^6 + 5d^2e(cd^2 + 2e(bd + a))x^9 + d(e^2d^2 + 2e^2(bd + 2ae))x^{12} + e^3d^2x^{15}) dx$$

$$= ad^5x + \frac{1}{4}d^4(bd + 5ae)x^4 + \frac{1}{7}d^3(cd^2 + 5e(bd + 2ae))x^7 + \frac{1}{2}d^2e(cd^2 + 2e(bd + a))x^{10} + \frac{1}{13}e^2d^2x^{13} + \frac{1}{16}e^3d^2x^{16}$$

Mathematica [A] time = 0.05, size = 164, normalized size = 1.01

$$\frac{5}{13}de^2x^{13}(ae^2 + 2bde + 2cd^2) + \frac{1}{2}d^2ex^{10}(2ae^2 + 2bde + cd^2) + \frac{1}{16}e^3x^{16}(ae^2 + 5bde + 10cd^2) + \frac{1}{7}d^3x^7(10ae^2 + 5bde + 2cd^2)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6), x]

[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22

fricas [A] time = 0.69, size = 182, normalized size = 1.12

$$\frac{1}{22}x^{22}e^5c + \frac{5}{19}x^{19}e^4dc + \frac{1}{19}x^{19}e^5b + \frac{5}{8}x^{16}e^3d^2c + \frac{5}{16}x^{16}e^4db + \frac{1}{16}x^{16}e^5a + \frac{10}{13}x^{13}e^2d^3c + \frac{10}{13}x^{13}e^3d^2b + \frac{5}{13}x^{13}e^4da + \frac{1}{2}x^{10}e^2d^3b + \frac{1}{2}x^{10}e^3d^2a + \frac{1}{7}x^7e^2d^3a + \frac{1}{7}x^7e^3d^2b + \frac{1}{7}x^7e^4da + \frac{1}{4}x^4e^2d^3a + \frac{1}{4}x^4e^3d^2b + \frac{1}{4}x^4e^4da + \frac{1}{19}x^4e^5c + \frac{1}{19}x^4e^4d^2c + \frac{1}{19}x^4e^5b^2 + \frac{1}{19}x^4e^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] 1/22*x^22*e^5*c + 5/19*x^19*e^4*d*c + 1/19*x^19*e^5*b + 5/8*x^16*e^3*d^2*c + 5/16*x^16*e^4*d*b + 1/16*x^16*e^5*a + 10/13*x^13*e^2*d^3*c + 10/13*x^13*e^3*d^2*b + 5/13*x^13*e^4*d*a + 1/2*x^10*e*d^4*c + x^10*e^2*d^3*b + x^10*e^3*d^2*a + 1/7*x^7*d^5*c + 5/7*x^7*e*d^4*b + 10/7*x^7*e^2*d^3*a + 1/4*x^4*d^5*b + 5/4*x^4*e*d^4*a + x*d^5*a

giac [A] time = 0.34, size = 173, normalized size = 1.06

$$\frac{1}{22}cx^{22}e^5 + \frac{5}{19}cdx^{19}e^4 + \frac{1}{19}bx^{19}e^5 + \frac{5}{8}cd^2x^{16}e^3 + \frac{5}{16}bdx^{16}e^4 + \frac{1}{16}ax^{16}e^5 + \frac{10}{13}cd^3x^{13}e^2 + \frac{10}{13}bd^2x^{13}e^3 + \frac{5}{13}adx^{13}e^4 + \frac{1}{2}x^{10}e^2d^3b + \frac{1}{2}x^{10}e^3d^2a + \frac{1}{7}x^7e^2d^3a + \frac{1}{7}x^7e^3d^2b + \frac{1}{7}x^7e^4da + \frac{1}{4}x^4e^2d^3a + \frac{1}{4}x^4e^3d^2b + \frac{1}{4}x^4e^4da + \frac{1}{19}x^4e^5c + \frac{1}{19}x^4e^4d^2c + \frac{1}{19}x^4e^5b^2 + \frac{1}{19}x^4e^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] $\frac{1}{22}c*x^{22}*e^5 + \frac{5}{19}c*d*x^{19}*e^4 + \frac{1}{19}b*x^{19}*e^5 + \frac{5}{8}c*d^2*x^{16}*e^3 + \frac{5}{16}b*d*x^{16}*e^4 + \frac{1}{16}a*x^{16}*e^5 + \frac{10}{13}c*d^3*x^{13}*e^2 + \frac{10}{13}b*d^2*x^{13}*e^3 + \frac{5}{13}a*d*x^{13}*e^4 + \frac{1}{2}c*d^4*x^{10}*e + b*d^3*x^{10}*e^2 + a*d^2*x^{10}*e^3 + \frac{1}{7}c*d^5*x^7 + \frac{5}{7}b*d^4*x^7*e + \frac{10}{7}a*d^3*x^7*e^2 + \frac{1}{4}b*d^5*x^4 + \frac{5}{4}a*d^4*x^4*e + a*d^5*x$

maple [A] time = 0.00, size = 169, normalized size = 1.04

$$\frac{c e^5 x^{22}}{22} + \frac{(e^5 b + 5 d e^4 c) x^{19}}{19} + \frac{(e^5 a + 5 d e^4 b + 10 d^2 e^3 c) x^{16}}{16} + \frac{(5 d e^4 a + 10 d^2 e^3 b + 10 d^3 e^2 c) x^{13}}{13} + \frac{(10 d^2 e^3 a + 10 d^3 e^2 b + 10 d^4 e c) x^{10}}{10} + \frac{1}{2} c d^4 x^7 + \frac{5}{7} b d^4 x^7 e + \frac{10}{7} a d^3 x^7 e^2 + \frac{1}{4} b d^5 x^4 + \frac{5}{4} a d^4 x^4 e + a d^5 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x)

[Out] $\frac{1}{22}c*e^5*x^{22} + \frac{1}{19}(b*e^5 + 5*c*d*e^4)*x^{19} + \frac{1}{16}(a*e^5 + 5*b*d*e^4 + 10*c*d^2*e^3)*x^{16} + \frac{1}{13}(5*a*d*e^4 + 10*b*d^2*e^3 + 10*c*d^3*e^2)*x^{13} + \frac{1}{10}(10*a*d^2*e^3 + 10*b*d^3*e^2 + 5*c*d^4*e)*x^{10} + \frac{1}{7}(10*a*d^3*e^2 + 5*b*d^4*e + c*d^5)*x^7 + \frac{1}{4}(5*a*d^4*e + b*d^5)*x^4 + a*d^5*x$

maxima [A] time = 0.81, size = 166, normalized size = 1.02

$$\frac{1}{22} c e^5 x^{22} + \frac{1}{19} (5 c d e^4 + b e^5) x^{19} + \frac{1}{16} (10 c d^2 e^3 + 5 b d e^4 + a e^5) x^{16} + \frac{5}{13} (2 c d^3 e^2 + 2 b d^2 e^3 + a d e^4) x^{13} + \frac{1}{2} (c d^4 e + 2 b d^3 e^2 + 2 a d^2 e^3) x^{10} + \frac{1}{7} (c d^5 + 5 b d^4 e + 10 a d^3 e^2) x^7 + \frac{1}{4} (b d^5 + 5 a d^4 e) x^4 + a d^5 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{22}c*e^5*x^{22} + \frac{1}{19}(5*c*d*e^4 + b*e^5)*x^{19} + \frac{1}{16}(10*c*d^2*e^3 + 5*b*d*e^4 + a*e^5)*x^{16} + \frac{5}{13}(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^{13} + \frac{1}{2}(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^{10} + \frac{1}{7}(c*d^5 + 5*b*d^4*e + 10*a*d^3*e^2)*x^7 + a*d^5*x + \frac{1}{4}(b*d^5 + 5*a*d^4*e)*x^4$

mupad [B] time = 1.60, size = 158, normalized size = 0.97

$$x^4 \left(\frac{b d^5}{4} + \frac{5 a e d^4}{4} \right) + x^{19} \left(\frac{b e^5}{19} + \frac{5 c d e^4}{19} \right) + x^7 \left(\frac{c d^5}{7} + \frac{5 b d^4 e}{7} + \frac{10 a d^3 e^2}{7} \right) + x^{16} \left(\frac{5 c d^2 e^3}{8} + \frac{5 b d e^4}{16} + \frac{a e^5}{16} \right) + \frac{1}{2} (c d^4 e + 2 b d^3 e^2 + 2 a d^2 e^3) x^{10} + \frac{1}{7} (c d^5 + 5 b d^4 e + 10 a d^3 e^2) x^7 + a d^5 x + \frac{1}{4} (b d^5 + 5 a d^4 e) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)

[Out] $x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^{19}*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7*((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^{16}*((a*e^5)/16 + (5*c*d^2*e^3)/16 + (5*b*d*e^4)/16 + (a*e^5)/16)$

$$2e^3)/8 + (5bd^4e)/16) + (ce^5x^{22})/22 + ad^5x + (d^2ex^{10}(2ae^2 + cd^2 + 2bde))/2 + (5d^2e^2x^{13}(ae^2 + 2cd^2 + 2bde))/13$$

sympy [A] time = 0.10, size = 187, normalized size = 1.15

$$ad^5x + \frac{ce^5x^{22}}{22} + x^{19} \left(\frac{be^5}{19} + \frac{5cde^4}{19} \right) + x^{16} \left(\frac{ae^5}{16} + \frac{5bde^4}{16} + \frac{5cd^2e^3}{8} \right) + x^{13} \left(\frac{5ade^4}{13} + \frac{10bd^2e^3}{13} + \frac{10cd^3e^2}{13} \right) + x^{10} \left(ad^2e^3 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)

[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e/4 + b*d**5/4)

3.2 $\int (d + ex^3)^4 (a + bx^3 + cx^6) dx$

Optimal. Leaf size=135

$$\frac{1}{13}e^2x^{13}(e(ae + 4bd) + 6cd^2) + \frac{1}{7}d^2x^7(6ae^2 + 4bde + cd^2) + \frac{1}{5}dex^{10}(e(2ae + 3bd) + 2cd^2) + \frac{1}{4}d^3x^4(4ae + bd) + ad^4x$$

[Out] a*d^4*x+1/4*d^3*(4*a*e+b*d)*x^4+1/7*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^7+1/5*d*e*(2*c*d^2+e*(2*a*e+3*b*d))*x^10+1/13*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^13+1/16*e^3*(b*e+4*c*d)*x^16+1/19*c*e^4*x^19

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{13}e^2x^{13}(e(ae + 4bd) + 6cd^2) + \frac{1}{7}d^2x^7(6ae^2 + 4bde + cd^2) + \frac{1}{5}dex^{10}(e(2ae + 3bd) + 2cd^2) + \frac{1}{4}d^3x^4(4ae + bd) + ad^4x$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6), x]

[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d + a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19

Rule 1407

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2*n_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (d + ex^3)^4 (a + bx^3 + cx^6) dx &= \int (ad^4 + d^3(bd + 4ae)x^3 + d^2(cd^2 + 4bde + 6ae^2)x^6 + 2de(2cd^2 + e(3bd + 4ae))x^9 + ad^4x^3 + \frac{1}{4}d^3(bd + 4ae)x^4 + \frac{1}{7}d^2(cd^2 + 4bde + 6ae^2)x^7 + \frac{1}{5}de(2cd^2 + e(3bd + 4ae))x^{10} + \frac{1}{13}e^2(6cd^2 + e(4bd + ae))x^{13} + \frac{1}{16}e^3(4cd + be)x^{16} + \frac{1}{19}ce^4x^{19}) dx \\ &= ad^4x + \frac{1}{4}d^3(bd + 4ae)x^4 + \frac{1}{7}d^2(cd^2 + 4bde + 6ae^2)x^7 + \frac{1}{5}de(2cd^2 + e(3bd + 4ae))x^{10} + \frac{1}{13}e^2(6cd^2 + e(4bd + ae))x^{13} + \frac{1}{16}e^3(4cd + be)x^{16} + \frac{1}{19}ce^4x^{19} \end{aligned}$$

Mathematica [A] time = 0.04, size = 135, normalized size = 1.00

$$\frac{1}{13}e^2x^{13}(ae^2 + 4bde + 6cd^2) + \frac{1}{5}dex^{10}(2ae^2 + 3bde + 2cd^2) + \frac{1}{7}d^2x^7(6ae^2 + 4bde + cd^2) + \frac{1}{4}d^3x^4(4ae + bd) + ad^4x$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6), x]

[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e + a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19

fricas [A] time = 0.90, size = 147, normalized size = 1.09

$$\frac{1}{19}x^{19}e^4c + \frac{1}{4}x^{16}e^3dc + \frac{1}{16}x^{16}e^4b + \frac{6}{13}x^{13}e^2d^2c + \frac{4}{13}x^{13}e^3db + \frac{1}{13}x^{13}e^4a + \frac{2}{5}x^{10}ed^3c + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}e^3da + \frac{1}{7}x^7d^4c + \frac{4}{7}x^7d^3e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] 1/19*x^19*e^4*c + 1/4*x^16*e^3*d*c + 1/16*x^16*e^4*b + 6/13*x^13*e^2*d^2*c + 4/13*x^13*e^3*d*b + 1/13*x^13*e^4*a + 2/5*x^10*e*d^3*c + 3/5*x^10*e^2*d^2*b + 2/5*x^10*e^3*d*a + 1/7*x^7*d^4*c + 4/7*x^7*e*d^3*b + 6/7*x^7*e^2*d^2*a + 1/4*x^4*d^4*b + x^4*e*d^3*a + x*d^4*a

giac [A] time = 0.34, size = 141, normalized size = 1.04

$$\frac{1}{19}cx^{19}e^4 + \frac{1}{4}cdx^{16}e^3 + \frac{1}{16}bx^{16}e^4 + \frac{6}{13}cd^2x^{13}e^2 + \frac{4}{13}bdx^{13}e^3 + \frac{1}{13}ax^{13}e^4 + \frac{2}{5}cd^3x^{10}e + \frac{3}{5}bd^2x^{10}e^2 + \frac{2}{5}adx^{10}e^3 + \frac{1}{7}cd^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2*x^13*e^2 + 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*e + 3/5*b*d^2*x^10*e^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b*d^3*x^7*e + 6/7*a*d^2*x^7*e^2 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*d^4*x

maple [A] time = 0.00, size = 136, normalized size = 1.01

$$\frac{ce^4x^{19}}{19} + \frac{(e^4b + 4de^3c)x^{16}}{16} + \frac{(e^4a + 4de^3b + 6e^2d^2c)x^{13}}{13} + \frac{(4de^3a + 6e^2d^2b + 4cd^3e)x^{10}}{10} + \frac{(6ad^2e^2 + 4bd^3e + cd^4)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a), x)

[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3+6*c*d^2*e^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/7*(6*a*d^2*e^2+4*b*d^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d^4*x

maxima [A] time = 0.72, size = 135, normalized size = 1.00

$$\frac{1}{19} ce^4 x^{19} + \frac{1}{16} (4cde^3 + be^4)x^{16} + \frac{1}{13} (6cd^2e^2 + 4bde^3 + ae^4)x^{13} + \frac{1}{5} (2cd^3e + 3bd^2e^2 + 2ade^3)x^{10} + \frac{1}{7} (cd^4 + 4bd^3e + 3ad^2e^2 + 2bde^3 + ae^4)x^7 + ad^4x + \frac{1}{4}(bd^4 + 4ad^3e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^4

mupad [B] time = 0.06, size = 130, normalized size = 0.96

$$x^4 \left(\frac{bd^4}{4} + aed^3 \right) + x^{16} \left(\frac{be^4}{16} + \frac{cde^3}{4} \right) + x^7 \left(\frac{cd^4}{7} + \frac{4bd^3e}{7} + \frac{6ad^2e^2}{7} \right) + x^{13} \left(\frac{6cd^2e^2}{13} + \frac{4bde^3}{13} + \frac{ae^4}{13} \right) + \frac{ce^4x^{19}}{19} + ad^4x + \frac{1}{4}(bd^4 + 4ad^3e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)

[Out] x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)/7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/13 + (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c*d^2 + 3*b*d*e))/5

sympy [A] time = 0.09, size = 151, normalized size = 1.12

$$ad^4x + \frac{ce^4x^{19}}{19} + x^{16} \left(\frac{be^4}{16} + \frac{cde^3}{4} \right) + x^{13} \left(\frac{ae^4}{13} + \frac{4bde^3}{13} + \frac{6cd^2e^2}{13} \right) + x^{10} \left(\frac{2ade^3}{5} + \frac{3bd^2e^2}{5} + \frac{2cd^3e}{5} \right) + x^7 \left(\frac{6ad^2e^2}{7} + \frac{4bde^3}{7} + \frac{ae^4}{7} \right) + ad^4x + \frac{1}{4}(bd^4 + 4ad^3e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)

[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e**2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x**4*(a*d**3*e + b*d**4/4)

3.3 $\int (d + ex^3)^3 (a + bx^3 + cx^6) dx$

Optimal. Leaf size=103

$$\frac{1}{10}ex^{10} (e(ae + 3bd) + 3cd^2) + \frac{1}{7}dx^7 (3e(ae + bd) + cd^2) + \frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

[Out] $a*d^3*x+1/4*d^2*(3*a*e+b*d)*x^4+1/7*d*(c*d^2+3*e*(a*e+b*d))*x^7+1/10*e*(3*c*d^2+e*(a*e+3*b*d))*x^{10}+1/13*e^2*(b*e+3*c*d)*x^{13}+1/16*c*e^3*x^{16}$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{10}ex^{10} (e(ae + 3bd) + 3cd^2) + \frac{1}{7}dx^7 (3e(ae + bd) + cd^2) + \frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6), x]

[Out] $a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 + (e*(3*c*d^2 + e*(3*b*d + a*e))*x^{10})/10 + (e^2*(3*c*d + b*e)*x^{13})/13 + (c*e^3*x^{16})/16$

Rule 1407

Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (d + ex^3)^3 (a + bx^3 + cx^6) dx &= \int (ad^3 + d^2(bd + 3ae)x^3 + d(cd^2 + 3e(bd + ae))x^6 + e(3cd^2 + e(3bd + ae))x^9) dx \\ &= ad^3x + \frac{1}{4}d^2(bd + 3ae)x^4 + \frac{1}{7}d(cd^2 + 3e(bd + ae))x^7 + \frac{1}{10}e(3cd^2 + e(3bd + ae))x^{10} \end{aligned}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 1.01

$$\frac{1}{10}ex^{10} (ae^2 + 3bde + 3cd^2) + \frac{1}{7}dx^7 (3ae^2 + 3bde + cd^2) + \frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6), x]

[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c*e^3*x^16)/16

fricas [A] time = 0.80, size = 112, normalized size = 1.09

$$\frac{1}{16}x^{16}e^3c + \frac{3}{13}x^{13}e^2dc + \frac{1}{13}x^{13}e^3b + \frac{3}{10}x^{10}ed^2c + \frac{3}{10}x^{10}e^2db + \frac{1}{10}x^{10}e^3a + \frac{1}{7}x^7d^3c + \frac{3}{7}x^7ed^2b + \frac{3}{7}x^7e^2da + \frac{1}{4}x^4d^3b + \frac{3}{4}x^4e^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] 1/16*x^16*e^3*c + 3/13*x^13*e^2*d*c + 1/13*x^13*e^3*b + 3/10*x^10*e*d^2*c + 3/10*x^10*e^2*d*b + 1/10*x^10*e^3*a + 1/7*x^7*d^3*c + 3/7*x^7*e*d^2*b + 3/7*x^7*e^2*d*a + 1/4*x^4*d^3*b + 3/4*x^4*e*d^2*a + x*d^3*a

giac [A] time = 0.30, size = 109, normalized size = 1.06

$$\frac{1}{16}cx^{16}e^3 + \frac{3}{13}cdx^{13}e^2 + \frac{1}{13}bx^{13}e^3 + \frac{3}{10}cd^2x^{10}e + \frac{3}{10}bdx^{10}e^2 + \frac{1}{10}ax^{10}e^3 + \frac{1}{7}cd^3x^7 + \frac{3}{7}bd^2x^7e + \frac{3}{7}adx^7e^2 + \frac{1}{4}bd^3x^4 + \frac{3}{4}e^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^2*x^10*e + 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 + 3/7*b*d^2*x^7*e + 3/7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4*e + a*d^3*x

maple [A] time = 0.00, size = 103, normalized size = 1.00

$$\frac{c e^3 x^{16}}{16} + \frac{(e^3 b + 3 c d e^2) x^{13}}{13} + \frac{(e^3 a + 3 b d e^2 + 3 d^2 e c) x^{10}}{10} + \frac{(3 a d e^2 + 3 b d^2 e + c d^3) x^7}{7} + a d^3 x + \frac{(3 a d^2 e + b d^3) x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a), x)

[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*x

maxima [A] time = 0.65, size = 102, normalized size = 0.99

$$\frac{1}{16} ce^3 x^{16} + \frac{1}{13} (3cde^2 + be^3)x^{13} + \frac{1}{10} (3cd^2e + 3bde^2 + ae^3)x^{10} + \frac{1}{7} (cd^3 + 3bd^2e + 3ade^2)x^7 + ad^3x + \frac{1}{4} (bd^3 + 3ad^2e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*(b*d^3 + 3*a*d^2*e)*x^4

mupad [B] time = 0.04, size = 102, normalized size = 0.99

$$x^4 \left(\frac{bd^3}{4} + \frac{3aed^2}{4} \right) + x^{13} \left(\frac{be^3}{13} + \frac{3cde^2}{13} \right) + x^7 \left(\frac{cd^3}{7} + \frac{3bd^2e}{7} + \frac{3ade^2}{7} \right) + x^{10} \left(\frac{3cd^2e}{10} + \frac{3bde^2}{10} + \frac{ae^3}{10} \right) + \frac{ce^3x^{16}}{16} + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)

[Out] x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7*((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e^2)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x

sympy [A] time = 0.09, size = 117, normalized size = 1.14

$$ad^3x + \frac{ce^3x^{16}}{16} + x^{13} \left(\frac{be^3}{13} + \frac{3cde^2}{13} \right) + x^{10} \left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10} \right) + x^7 \left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7} \right) + x^4 \left(\frac{3ad^2e}{4} + \frac{bd^3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)

[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)

3.4 $\int (d + ex^3)^2 (a + bx^3 + cx^6) dx$

Optimal. Leaf size=73

$$\frac{1}{7}x^7 (e(ae + 2bd) + cd^2) + \frac{1}{4}dx^4(2ae + bd) + ad^2x + \frac{1}{10}ex^{10}(be + 2cd) + \frac{1}{13}ce^2x^{13}$$

[Out] $a*d^2*x+1/4*d*(2*a*e+b*d)*x^4+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/10*e*(b*e+2*c*d)*x^{10}+1/13*c*e^2*x^{13}$

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1407}

$$\frac{1}{7}x^7 (e(ae + 2bd) + cd^2) + \frac{1}{4}dx^4(2ae + bd) + ad^2x + \frac{1}{10}ex^{10}(be + 2cd) + \frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^3)^2*(a + b*x^3 + c*x^6), x]$

[Out] $a*d^2*x + (d*(b*d + 2*a*e))*x^4/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*(2*c*d + b*e))*x^{10}/10 + (c*e^2*x^{13})/13$

Rule 1407

$\text{Int}[(d + e*x^3)^2*(a + b*x^3 + c*x^6), x]$ \rightarrow $\text{Int}[\text{ExpandIntegrand}[(d + e*x^3)^2*(a + b*x^3 + c*x^6), x], x]$ $;$ $\text{FreeQ}[\{a, b, c, d, e, n\}, x]$ $\&\&$ $\text{EqQ}[n^2, 2*n]$ $\&\&$ $\text{NeQ}[b^2 - 4*a*c, 0]$ $\&\&$ $\text{NeQ}[c*d^2 - b*d*e + a*e^2, 0]$ $\&\&$ $\text{IGtQ}[q, 0]$

Rubi steps

$$\begin{aligned} \int (d + ex^3)^2 (a + bx^3 + cx^6) dx &= \int (ad^2 + d(bd + 2ae)x^3 + (cd^2 + e(2bd + ae))x^6 + e(2cd + be)x^9 + ce^2x^{12}) dx \\ &= ad^2x + \frac{1}{4}d(bd + 2ae)x^4 + \frac{1}{7}(cd^2 + e(2bd + ae))x^7 + \frac{1}{10}e(2cd + be)x^{10} + \frac{1}{13}ce^2x^{13} \end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{1}{7}x^7 (ae^2 + 2bde + cd^2) + \frac{1}{4}dx^4(2ae + bd) + ad^2x + \frac{1}{10}ex^{10}(be + 2cd) + \frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]

[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13

fricas [A] time = 0.83, size = 76, normalized size = 1.04

$$\frac{1}{13}x^{13}e^2c + \frac{1}{5}x^{10}edc + \frac{1}{10}x^{10}e^2b + \frac{1}{7}x^7d^2c + \frac{2}{7}x^7edb + \frac{1}{7}x^7e^2a + \frac{1}{4}x^4d^2b + \frac{1}{2}x^4eda + xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] 1/13*x^13*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/7*x^7*d^2*c + 2/7*x^7*e*d*b + 1/7*x^7*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + x*d^2*a

giac [A] time = 0.33, size = 76, normalized size = 1.04

$$\frac{1}{13}cx^{13}e^2 + \frac{1}{5}cdx^{10}e + \frac{1}{10}bx^{10}e^2 + \frac{1}{7}cd^2x^7 + \frac{2}{7}bdx^7e + \frac{1}{7}ax^7e^2 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adx^4e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + a*d^2*x

maple [A] time = 0.00, size = 70, normalized size = 0.96

$$\frac{ce^2x^{13}}{13} + \frac{(be^2 + 2cde)x^{10}}{10} + \frac{(ae^2 + 2deb + cd^2)x^7}{7} + ad^2x + \frac{(2dea + bd^2)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x)

[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4*(2*a*d*e+b*d^2)*x^4+a*d^2*x

maxima [A] time = 0.75, size = 69, normalized size = 0.95

$$\frac{1}{13}ce^2x^{13} + \frac{1}{10}(2cde + be^2)x^{10} + \frac{1}{7}(cd^2 + 2bde + ae^2)x^7 + \frac{1}{4}(bd^2 + 2ade)x^4 + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] $\frac{1}{13}c^2e^2x^{13} + \frac{1}{10}(2cd^2e + b^2e^2)x^{10} + \frac{1}{7}(cd^2 + 2bd^2e + a^2e^2)x^7 + \frac{1}{4}(bd^2 + 2ad^2e)x^4 + ad^2x$

mupad [B] time = 0.04, size = 70, normalized size = 0.96

$$x^7 \left(\frac{cd^2}{7} + \frac{2bde}{7} + \frac{ae^2}{7} \right) + x^4 \left(\frac{bd^2}{4} + \frac{aed}{2} \right) + x^{10} \left(\frac{be^2}{10} + \frac{cde}{5} \right) + \frac{ce^2x^{13}}{13} + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^3)^2*(a + b*x^3 + c*x^6), x)`

[Out] $x^7*((a^2e^2)/7 + (cd^2)/7 + (2bd^2e)/7) + x^4*((bd^2)/4 + (ad^2e)/2) + x^{10}*((b^2e^2)/10 + (cde)/5) + (ce^2x^{13})/13 + ad^2x$

sympy [A] time = 0.08, size = 75, normalized size = 1.03

$$ad^2x + \frac{ce^2x^{13}}{13} + x^{10} \left(\frac{be^2}{10} + \frac{cde}{5} \right) + x^7 \left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7} \right) + x^4 \left(\frac{ade}{2} + \frac{bd^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**3+d)**2*(c*x**6+b*x**3+a), x)`

[Out] $a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 + 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)$

3.5 $\int (d + ex^3)(a + bx^3 + cx^6) dx$

Optimal. Leaf size=42

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1407}

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]

[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n)), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int (d + ex^3)(a + bx^3 + cx^6) dx &= \int (ad + (bd + ae)x^3 + (cd + be)x^6 + cex^9) dx \\ &= adx + \frac{1}{4}(bd + ae)x^4 + \frac{1}{7}(cd + be)x^7 + \frac{1}{10}cex^{10} \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$\frac{1}{4}x^4(ae + bd) + adx + \frac{1}{7}x^7(be + cd) + \frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]

[Out] $a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^{10})/10$

fricas [A] time = 0.91, size = 40, normalized size = 0.95

$$\frac{1}{10}x^{10}ec + \frac{1}{7}x^7dc + \frac{1}{7}x^7eb + \frac{1}{4}x^4db + \frac{1}{4}x^4ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")`

[Out] $1/10*x^{10}*e*c + 1/7*x^7*d*c + 1/7*x^7*e*b + 1/4*x^4*d*b + 1/4*x^4*e*a + x*d*a$

giac [A] time = 0.33, size = 43, normalized size = 1.02

$$\frac{1}{10}cx^{10}e + \frac{1}{7}cdx^7 + \frac{1}{7}bx^7e + \frac{1}{4}bdx^4 + \frac{1}{4}ax^4e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")`

[Out] $1/10*c*x^{10}*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x^4*e + a*d*x$

maple [A] time = 0.00, size = 37, normalized size = 0.88

$$\frac{ce x^{10}}{10} + \frac{(be + cd)x^7}{7} + \frac{(ae + bd)x^4}{4} + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^3+d)*(c*x^6+b*x^3+a),x)`

[Out] $a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^{10}$

maxima [A] time = 0.55, size = 36, normalized size = 0.86

$$\frac{1}{10}cex^{10} + \frac{1}{7}(cd + be)x^7 + \frac{1}{4}(bd + ae)x^4 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")`

[Out] $1/10*c*e*x^{10} + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x$

mupad [B] time = 0.04, size = 38, normalized size = 0.90

$$\frac{cex^{10}}{10} + \left(\frac{be}{7} + \frac{cd}{7}\right)x^7 + \left(\frac{ae}{4} + \frac{bd}{4}\right)x^4 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^3)*(a + b*x^3 + c*x^6),x)`

[Out] `x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10`

sympy [A] time = 0.07, size = 39, normalized size = 0.93

$$adx + \frac{cex^{10}}{10} + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)`

[Out] `a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)`

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3} dx$

Optimal. Leaf size=188

$$\frac{\log\left(d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex} + e^{2/3}x^2\right)(ae^2 - bde + cd^2)}{6d^{2/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d} + \sqrt[3]{ex}\right)(ae^2 - bde + cd^2)}{3d^{2/3}e^{7/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{3}d^{2/3}e^{7/3}}$$

[Out] $-(b*e+c*d)*x/e^2+1/4*c*x^4/e+1/3*(a*e^2-b*d*e+c*d^2)*\ln(d^{(1/3)+e^{(1/3)*x}}/d^{(2/3)/e^{(7/3)}}-1/6*(a*e^2-b*d*e+c*d^2)*\ln(d^{(2/3)}-d^{(1/3)}*e^{(1/3)*x}+e^{(2/3)*x^2})/d^{(2/3)/e^{(7/3)}}-1/3*(a*e^2-b*d*e+c*d^2)*\arctan(1/3*(d^{(1/3)}-2*e^{(1/3)*x})/d^{(1/3)*3^{(1/2)}})/d^{(2/3)/e^{(7/3)}}*3^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1411, 388, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex} + e^{2/3}x^2\right)(ae^2 - bde + cd^2)}{6d^{2/3}e^{7/3}} + \frac{\log\left(\sqrt[3]{d} + \sqrt[3]{ex}\right)(ae^2 - bde + cd^2)}{3d^{2/3}e^{7/3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{ex}}{\sqrt{3}\sqrt[3]{d}}\right)(ae^2 - bde + cd^2)}{\sqrt{3}d^{2/3}e^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3), x]

[Out] $-(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*\text{ArcTan}[(d^{(1/3)} - 2*e^{(1/3)*x})/(\text{Sqrt}[3]*d^{(1/3)})]) / (\text{Sqrt}[3]*d^{(2/3)}*e^{(7/3)}) + ((c*d^2 - b*d*e + a*e^2)*\text{Log}[d^{(1/3)} + e^{(1/3)*x}]) / (3*d^{(2/3)}*e^{(7/3)}) - ((c*d^2 - b*d*e + a*e^2)*\text{Log}[d^{(2/3)} - d^{(1/3)}*e^{(1/3)*x} + e^{(2/3)*x^2}]) / (6*d^{(2/3)}*e^{(7/3)})$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1)), x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) - (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3 + cx^6}{d + ex^3} dx &= \frac{cx^4}{4e} + \frac{\int \frac{4ae - (4cd - 4be)x^3}{d + ex^3} dx}{4e} \\
&= -\frac{(cd - be)x}{e^2} + \frac{cx^4}{4e} - \left(-a - \frac{d(cd - be)}{e^2} \right) \int \frac{1}{d + ex^3} dx \\
&= -\frac{(cd - be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(a + \frac{d(cd - be)}{e^2} \right) \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{e}x} dx}{3d^{2/3}} + \frac{\left(a + \frac{d(cd - be)}{e^2} \right) \int \frac{2\sqrt[3]{d} - \sqrt[3]{e}x}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2} dx}{3d^{2/3}} \\
&= -\frac{(cd - be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2 - bde + ae^2) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{3d^{2/3}e^{7/3}} - \frac{(cd^2 - bde + ae^2) \int \frac{-\sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}}{d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2} dx}{6d^{2/3}e^{7/3}} \\
&= -\frac{(cd - be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2 - bde + ae^2) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{3d^{2/3}e^{7/3}} - \frac{(cd^2 - bde + ae^2) \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2)}{6d^{2/3}e^{7/3}} \\
&= -\frac{(cd - be)x}{e^2} + \frac{cx^4}{4e} - \frac{(cd^2 - bde + ae^2) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}} + \frac{(cd^2 - bde + ae^2) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{3d^{2/3}e^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 176, normalized size = 0.94

$$\frac{-\frac{2 \log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2\right)(e(ae - bd) + cd^2)}{d^{2/3}} + \frac{4 \log\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)(e(ae - bd) + cd^2)}{d^{2/3}} - \frac{4\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)(e(ae - bd) + cd^2)}{d^{2/3}} + 12\sqrt[3]{e}x(be - cd)}{12e^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3), x]

[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*sqrt[3]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/sqrt[3]])/d^(2/3) + (4*(c*d^2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e^(7/3))

fricas [A] time = 0.90, size = 465, normalized size = 2.47

$$\left[\frac{3cd^2e^2x^4 + 6\sqrt{\frac{1}{3}}(cd^3e - bd^2e^2 + ade^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}} \log\left(\frac{2dex^3 - 3(d^2e)^{\frac{1}{3}}dx - d^2 + 3\sqrt{\frac{1}{3}}\left(2dex^2 + (d^2e)^{\frac{2}{3}}x - (d^2e)^{\frac{1}{3}}d\right)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}}}{ex^3 + d}\right)}{\dots} \right] - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")

[Out] [1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x/(d^2*e^3), 1/12*(3*c*d^2*e^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x/(d^2*e^3)]

giac [A] time = 0.37, size = 173, normalized size = 0.92

$$\frac{\sqrt{3}(cd^2 - bde + ae^2) \arctan\left(\frac{\sqrt{3}\left(2x + (-de^{(-1)})^{\frac{1}{3}}\right)}{3(-de^{(-1)})^{\frac{1}{3}}}\right) e^{(-1)}}{3(-de^2)^{\frac{2}{3}}} + \frac{(cd^2 - bde + ae^2)e^{(-1)} \log\left(x^2 + (-de^{(-1)})^{\frac{1}{3}}x + (-de^{(-1)})^{\frac{2}{3}}\right)}{6(-de^2)^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")

[Out] -1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1)))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c*d^2 - b*d*e + a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3) - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e^(-1))^(1/3)*e^(-4)*log(abs(x - (-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4)

maple [B] time = 0.01, size = 313, normalized size = 1.66

$$\frac{cx^4}{4e} + \frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e} + \frac{a \ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e} - \frac{a \ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{e}\right)^{\frac{2}{3}}e} - \frac{\sqrt{3} bd \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2} - \frac{bd}{3\left(\frac{d}{e}\right)^{\frac{2}{3}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d), x)

[Out] $\frac{1}{4}cx^4/e + 1/e * bx - 1/e^2 * cd * x + 1/3/e / (d/e)^{(2/3)} * \ln(x + (d/e)^{(1/3)}) * a - 1/3/e^{2/3} / (d/e)^{(2/3)} * \ln(x + (d/e)^{(1/3)}) * d * b + 1/3/e^3 / (d/e)^{(2/3)} * \ln(x + (d/e)^{(1/3)}) * c * d^2 - 1/6/e / (d/e)^{(2/3)} * \ln(x^2 - (d/e)^{(1/3)} * x + (d/e)^{(2/3)}) * a + 1/6/e^2 / (d/e)^{(2/3)} * \ln(x^2 - (d/e)^{(1/3)} * x + (d/e)^{(2/3)}) * d * b - 1/6/e^3 / (d/e)^{(2/3)} * \ln(x^2 - (d/e)^{(1/3)} * x + (d/e)^{(2/3)}) * c * d^2 + 1/3/e / (d/e)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (d/e)^{(1/3)} * x - 1)) * a - 1/3/e^2 / (d/e)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (d/e)^{(1/3)} * x - 1)) * d * b + 1/3/e^3 / (d/e)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2 / (d/e)^{(1/3)} * x - 1)) * c * d^2$

maxima [A] time = 1.52, size = 169, normalized size = 0.90

$$\frac{cex^4 - 4(cd - be)x}{4e^2} + \frac{\sqrt{3}(cd^2 - bde + ae^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} - \frac{(cd^2 - bde + ae^2) \log\left(x^2 - x\left(\frac{d}{e}\right)^{\frac{1}{3}} + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d), x, algorithm="maxima")

[Out] $\frac{1}{4}(c * e * x^4 - 4 * (c * d - b * e) * x) / e^2 + \frac{1}{3} * \sqrt{3} * (c * d^2 - b * d * e + a * e^2) * \arctan\left(\frac{1}{3} * \sqrt{3} * \left(\frac{2 * x - \left(\frac{d}{e}\right)^{\frac{1}{3}}}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)\right) / (e^3 * \left(\frac{d}{e}\right)^{\frac{2}{3}}) - \frac{1}{6} * (c * d^2 - b * d * e + a * e^2) * \log\left(x^2 - x * \left(\frac{d}{e}\right)^{\frac{1}{3}} + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right) / (e^3 * \left(\frac{d}{e}\right)^{\frac{2}{3}}) + \frac{1}{3} * (c * d^2 - b * d * e + a * e^2) * \log\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right) / (e^3 * \left(\frac{d}{e}\right)^{\frac{2}{3}})$

mupad [B] time = 0.27, size = 165, normalized size = 0.88

$$x \left(\frac{b}{e} - \frac{cd}{e^2}\right) + \frac{cx^4}{4e} + \frac{\ln\left(e^{1/3}x + d^{1/3}\right) (cd^2 - bde + ae^2)}{3d^{2/3}e^{7/3}} + \frac{\ln\left(2e^{1/3}x - d^{1/3} + \sqrt{3}d^{1/3}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) (cd^2 - bde + ae^2)}{3d^{2/3}e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

sympy [A] time = 0.90, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e} + x \left(\frac{b}{e} - \frac{cd}{e^2} \right) + \text{RootSum} \left(27t^3d^2e^7 - a^3e^6 + 3a^2bde^5 - 3a^2cd^2e^4 - 3ab^2d^2e^4 + 6abcd^3e^3 - 3ac^2d^4e^2 + b^3d^3e^3 - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6 + 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2) + x)))
```

$$3.7 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^2} dx$$

Optimal. Leaf size=213

$$\frac{x(ae^2 - bde + cd^2)}{3de^2(d + ex^3)} + \frac{\log(d^{2/3} - \sqrt[3]{d} \sqrt[3]{e} x + e^{2/3} x^2)(4cd^2 - e(2ae + bd))}{18d^{5/3}e^{7/3}} - \frac{\log(\sqrt[3]{d} + \sqrt[3]{e} x)(4cd^2 - e(2ae + bd))}{9d^{5/3}e^{7/3}}$$

[Out] $c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)-1/9*(4*c*d^2-e*(2*a*e+b*d))*\ln(d^{1/3}+e^{1/3}*x)/d^{5/3}/e^{7/3}+1/18*(4*c*d^2-e*(2*a*e+b*d))*\ln(d^{2/3}-d^{1/3}*e^{1/3}*x+e^{2/3}*x^2)/d^{5/3}/e^{7/3}+1/9*(4*c*d^2-e*(2*a*e+b*d))*\arctan(1/3*(d^{1/3}-2*e^{1/3}*x)/d^{1/3}*3^{1/2})/d^{5/3}/e^{7/3}*3^{1/2}$

Rubi [A] time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x(ae^2 - bde + cd^2)}{3de^2(d + ex^3)} + \frac{\log(d^{2/3} - \sqrt[3]{d} \sqrt[3]{e} x + e^{2/3} x^2)(4cd^2 - e(2ae + bd))}{18d^{5/3}e^{7/3}} - \frac{\log(\sqrt[3]{d} + \sqrt[3]{e} x)(4cd^2 - e(2ae + bd))}{9d^{5/3}e^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2, x]

[Out] $(c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*\text{ArcTan}[(d^{1/3} - 2*e^{1/3}*x)/(\text{Sqrt}[3]*d^{1/3})])/(3*\text{Sqrt}[3]*d^{5/3}*e^{7/3}) - ((4*c*d^2 - e*(b*d + 2*a*e))*\text{Log}[d^{1/3} + e^{1/3}*x])/(9*d^{5/3}*e^{7/3}) + ((4*c*d^2 - e*(b*d + 2*a*e))*\text{Log}[d^{2/3} - d^{1/3}*e^{1/3}*x + e^{2/3}*x^2])/(18*d^{5/3}*e^{7/3})$

Rule 31

Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3 + cx^6}{(d + ex^3)^2} dx &= \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} - \frac{\int \frac{cd^2 - e(bd + 2ae) - 3cdex^3}{d + ex^3} dx}{3de^2} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} - \frac{(4cd^2 - e(bd + 2ae)) \int \frac{1}{d + ex^3} dx}{3de^2} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} - \frac{(4cd^2 - e(bd + 2ae)) \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{e}x} dx}{9d^{5/3}e^2} - \frac{(4cd^2 - e(bd + 2ae)) \int \frac{1}{d + ex^3} dx}{9d^{5/3}e} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} - \frac{(4cd^2 - e(bd + 2ae)) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2 - e(bd + 2ae))}{18d^{5/3}e} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} - \frac{(4cd^2 - e(bd + 2ae)) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2 - e(bd + 2ae))}{18d^{5/3}e} \\
&= \frac{cx}{e^2} + \frac{(cd^2 - bde + ae^2)x}{3de^2(d + ex^3)} + \frac{(4cd^2 - e(bd + 2ae)) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}} - \frac{(4cd^2 - e(bd + 2ae))}{9d^{5/3}e}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 199, normalized size = 0.93

$$\frac{6\sqrt[3]{e}x(e(ae-bd)+cd^2)}{d(d+ex^3)} + \frac{\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2\right)(4cd^2 - e(2ae+bd))}{d^{5/3}} - \frac{2\log\left(\sqrt[3]{d} + \sqrt[3]{e}x\right)(4cd^2 - e(2ae+bd))}{d^{5/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)(4cd^2 - e(2ae+bd))}{d^{5/3}}$$

$18e^{7/3}$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2, x]

[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3)) + (2*sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))

fricas [A] time = 0.97, size = 697, normalized size = 3.27

$$\left[\frac{18cd^3e^2x^4 - 3\sqrt{\frac{1}{3}}(4cd^4e - bd^3e^2 - 2ad^2e^3 + (4cd^3e^2 - bd^2e^3 - 2ade^4)x^3)\sqrt{-\frac{(d^2e)^{\frac{1}{3}}}{e}} \log\left(\frac{2dex^3 - 3(d^2e)^{\frac{1}{3}}dx - d^2 + 3\sqrt{\frac{1}{3}}(4cd^3e^2 - bd^2e^3 - 2ade^4)x^3}{(d^2e)^{\frac{1}{3}}}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")

[Out] [1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3))*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d) + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x/(d^3*e^4*x^3 + d^4*e^3), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x/(d^3*e^4*x^3 + d^4*e^3)]

giac [A] time = 0.38, size = 199, normalized size = 0.93

$$cx^{e^{-2}} + \frac{\sqrt{3}(4cd^2 - bde - 2ae^2) \arctan\left(\frac{\sqrt{3}\left(2x + (-de^{(-1)})^{\frac{1}{3}}\right)}{3(-de^{(-1)})^{\frac{1}{3}}}\right)}{9(-de^2)^{\frac{2}{3}}d} e^{(-1)} + \frac{(4cd^2 - bde - 2ae^2)e^{(-1)} \log\left(x^2 + (-de^{(-1)})^{\frac{1}{3}}x\right)}{18(-de^2)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")

[Out] $c*x*e^{-2} + 1/9*\sqrt{3}*(4*c*d^2 - b*d*e - 2*a*e^2)*\arctan(1/3*\sqrt{3}*(2*x + (-d*e^{-1})^{1/3})/(-d*e^{-1})^{1/3})*e^{-1}/((-d*e^2)^{2/3}*d) + 1/18*(4*c*d^2 - b*d*e - 2*a*e^2)*e^{-1}*\log(x^2 + (-d*e^{-1})^{1/3}*x + (-d*e^{-1})^{2/3})/((-d*e^2)^{2/3}*d) + 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d*e^{-1})^{1/3}*e^{-2}*\log(\text{abs}(x - (-d*e^{-1})^{1/3}))/d^2 + 1/3*(c*d^2*x - b*d*x*e + a*x*e^2)*e^{-2}/((x^3*e + d)*d)$

maple [A] time = 0.01, size = 345, normalized size = 1.62

$$\frac{\frac{ax}{3(ex^3+d)d} - \frac{bx}{3(ex^3+d)e} + \frac{cdx}{3(ex^3+d)e^2} + \frac{2\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de} + \frac{2a \ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de} - \frac{a \ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x\right)}{9\left(\frac{d}{e}\right)^{\frac{2}{3}}de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x)`

[Out] $c*x/e^2+1/3/d*x/(e*x^3+d)*a-1/3/e*x/(e*x^3+d)*b+1/3/e^2*d*x/(e*x^3+d)*c+2/9/e/d/(d/e)^{2/3}*\ln(x+(d/e)^{1/3})*a+1/9/e^2/(d/e)^{2/3}*\ln(x+(d/e)^{1/3})*b-4/9/e^3*d/(d/e)^{2/3}*\ln(x+(d/e)^{1/3})*c-1/9/e/d/(d/e)^{2/3}*\ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*a-1/18/e^2/(d/e)^{2/3}*\ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*b+2/9/e^3*d/(d/e)^{2/3}*\ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*c+2/9/e/d/(d/e)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a+1/9/e^2/(d/e)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*b-4/9/e^3*d/(d/e)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*c$

maxima [A] time = 1.60, size = 204, normalized size = 0.96

$$\frac{\frac{(cd^2 - bde + ae^2)x}{3(de^3x^3 + d^2e^2)} + \frac{cx}{e^2} - \frac{\sqrt{3}(4cd^2 - bde - 2ae^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{9de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} + \frac{(4cd^2 - bde - 2ae^2) \log\left(x^2 - x\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{18de^3\left(\frac{d}{e}\right)^{\frac{2}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")`

[Out] $\frac{1}{3}*(c*d^2 - b*d*e + a*e^2)*x/(d*e^3*x^3 + d^2*e^2) + c*x/e^2 - 1/9*\sqrt{3}*(4*c*d^2 - b*d*e - 2*a*e^2)*\arctan(1/3*\sqrt{3}*(2*x - (d/e)^{1/3}))/((d/e)^{1/3})/(d*e^3*(d/e)^{2/3}) + 1/18*(4*c*d^2 - b*d*e - 2*a*e^2)*\log(x^2 - x*(d/e)^{1/3} + (d/e)^{2/3})/(d*e^3*(d/e)^{2/3}) - 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*\log(x + (d/e)^{1/3})/(d*e^3*(d/e)^{2/3})$

mupad [B] time = 1.80, size = 187, normalized size = 0.88

$$\frac{cx}{e^2} + \frac{\ln(e^{1/3}x + d^{1/3})(-4cd^2 + bde + 2ae^2)}{9d^{5/3}e^{7/3}} + \frac{x(cd^2 - bde + ae^2)}{3d(e^3x^3 + de^2)} + \frac{\ln(2e^{1/3}x - d^{1/3} + \sqrt{3}d^{1/3}1i)\left(-\frac{1}{2} + \frac{\sqrt{3}1}{2}\right)}{9d^{5/3}e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)`

[Out] $(c*x)/e^2 + (\log(e^{1/3}*x + d^{1/3})*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^{5/3}*e^{7/3}) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (\log(3^{1/2}*d^{1/3}*1i + 2*e^{1/3}*x - d^{1/3})*((3^{1/2}*1i)/2 - 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^{5/3}*e^{7/3}) - (\log(3^{1/2}*d^{1/3}*1i - 2*e^{1/3}*x + d^{1/3})*((3^{1/2}*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^{5/3}*e^{7/3})$

sympy [A] time = 1.68, size = 206, normalized size = 0.97

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{3d^2e^2 + 3de^3x^3} + \text{RootSum}\left(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^3e^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)`

[Out] $c*x/e^2 + x*(a*e^2 - b*d*e + c*d^2)/(3*d^2*e^2 + 3*d*e^3*x^3) + \text{RootSum}(729*_t^3*d^5*e^7 - 8*a^3*e^6 - 12*a^2*b*d*e^5 + 48*a^2*c*d^2*e^4 - 6*a*b^2*d^2*e^4 + 48*a*b*c*d^3*e^3 - 96*a*c^2*d^3*e^3 - b^3*d^3*e^3 + 12*b^2*c*d^4*e^2 - 48*b*c^2*d^5*e + 64*c^3*d^6, \text{Lambda}(_t, _t*\log(9*_t*d^2*e^2/(2*a*e^2 + b*d*e - 4*c*d^2) + x)))$

$$3.8 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^3} dx$$

Optimal. Leaf size=242

$$\frac{x(7cd^2 - e(5ae + bd))}{18d^2e^2(d + ex^3)} + \frac{x(ae^2 - bde + cd^2)}{6de^2(d + ex^3)^2} - \frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)(e(5ae + bd) + 2cd^2)}{54d^{8/3}e^{7/3}} + \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex})}{54d^{8/3}e^{7/3}}$$

[Out] $1/6*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^2-1/18*(7*c*d^2-e*(5*a*e+b*d))*x/d^2/e^2/(e*x^3+d)+1/27*(2*c*d^2+e*(5*a*e+b*d))*\ln(d^{1/3}+e^{1/3}*x)/d^{8/3}/e^{7/3}-1/54*(2*c*d^2+e*(5*a*e+b*d))*\ln(d^{2/3}-d^{1/3}*e^{1/3}*x+e^{2/3}*x^2)/d^{8/3}/e^{7/3}-1/27*(2*c*d^2+e*(5*a*e+b*d))*\arctan(1/3*(d^{1/3}-2*e^{1/3}*x)/d^{1/3}*3^{1/2})/d^{8/3}/e^{7/3}*3^{1/2}$

Rubi [A] time = 0.26, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1409, 385, 200, 31, 634, 617, 204, 628}

$$\frac{x(7cd^2 - e(5ae + bd))}{18d^2e^2(d + ex^3)} + \frac{x(ae^2 - bde + cd^2)}{6de^2(d + ex^3)^2} - \frac{\log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{ex} + e^{2/3}x^2)(e(5ae + bd) + 2cd^2)}{54d^{8/3}e^{7/3}} + \frac{\log(\sqrt[3]{d} + \sqrt[3]{ex})}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3, x]

[Out] $((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d + 5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*\text{ArcTan}[(d^{1/3} - 2*e^{1/3}*x)/(\text{Sqrt}[3]*d^{1/3})])/(9*\text{Sqrt}[3]*d^{8/3}*e^{7/3}) + ((2*c*d^2 + e*(b*d + 5*a*e))*\text{Log}[d^{1/3} + e^{1/3}*x])/(27*d^{8/3}*e^{7/3}) - ((2*c*d^2 + e*(b*d + 5*a*e))*\text{Log}[d^{2/3} - d^{1/3}*e^{1/3}*x + e^{2/3}*x^2])/(54*d^{8/3}*e^{7/3})$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F

reeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1409

Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3 + cx^6}{(d + ex^3)^3} dx &= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{\int \frac{cd^2 - e(bd + 5ae) - 6cdex^3}{(d + ex^3)^2} dx}{6de^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{(7cd^2 - e(bd + 5ae))x}{18d^2e^2(d + ex^3)} + \frac{(2cd^2 + e(bd + 5ae)) \int \frac{1}{d + ex^3} dx}{9d^2e^2} \\
&= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{(7cd^2 - e(bd + 5ae))x}{18d^2e^2(d + ex^3)} + \frac{(2cd^2 + e(bd + 5ae)) \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{e}x} dx}{27d^{8/3}e^2} + \frac{(2cd^2 + e(bd + 5ae)) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{27d^{8/3}e^{7/3}} \\
&= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{(7cd^2 - e(bd + 5ae))x}{18d^2e^2(d + ex^3)} + \frac{(2cd^2 + e(bd + 5ae)) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{27d^{8/3}e^{7/3}} \\
&= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{(7cd^2 - e(bd + 5ae))x}{18d^2e^2(d + ex^3)} + \frac{(2cd^2 + e(bd + 5ae)) \log(\sqrt[3]{d} + \sqrt[3]{e}x)}{27d^{8/3}e^{7/3}} \\
&= \frac{(cd^2 - bde + ae^2)x}{6de^2(d + ex^3)^2} - \frac{(7cd^2 - e(bd + 5ae))x}{18d^2e^2(d + ex^3)} - \frac{(2cd^2 + e(bd + 5ae)) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}d^{8/3}e^{7/3}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.27, size = 209, normalized size = 0.86

$$\frac{2 \log(\sqrt[3]{d} + \sqrt[3]{e}x) (e(5ae + bd) + 2cd^2) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) (e(5ae + bd) + 2cd^2) - \log(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3})}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3, x]

[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e*(8*d + 5*e*x^3)))/(d + e*x^3)^2 - 2*sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))

fricas [B] time = 0.56, size = 941, normalized size = 3.89

$$3(7cd^4e^2 - bd^3e^3 - 5ad^2e^4)x^4 - 3\sqrt{\frac{1}{3}}(2cd^5e + bd^4e^2 + 5ad^3e^3 + (2cd^3e^3 + bd^2e^4 + 5ade^5)x^6 + 2(2cd^4e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*\sqrt{1/3}*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + \\ & 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*\sqrt{-(d^2*e)^{(1/3)}/e}*\log(\\ & (2*d*e*x^3 - 3*(d^2*e)^{(1/3)}*d*x - d^2 + 3*\sqrt{1/3}*(2*d*e*x^2 + (d^2*e)^{(2/3)}*x - \\ & (d^2*e)^{(1/3)}*d)*\sqrt{-(d^2*e)^{(1/3)}/e}))/ (e*x^3 + d)) + ((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^{(2/3)}*\log(d*e*x^2 - (d^2*e)^{(2/3)}*x + (d^2*e)^{(1/3)}*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^{(2/3)}*\log(d*e*x + (d^2*e)^{(2/3)}) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 6*\sqrt{1/3}*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*\sqrt{(d^2*e)^{(1/3)}/e}*\arctan(\sqrt{1/3}*(2*(d^2*e)^{(2/3)}*x - (d^2*e)^{(1/3)}*d)*\sqrt{(d^2*e)^{(1/3)}/e}/d^2) + ((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^{(2/3)}*\log(d*e*x^2 - (d^2*e)^{(2/3)}*x + (d^2*e)^{(1/3)}*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^{(2/3)}*\log(d*e*x + (d^2*e)^{(2/3)}) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3)] \end{aligned}$$

giac [A] time = 0.40, size = 224, normalized size = 0.93

$$\frac{\sqrt{3}(2cd^2 + bde + 5ae^2) \arctan\left(\frac{\sqrt{3}\left(2x + (-de^{(-1)})^{\frac{1}{3}}\right)}{3(-de^{(-1)})^{\frac{1}{3}}}\right) e^{(-1)}}{27(-de^2)^{\frac{2}{3}}d^2} - \frac{(2cd^2 + bde + 5ae^2)e^{(-1)} \log\left(x^2 + (-de^{(-1)})^{\frac{1}{3}}x + (-d\right)}{54(-de^2)^{\frac{2}{3}}d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")

[Out]
$$\frac{-1/27*\sqrt{3}*(2*c*d^2 + b*d*e + 5*a*e^2)*\arctan(1/3*\sqrt{3}*(2*x + (-d*e^(-1))^{1/3}))/(-d*e^(-1))^{1/3}}{e^(-1)/((-d*e^2)^{2/3}*d^2) - 1/54*(2*c*d^2 + b*d*e + 5*a*e^2)*e^(-1)*\log(x^2 + (-d*e^(-1))^{1/3}*x + (-d*e^(-1))^{2/3})} + \frac{1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d*e^(-1))^{1/3}}{e^(-2)*\log(\text{abs}(x - (-d*e^(-1))^{1/3}))}/d^3 - \frac{1/18*(7*c*d^2*x^4*e - b*d*x^4*e^2 - 5*a*x^4*e^3 + 4*c*d^3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)}{(x^3*e + d)^2*d^2}$$

maple [A] time = 0.01, size = 362, normalized size = 1.50

$$\frac{5\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e} + \frac{5a \ln\left(x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e} - \frac{5a \ln\left(x^2 - \left(\frac{d}{e}\right)^{\frac{1}{3}}x + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{54\left(\frac{d}{e}\right)^{\frac{2}{3}}d^2e} + \frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}de^2} + \frac{b \ln\left(\dots\right)}{27\left(\frac{d}{e}\right)^{\frac{2}{3}}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x)

[Out]
$$\frac{(1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x)}{(e*x^3+d)^2+5/27/e/d^2/(d/e)^{2/3}*ln(x+(d/e)^{1/3})*a+1/27/e^2/d/(d/e)^{2/3}*ln(x+(d/e)^{1/3})*b+2/27/e^3/(d/e)^{2/3}*ln(x+(d/e)^{1/3})*c-5/54/e/d^2/(d/e)^{2/3}*ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*a-1/54/e^2/d/(d/e)^{2/3}*ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*b-1/27/e^3/(d/e)^{2/3}*ln(x^2-(d/e)^{1/3}*x+(d/e)^{2/3})*c+5/27/e/d^2/(d/e)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*a+1/27/e^2/d/(d/e)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*b+2/27/e^3/(d/e)^{2/3}*3^{1/2}*arctan(1/3*3^{1/2}*(2/(d/e)^{1/3}*x-1))*c}$$

maxima [A] time = 1.69, size = 240, normalized size = 0.99

$$\frac{(7cd^2e - bde^2 - 5ae^3)x^4 + 2(2cd^3 + bd^2e - 4ade^2)x}{18(d^2e^4x^6 + 2d^3e^3x^3 + d^4e^2)} + \frac{\sqrt{3}(2cd^2 + bde + 5ae^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{27d^2e^3\left(\frac{d}{e}\right)^{\frac{2}{3}}} (2cd^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")

[Out]
$$-1/18*((7*c*d^2*e - b*d*e^2 - 5*a*e^3)*x^4 + 2*(2*c*d^3 + b*d^2*e - 4*a*d*e^2)*x)/(d^2*e^4*x^6 + 2*d^3*e^3*x^3 + d^4*e^2) + 1/27*\sqrt{3}*(2*c*d^2 + b*d*e + 5*a*e^2)*\arctan(1/3*\sqrt{3}*(2*x - (d/e)^{(1/3)})/(d/e)^{(1/3)})/(d^2*e^3*(d/e)^{(2/3)}) - 1/54*(2*c*d^2 + b*d*e + 5*a*e^2)*\log(x^2 - x*(d/e)^{(1/3)} + (d/e)^{(2/3)})/(d^2*e^3*(d/e)^{(2/3)}) + 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*\log(x + (d/e)^{(1/3)})/(d^2*e^3*(d/e)^{(2/3)})$$

mupad [B] time = 0.29, size = 221, normalized size = 0.91

$$\frac{\ln\left(e^{1/3}x + d^{1/3}\right)\left(2cd^2 + bde + 5ae^2\right)}{27d^{8/3}e^{7/3}} - \frac{x(2cd^2 + bde - 4ae^2)}{9de^2} - \frac{x^4(-7cd^2 + bde + 5ae^2)}{18d^2e} + \frac{\ln\left(2e^{1/3}x - d^{1/3} + \sqrt{3}d^{1/3}1i\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)

[Out]
$$\frac{(\log(e^{1/3}*x + d^{1/3})*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^{(8/3)}*e^{(7/3)}) - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 + b*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (\log(3^{(1/2)}*d^{(1/3)}*1i + 2*e^{(1/3)}*x - d^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^{(8/3)}*e^{(7/3)}) - (\log(3^{(1/2)}*d^{(1/3)}*1i - 2*e^{(1/3)}*x + d^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^{(8/3)}*e^{(7/3)})$$

sympy [A] time = 5.23, size = 246, normalized size = 1.02

$$\frac{x^4(5ae^3 + bde^2 - 7cd^2e) + x(8ade^2 - 2bd^2e - 4cd^3)}{18d^4e^2 + 36d^3e^3x^3 + 18d^2e^4x^6} + \text{RootSum}\left(19683t^3d^8e^7 - 125a^3e^6 - 75a^2bde^5 - 150a^2cd^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)

[Out]
$$(x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + \text{RootSum}(19683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e**4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, \text{Lambda}(_t, _t*\log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))$$

$$3.9 \quad \int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=132

$$\frac{(ace + b^2(-e) + bcd) \log(a + bx^3 + cx^6)}{6c^3} - \frac{(3abce - 2ac^2d + b^3(-e) + b^2cd) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}} + \frac{x^3(cd - be)}{3c^2} + \frac{ex^6}{6c}$$

[Out] 1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/6*(a*c*e-b^2*e+b*c*d)*ln(c*x^6+b*x^3+a)/c^3-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.22, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(ace + b^2(-e) + bcd) \log(a + bx^3 + cx^6)}{6c^3} - \frac{(3abce - 2ac^2d + b^2cd + b^3(-e)) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}} + \frac{x^3(cd - be)}{3c^2} + \frac{ex^6}{6c}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2(d+ex)}{a+bx+cx^2} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{cd-be}{c^2} + \frac{ex}{c} - \frac{a(cd-be) + (bcd-b^2e+ace)x}{c^2(a+bx+cx^2)} \right) dx, x, x^3 \right) \\
&= \frac{(cd-be)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{\text{Subst} \left(\int \frac{a(cd-be) + (bcd-b^2e+ace)x}{a+bx+cx^2} dx, x, x^3 \right)}{3c^2} \\
&= \frac{(cd-be)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{(bcd-b^2e+ace) \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6c^3} + \frac{(b^2cd-2ac^2d-b^3e+3abce)}{6c^3} \\
&= \frac{(cd-be)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{(bcd-b^2e+ace) \log(a+bx^3+cx^6)}{6c^3} - \frac{(b^2cd-2ac^2d-b^3e+3abce)}{6c^3} \\
&= \frac{(cd-be)x^3}{3c^2} + \frac{ex^6}{6c} - \frac{(b^2cd-2ac^2d-b^3e+3abce) \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2-4ac}} \right)}{3c^3 \sqrt{b^2-4ac}} - \frac{(bcd-b^2e+ace) \log(a+bx^3+cx^6)}{6c^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 126, normalized size = 0.95

$$\frac{(-ace + b^2e - bcd) \log(a + bx^3 + cx^6) + \frac{2(3abce - 2ac^2d + b^3(-e) + b^2cd) \tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + 2cx^3(cd - be) + c^2ex^6}{6c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (- (b*c*d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)

fricas [A] time = 1.84, size = 430, normalized size = 3.26

$$\left[\frac{(b^2c^2 - 4ac^3)ex^6 + 2((b^2c^2 - 4ac^3)d - (b^3c - 4abc^2)e)x^3 + \sqrt{b^2 - 4ac}((b^2c - 2ac^2)d - (b^3 - 3abc)e) \log\left(\frac{2}{\sqrt{-b^2 + 4ac}}\right)}{6(b^2c^3 - 4ac^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] [1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a)]/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*sqrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a)]/(b^2*c^3 - 4*a*c^4)]

giac [A] time = 1.00, size = 131, normalized size = 0.99

$$\frac{cx^6e + 2cdx^3 - 2bx^3e}{6c^2} - \frac{(bcd - b^2e + ace) \log(cx^6 + bx^3 + a)}{6c^3} + \frac{(b^2cd - 2ac^2d - b^3e + 3abce) \arctan\left(\frac{2cx^3+b}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] $\frac{1}{6}*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - \frac{1}{6}*(b*c*d - b^2*e + a*c*e)*\log(c*x^6 + b*x^3 + a)/c^3 + \frac{1}{3}*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*\arctan((2*c*x^3 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*c^3)$

maple [B] time = 0.01, size = 260, normalized size = 1.97

$$\frac{e x^6}{6c} - \frac{b e x^3}{3c^2} + \frac{d x^3}{3c} + \frac{a b e \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2} c^2} - \frac{2 a d \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2} c} - \frac{b^3 e \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2} c^3} + \frac{b^2 d \arctan\left(\frac{2c x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2} c^2} - \frac{a e}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)`

[Out] $\frac{1}{6}*e*x^6/c - \frac{1}{3}/c^2*b*e*x^3 + \frac{1}{3}/c*d*x^3 - \frac{1}{6}/c^2*\ln(c*x^6+b*x^3+a)*a*e + \frac{1}{6}/c^3*\ln(c*x^6+b*x^3+a)*b^2*e - \frac{1}{6}/c^2*\ln(c*x^6+b*x^3+a)*b*d + \frac{1}{c^2}/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*a*b*e - \frac{2}{3}/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*a*d - \frac{1}{3}/c^3/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*b^3*e + \frac{1}{3}/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*b^2*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.40, size = 3586, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)`

[Out] $x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (\log(a + b*x^3 + c*x^6)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (\operatorname{atan}((4*c^6*(4*a*c - b^2)^{(3/2)}*(x^3*((b*((b^5*c^3*d^3 - b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2*d^2*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d*e^2$

$$\begin{aligned}
& + 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*b^2*c^4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (3*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(4*c^3*(4*a*c - b^2)*(36*a*c^4 - 9*b^2*c^3)))/(4*a^2*c) - ((2*a*c - b^2)*((((((45*b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^3)/(4*c^6*(4*a*c - b^2)^(3/2)))/(4*a^2*c*(4*a*c - b^2)^(1/2))) - (b*((a*b^7*e^3 - a*b^4*c^3*d^3 - 4*a^2*b^5*c*e^3 - 2*a^4*b*c^3*e^3 + a^4*c^4*d*e^2 + a^2*b^2*c^4*d^3 + 5*a^3*b^3*c^2*e^3 - 3*a*b^6*c*d*e^2 + 3*a*b^5*c^2*d^2*e + 2*a^3*b*c^4*d^2*e - 6*a^2*b^3*c^3*d^2*e + 9*a^2*b^4*c^2*d*e^2 - 7*a^3*b^2*c^3*d*e^2)/c^6 + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*a*b^5*c^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30*a*b^4*c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*
\end{aligned}$$

$$\begin{aligned} & (b^3cd + 12abc^2d - 15ab^2ce)/(36a^4c - 9b^2c^3)(b^3e + 2a^2c^2d - b^2cd - 3abc^2e)/(6c^3(4ac - b^2)^{1/2}) + (9ab(b^3e + 2a^2c^2d - b^2cd - 3abc^2e)(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/((4ac - b^2)^{1/2}(36a^4c - 9b^2c^3)) * \\ & (b^3e + 2a^2c^2d - b^2cd - 3abc^2e)/(6c^3(4ac - b^2)^{1/2}) - (3ab(b^3e + 2a^2c^2d - b^2cd - 3abc^2e)^2(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/(2c^3(4ac - b^2)(36a^4c - 9b^2c^3)))/((4a^2c) + ((2ac - b^2)*(((36a^2c^8d - 72ab^2c^7d + 72ab^3c^6e - 108a^2b^2c^7e)/c^6 + (54abc^3(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/(36a^4c - 9b^2c^3))(b^3e + 2a^2c^2d - b^2cd - 3abc^2e))/(6c^3(4ac - b^2)^{1/2}) + (9ab(b^3e + 2a^2c^2d - b^2cd - 3abc^2e)(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/((4ac - b^2)^{1/2}(36a^4c - 9b^2c^3)))(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/(2(36a^4c - 9b^2c^3)) + (((15ab^3c^5d^2 - 12a^2b^2c^6d^2 + 15ab^5c^3e^2 + 27a^3b^2c^5e^2 - 42a^2b^3c^4e^2 - 12a^3c^6d^2e - 30ab^4c^4d^2e + 54a^2b^2c^5d^2e)/c^6 + (((36a^2c^8d - 72ab^2c^7d + 72ab^3c^6e - 108a^2b^2c^7e)/c^6 + (54abc^3(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/(36a^4c - 9b^2c^3))(3b^4e + 12a^2c^2e - 3b^3cd + 12abc^2d - 15ab^2ce))/(2(36a^4c - 9b^2c^3)))(b^3e + 2a^2c^2d - b^2cd - 3abc^2e))/(6c^3(4ac - b^2)^{1/2}) - (ab(b^3e + 2a^2c^2d - b^2cd - 3abc^2e)^3)/(2c^6(4ac - b^2)^{3/2}))/((4a^2c(4ac - b^2)^{1/2}))/((b^9e^3 + 8a^3c^6d^3 - b^6c^3d^3 + 6ab^4c^4d^3 + 3b^7c^2d^2e - 12a^2b^2c^5d^3 + 27a^2b^5c^2e^3 - 27a^3b^3c^3e^3 - 9ab^7c^2e^3 - 3b^8c^2d^2e - 21ab^5c^3d^2e + 24ab^6c^2d^2e - 36a^3b^2c^5d^2e + 48a^2b^3c^4d^2e - 63a^2b^4c^3d^2e + 54a^3b^2c^4d^2e)))(b^3e + 2a^2c^2d - b^2cd - 3abc^2e))/(3c^3(4ac - b^2)^{1/2}) \end{aligned}$$

sympy [B] time = 55.47, size = 620, normalized size = 4.70

$$x^3 \left(-\frac{be}{3c^2} + \frac{d}{3c} \right) + \left(-\frac{\sqrt{-4ac + b^2} (3abce - 2ac^2d - b^3e + b^2cd)}{6c^3(4ac - b^2)} - \frac{ace - b^2e + bcd}{6c^3} \right) \log \left(x^3 + \frac{2a^2ce - ab^2e + abcd}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a), x)

[Out] x**3*(-b*e/(3*c**2) + d/(3*c)) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(-sqrt(-4*a*c +

$$\begin{aligned}
& b^{**2}*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) \\
& - (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b* \\
& *2*c*d) + (\text{sqrt}(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d \\
&)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*\log(x**3 + (\\
& 2*a**2*c*e - a*b**2*e + a*b*c*d + 12*a*c**3*(\text{sqrt}(-4*a*c + b**2)*(3*a*b*c*e \\
& - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2* \\
& e + b*c*d)/(6*c**3)) - 3*b**2*c**2*(\text{sqrt}(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c* \\
& *2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d \\
&)/(6*c**3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d) + e*x**6/(6*c)
\end{aligned}$$

$$3.10 \quad \int \frac{x^5(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=97

$$\frac{(2ace + b^2(-e) + bcd) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} + \frac{(cd - be) \log(a + bx^3 + cx^6)}{6c^2} + \frac{ex^3}{3c}$$

[Out] 1/3*e*x^3/c+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2+1/3*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1474, 773, 634, 618, 206, 628}

$$\frac{(2ace + b^2(-e) + bcd) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} + \frac{(cd - be) \log(a + bx^3 + cx^6)}{6c^2} + \frac{ex^3}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_.) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (d + ex^3)}{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(d + ex)}{a + bx + cx^2} dx, x, x^3 \right) \\
&= \frac{ex^3}{3c} + \frac{\text{Subst} \left(\int \frac{-ae + (cd - be)x}{a + bx + cx^2} dx, x, x^3 \right)}{3c} \\
&= \frac{ex^3}{3c} + \frac{(cd - be) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^3 \right)}{6c^2} - \frac{(bcd - b^2e + 2ace) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^3 \right)}{6c^2} \\
&= \frac{ex^3}{3c} + \frac{(cd - be) \log(a + bx^3 + cx^6)}{6c^2} + \frac{(bcd - b^2e + 2ace) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right)}{3c^2} \\
&= \frac{ex^3}{3c} + \frac{(bcd - b^2e + 2ace) \tanh^{-1} \left(\frac{b + 2cx^3}{\sqrt{b^2 - 4ac}} \right)}{3c^2 \sqrt{b^2 - 4ac}} + \frac{(cd - be) \log(a + bx^3 + cx^6)}{6c^2}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 93, normalized size = 0.96

$$\frac{2(-2ace + b^2e - bcd) \tan^{-1} \left(\frac{b + 2cx^3}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + \frac{(cd - be) \log(a + bx^3 + cx^6) + 2cex^3}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2)

fricas [A] time = 1.36, size = 305, normalized size = 3.14

$$\frac{2(b^2c - 4ac^2)ex^3 + (bcd - (b^2 - 2ac)e)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) + ((b^2c - 4ac^2)d - (b^3 - 4a^2bc)e)\sqrt{b^2 - 4ac}}{6(b^2c^2 - 4ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] [1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*x^6 + b*x^3 + a)/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + 2*(b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*x^6 + b*x^3 + a)/(b^2*c^2 - 4*a*c^3)]

giac [A] time = 1.07, size = 95, normalized size = 0.98

$$\frac{x^3e}{3c} + \frac{(cd - be) \log(cx^6 + bx^3 + a)}{6c^2} - \frac{(bcd - b^2e + 2ace) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

maple [A] time = 0.00, size = 175, normalized size = 1.80

$$\frac{ex^3}{3c} - \frac{2ae \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c} + \frac{b^2e \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c^2} - \frac{bd \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c} - \frac{be \ln(cx^6 + bx^3 + a)}{6c^2} + \frac{d \ln(cx^6 + bx^3 + a)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)$

[Out] $\frac{1}{3}e*x^3/c - \frac{1}{6}/c^2*\ln(c*x^6+b*x^3+a)*b*e + \frac{1}{6}/c*\ln(c*x^6+b*x^3+a)*d - \frac{2}{3}/c/((4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*a*e + \frac{1}{3}/c^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*b^2*e - \frac{1}{3}/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*b*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, \text{algorithm}=\text{"maxima"})$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 2.95, size = 2624, normalized size = 27.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)$

[Out] $(e*x^3)/(3*c) + (\log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (\text{atan}((4*c^3*(4*a*c - b^2)^{(3/2)}*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^3*c^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^2*d*e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^{(1/2)}) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^{(1/2)}*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^{(1/2)}) + (3*b^2*(2*a*c*e - b^2*e + b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2)))/(4*a^2*c) + ((2*a*c - b^2)*((((45*b^2*c^5*d - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d)))/$

$$\begin{aligned}
& (6c^2(4ac - b^2)^{1/2}) - (9b^2c(2ace - b^2e + bcd)(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/(2(4ac - b^2)^{1/2}(36ac^3 - 9b^2c^2)) * (3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)/(2(36ac^3 - 9b^2c^2)) + (b^2(2ace - b^2e + bcd)^3)/(4c^3(4ac - b^2)^{3/2}) \\
&) - (((6a^2c^4e^2 + 12b^2c^4d^2 + 12b^4c^2e^2 - 18ab^2c^3e^2 - 24b^3c^3d^2e + 18abc^4d^2e)/c^3 - (((45b^2c^5d - 45b^3c^4e + 36abc^5e)/c^3 - (27b^2c^3(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((36ac^3 - 9b^2c^2))(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)))/(2(36ac^3 - 9b^2c^2)) * (2ace - b^2e + bcd))/(6c^2(4ac - b^2)^{1/2}))) / (4a^2c(4ac - b^2)^{1/2})) + (b((a^2b^2c^3e^3 - ab^4e^3 + a^2c^3d^2e + abc^3d^3 + 3ab^3cd^2e^2 - 3ab^2c^2d^2e - 2a^2b^2c^2d^2e)/c^3 - (((15ab^3c^2e^2 - 12a^2b^3c^3e^2 + 15abc^4d^2 + 12a^2c^4d^2e - 30ab^2c^3d^2e)/c^3 - (((36a^2c^5e + 72abc^5d - 72ab^2c^4e)/c^3 - (54abc^3(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((36ac^3 - 9b^2c^2))(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)))/(2(36ac^3 - 9b^2c^2)) * (3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)))/(2(36ac^3 - 9b^2c^2)) - (((((36a^2c^5e + 72abc^5d - 72ab^2c^4e)/c^3 - (54abc^3(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((36ac^3 - 9b^2c^2))(2ace - b^2e + bcd))/(6c^2(4ac - b^2)^{1/2}) - (9abc(2ace - b^2e + bcd)(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((4ac - b^2)^{1/2}(36ac^3 - 9b^2c^2)) * (2ace - b^2e + bcd))/(6c^2(4ac - b^2)^{1/2}) + (3ab(2ace - b^2e + bcd)^2(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/(2c(4ac - b^2)(36ac^3 - 9b^2c^2)))/((4a^2c) + ((2ac - b^2) * (((((36a^2c^5e + 72abc^5d - 72ab^2c^4e)/c^3 - (54abc^3(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((36ac^3 - 9b^2c^2))(2ace - b^2e + bcd)))/(6c^2(4ac - b^2)^{1/2}) - (9abc(2ace - b^2e + bcd)(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((4ac - b^2)^{1/2}(36ac^3 - 9b^2c^2)) * (3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)))/(2(36ac^3 - 9b^2c^2)) - (((15ab^3c^2e^2 - 12a^2b^3c^3e^2 + 15abc^4d^2 + 12a^2c^4d^2e - 30ab^2c^3d^2e)/c^3 - (((36a^2c^5e + 72abc^5d - 72ab^2c^4e)/c^3 - (54abc^3(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e))/((36ac^3 - 9b^2c^2))(3b^3e + 12ac^2d - 3b^2cd - 12abc^2e)))/(2(36ac^3 - 9b^2c^2)) * (2ace - b^2e + bcd))/(6c^2(4ac - b^2)^{1/2}) + (ab(2ace - b^2e + bcd)^3)/(2c^3(4ac - b^2)^{3/2}))) / (4a^2c(4ac - b^2)^{1/2}))) / (8a^3c^3e^3 - b^6e^3 + b^3c^3d^3 - 3b^4c^2d^2e - 12a^2b^2c^2e^3 + 6ab^4c^3e^3 + 3b^5cd^2e^2 + 6ab^2c^3d^2e - 12ab^3c^2d^2e^2 + 12a^2b^3c^3d^2e^2) * (2ace - b^2e + bcd))/(3c^2(4ac - b^2)^{1/2})
\end{aligned}$$

sympy [B] time = 17.49, size = 434, normalized size = 4.47

$$\left(-\frac{\sqrt{-4ac + b^2} (2ace - b^2e + bcd)}{6c^2(4ac - b^2)} - \frac{be - cd}{6c^2} \right) \log \left(x^3 + \frac{-abe - 12ac^2 \left(-\frac{\sqrt{-4ac + b^2} (2ace - b^2e + bcd)}{6c^2(4ac - b^2)} - \frac{be - cd}{6c^2} \right) + 2acd + 2ace - b^2e + bcd}{2ace - b^2e + bcd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out]
$$\begin{aligned} & (-\sqrt{-4*a*c + b**2}*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - \\ & (b*e - c*d)/(6*c**2))*\log(x**3 + (-a*b*e - 12*a*c**2*(-\sqrt{-4*a*c + b**2})* \\ & (2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) \\ & + 2*a*c*d + 3*b**2*c*(-\sqrt{-4*a*c + b**2}*(2*a*c*e - b**2*e + b*c*d)/(6*c** \\ & *2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d) + (\\ & \sqrt{-4*a*c + b**2}*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b \\ & *e - c*d)/(6*c**2))*\log(x**3 + (-a*b*e - 12*a*c**2*(\sqrt{-4*a*c + b**2})*(2* \\ & a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2 \\ & *a*c*d + 3*b**2*c*(\sqrt{-4*a*c + b**2}*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(\\ & 4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d) + e*x** \\ & 3/(3*c) \end{aligned}$$

$$3.11 \quad \int \frac{x^2(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=72

$$\frac{e \log(a + bx^3 + cx^6)}{6c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e \log(a + bx^3 + cx^6)}{6c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]

[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 (d + ex^3)}{a + bx^3 + cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{d + ex}{a + bx + cx^2} dx, x, x^3 \right) \\ &= \frac{e \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right)}{6c} + \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^3 \right)}{6c} \\ &= \frac{e \log(a + bx^3 + cx^6)}{6c} - \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right)}{3c} \\ &= -\frac{(2cd - be) \tanh^{-1} \left(\frac{b+2cx^3}{\sqrt{b^2 - 4ac}} \right)}{3c\sqrt{b^2 - 4ac}} + \frac{e \log(a + bx^3 + cx^6)}{6c} \end{aligned}$$

Mathematica [A] time = 0.05, size = 71, normalized size = 0.99

$$\frac{e \log(a + bx^3 + cx^6) - \frac{2(be - 2cd) \tan^{-1} \left(\frac{b+2cx^3}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6), x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

fricas [A] time = 1.09, size = 216, normalized size = 3.00

$$\left[\frac{(b^2 - 4ac)e \log(cx^6 + bx^3 + a) - \sqrt{b^2 - 4ac} (2cd - be) \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right)}{6(b^2c - 4ac^2)}, (b^2 - 4ac)e \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]

giac [A] time = 1.21, size = 70, normalized size = 0.97

$$\frac{e \log(cx^6 + bx^3 + a)}{6c} + \frac{(2cd - be) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}c} + \frac{2d \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}} + \frac{e \ln(cx^6 + bx^3 + a)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)

[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*d-1/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e*b/c

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 2.63, size = 1632, normalized size = 22.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] - (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
(atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)*
((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 1
2*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 -
12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + (((b*e - 2*c*d)*(72*a*b*c^2*e -
36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c
(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*
a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2)))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*
d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e)*(6*c^3*d^
2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e
- 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^
2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - (((b*e - 2*c*d
)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2
- 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*e - 12*a*c*e)*(b
*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d)/(
6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(
4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))/(4*a^2*c) - ((2*a*c - b^2)*((3*b^2
*e - 12*a*c*e)*((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*
b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^
2*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c -
b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b^2*(b*e - 2*c*d)^3)/(4*(4*a*c -
b^2)^(3/2)) + ((b*e - 2*c*d)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2
*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/
(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^2 - 18*b*c^2*d*e))/(6*c*(4*a*c - b^2)
^(1/2)))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/(b^3*e^3 - 8*
c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2) + ((2*a*c - b^2)*(4*a*c - b^2)*((
3*b^2*e - 12*a*c*e)*((b*e - 2*c*d)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c
```

$$\begin{aligned} &^3(3*b^2*e - 12*a*c*e)/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^{(1/2)} + \\ &(9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*a*c^2 - 9*b^2*c)*(4*a* \\ &c - b^2)^{(1/2}))/((2*(36*a*c^2 - 9*b^2*c)) + ((b*e - 2*c*d)*((3*b^2*e - 12 \\ &*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36* \\ &a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 - 12*a*c^2*d*e)) \\ &)/(6*c*(4*a*c - b^2)^{(1/2)} - (a*b*(b*e - 2*c*d)^3)/(2*(4*a*c - b^2)^{(3/2}))) \\ &)/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2* \\ &c*d))/(3*c*(4*a*c - b^2)^{(1/2)}) \end{aligned}$$

sympy [B] time = 6.55, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{6c(4ac - b^2)} \right) \log \left(x^3 + \frac{-12ac \left(\frac{e}{6c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{6c(4ac - b^2)} \right) + 2ae + 3b^2 \left(\frac{e}{6c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{6c(4ac - b^2)} \right) - bd}{be - 2cd} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))

$$3.12 \quad \int \frac{d+ex^3}{x(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=78

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} - \frac{d \log(a + bx^3 + cx^6)}{6a} + \frac{d \log(x)}{a}$$

[Out] $d*\ln(x)/a-1/6*d*\ln(c*x^6+b*x^3+a)/a+1/3*(-2*a*e+b*d)*\operatorname{arctanh}((2*c*x^3+b)/(-4*a*c+b^2)^{(1/2)})/a/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}} - \frac{d \log(a + bx^3 + cx^6)}{6a} + \frac{d \log(x)}{a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)), x]$

[Out] $((b*d - 2*a*e)*\operatorname{ArcTanh}[(b + 2*c*x^3)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(3*a*\operatorname{Sqrt}[b^2 - 4*a*c]) + (d*\operatorname{Log}[x])/a - (d*\operatorname{Log}[a + b*x^3 + c*x^6])/(6*a)$

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\operatorname{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

```
Int[((d._) + (e._)*(x_))/((a._) + (b._)*(x_) + (c._)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_)))/((a._) + (b._)*(x_) + (c._)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_)*((a_) + (c._)*(x_)^(n2_)) + (b._)*(x_)^(n_)]^(p_)*((d_) + (e._)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^3}{x(a + bx^3 + cx^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{d + ex}{x(a + bx + cx^2)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{d}{ax} + \frac{-bd + ae - cdx}{a(a + bx + cx^2)} \right) dx, x, x^3 \right) \\
 &= \frac{d \log(x)}{a} + \frac{\text{Subst} \left(\int \frac{-bd + ae - cdx}{a + bx + cx^2} dx, x, x^3 \right)}{3a} \\
 &= \frac{d \log(x)}{a} - \frac{d \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^3 \right)}{6a} + \frac{(-bd + 2ae) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^3 \right)}{6a} \\
 &= \frac{d \log(x)}{a} - \frac{d \log(a + bx^3 + cx^6)}{6a} - \frac{(-bd + 2ae) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^3 \right)}{3a} \\
 &= \frac{(bd - 2ae) \tanh^{-1} \left(\frac{b + 2cx^3}{\sqrt{b^2 - 4ac}} \right)}{3a\sqrt{b^2 - 4ac}} + \frac{d \log(x)}{a} - \frac{d \log(a + bx^3 + cx^6)}{6a}
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 1.03

$$\frac{d \log(x)}{a} - \frac{\text{RootSum}\left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^3 c d \log(x - \#1) - a e \log(x - \#1) + b d \log(x - \#1)}{2 \#1^3 c + b} \&\right]}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)), x]

[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 &, (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) &]/(3*a)

fricas [A] time = 1.43, size = 240, normalized size = 3.08

$$\left[\frac{(b^2 - 4ac)d \log(cx^6 + bx^3 + a) - 6(b^2 - 4ac)d \log(x) + \sqrt{b^2 - 4ac}(bd - 2ae) \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac - (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right)}{6(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) + sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), - 1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]

giac [A] time = 1.05, size = 76, normalized size = 0.97

$$-\frac{d \log(cx^6 + bx^3 + a)}{6a} + \frac{d \log(|x|)}{a} - \frac{(bd - 2ae) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}a} + \frac{2e \arctan\left(\frac{2cx^3 + b}{\sqrt{4ac - b^2}}\right)}{3\sqrt{4ac - b^2}} + \frac{d \ln(x)}{a} - \frac{d \ln(cx^6 + bx^3 + a)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/a*d*ln(x)-1/6*d*ln(c*x^6+b*x^3+a)/a+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 6.76, size = 4149, normalized size = 53.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*(((3*b^2*d - 12*a*c*d)*((2*a*e - b*d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2)))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e)/(6*a*(4*a*c - b^2)^(1/2))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^2)^(1/2)) - (((2*a*e - b*d)*(((2*a*e - b*d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378
```

$$\begin{aligned}
& *a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^{(1/2)})) \\
&)/(6*a*(4*a*c - b^2)^{(1/2)} + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)* \\
& 2*c^4)*(2*a*e - b*d)^2)/(72*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(2*a*e \\
& - b*d))/(6*a*(4*a*c - b^2)^{(1/2)} - ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 3 \\
& 78*a*b^2*c^4)*(2*a*e - b*d)^3)/(432*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^{(3/2)})) \\
& *(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e))/(16 \\
& *a^4*c^3*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) - ((c^3*e^4 - ((3*b \\
& ^2*d - 12*a*c*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2 \\
& *c^3*e^2 - ((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378* \\
& a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a* \\
& b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c) \\
&) + 7*c^4*d*e^2))/(2*(9*a*b^2 - 36*a^2*c)) + (((2*a*e - b*d)*((2*a*e - b* \\
& d)*((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a \\
& ^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^{(1/2)} \\
&) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/ \\
& (12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^{(1/2)})))/(6*a*(4*a*c - b^2)^{(1/2)} \\
& + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d)^2)/(72 \\
& *a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 \\
& - 36*a^2*c)) + (((3*b^2*d - 12*a*c*d)*((2*a*e - b*d)*((3*b^2*d - 12*a*c \\
& d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d \\
& - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^{(1/2)} + ((3*b^2*d - 12 \\
& *a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^ \\
& 2*c)*(4*a*c - b^2)^{(1/2)})))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a \\
& *c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*(10 \\
& 8*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^ \\
& 3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4 \\
& *a*c - b^2)^{(1/2)}))*(2*a*e - b*d))/(6*a*(4*a*c - b^2)^{(1/2)} - ((108*b^4*c^ \\
& 3 - 378*a*b^2*c^4)*(2*a*e - b*d)^4)/(1296*a^4*(4*a*c - b^2)^2)))*(4*b^5*d - \\
& 2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e))/(16 \\
& *a^4*c^3*(4*a*c - b^2)^{(1/2)}*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) \\
&))/(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) - \\
& (3*(4*a*c - b^2)^{(3/2)}*(c^3*d*e^3 + ((3*b^2*d - 12*a*c*d)*((2*a*e - b*d)* \\
& ((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 1 \\
& 2*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)))))/(6*a*(4*a*c - b^2)^{(1/2)} + (9*b^3*c^3 \\
& *(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^ \\
& (1/2)))))/(6*a*(4*a*c - b^2)^{(1/2)} + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e \\
& - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))/(2*(9*a*b^2 - 36*a^2* \\
& c)) - ((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*((3*b^2*d - 12*a*c*d)*(\\
& 27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a \\
& *b^2 - 36*a^2*c)))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d \\
& *e))/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(2*(9*a*b^2 - 3 \\
& 6*a^2*c)) + (((3*b^2*d - 12*a*c*d)*((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^ \\
& 2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c)))))/(6 \\
& *a*(4*a*c - b^2)^{(1/2)} + (9*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4 \\
& *(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^{(1/2)})))/(2*(9*a*b^2 - 36*a^2*c)) + ((2
\end{aligned}$$

$$\begin{aligned}
& *a*e - b*d) * (((3*b^2*d - 12*a*c*d) * (27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d)) / (2*(9*a*b^2 - 36*a^2*c))) / (2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e)) / (6*a*(4*a*c - b^2)^{(1/2)}) * (2*a*e - b*d) / (6*a*(4*a*c - b^2)^{(1/2)}) - (b^3*c^3*(2*a*e - b*d)^4) / (48*a^3*(4*a*c - b^2)^2) * (4*b^5*d - 2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e)) / (c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) * (a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) + (3*(4*a*c - b^2)^2 * (((3*b^2*d - 12*a*c*d) * (((3*b^2*d - 12*a*c*d) * ((2*a*e - b*d) * (27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d)) / (2*(9*a*b^2 - 36*a^2*c)))) / (6*a*(4*a*c - b^2)^{(1/2)}) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d) * (2*a*e - b*d)) / (4*(9*a*b^2 - 36*a^2*c) * (4*a*c - b^2)^{(1/2))))) / (2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d) * (((3*b^2*d - 12*a*c*d) * (27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d)) / (2*(9*a*b^2 - 36*a^2*c)))) / (2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e)) / (6*a*(4*a*c - b^2)^{(1/2)})) / (2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d) * (((2*a*e - b*d) * ((2*a*e - b*d) * (27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d)) / (2*(9*a*b^2 - 36*a^2*c)))) / (6*a*(4*a*c - b^2)^{(1/2)}) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d) * (2*a*e - b*d)) / (4*(9*a*b^2 - 36*a^2*c) * (4*a*c - b^2)^{(1/2))))) / (6*a*(4*a*c - b^2)^{(1/2)}) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d) * (2*a*e - b*d)^2) / (8*a*(9*a*b^2 - 36*a^2*c) * (4*a*c - b^2))) / (6*a*(4*a*c - b^2)^{(1/2)}) + ((2*a*e - b*d) * (((3*b^2*d - 12*a*c*d) * (((3*b^2*d - 12*a*c*d) * (27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d)) / (2*(9*a*b^2 - 36*a^2*c)))) / (2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e)) / (2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2)) / (6*a*(4*a*c - b^2)^{(1/2)}) - (b^3*c^3*(3*b^2*d - 12*a*c*d) * (2*a*e - b*d)^3) / (16*a^2*(9*a*b^2 - 36*a^2*c) * (4*a*c - b^2)^{(3/2)}) * (4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e)) / (c^3*(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) * (a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) * (2*a*e - b*d) / (3*a*(4*a*c - b^2)^{(1/2)})
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.13 \quad \int \frac{d+ex^3}{x^4(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=112

$$\frac{(-abe - 2acd + b^2d) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} + \frac{(bd - ae) \log(a + bx^3 + cx^6)}{6a^2} - \frac{\log(x)(bd - ae)}{a^2} - \frac{d}{3ax^3}$$

[Out] $-1/3*d/a/x^3 - (-a*e+b*d)*\ln(x)/a^2 + 1/6*(-a*e+b*d)*\ln(c*x^6+b*x^3+a)/a^2 - 1/3*(-a*b*e-2*a*c*d+b^2*d)*\operatorname{arctanh}((2*c*x^3+b)/(-4*a*c+b^2)^{(1/2)})/a^2 - (-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.20, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(-abe - 2acd + b^2d) \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} + \frac{(bd - ae) \log(a + bx^3 + cx^6)}{6a^2} - \frac{\log(x)(bd - ae)}{a^2} - \frac{d}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)), x]

[Out] $-d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*\operatorname{ArcTanh}[(b + 2*c*x^3)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(3*a^2*\operatorname{Sqrt}[b^2 - 4*a*c]) - ((b*d - a*e)*\operatorname{Log}[x])/a^2 + ((b*d - a*e)*\operatorname{Log}[a + b*x^3 + c*x^6])/(6*a^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1474

Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_.) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^3}{x^4(a + bx^3 + cx^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{d + ex}{x^2(a + bx + cx^2)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{d}{ax^2} + \frac{-bd + ae}{a^2x} + \frac{b^2d - acd - abe + c(bd - ae)x}{a^2(a + bx + cx^2)} \right) dx, x, x^3 \right) \\
 &= \frac{d}{3ax^3} - \frac{(bd - ae) \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2d - acd - abe + c(bd - ae)x}{a + bx + cx^2} dx, x, x^3 \right)}{3a^2} \\
 &= \frac{d}{3ax^3} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^3 \right)}{6a^2} + \frac{(b^2d - 2acd - abe) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^3 \right)}{6a^2} \\
 &= \frac{d}{3ax^3} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \log(a + bx^3 + cx^6)}{6a^2} - \frac{(b^2d - 2acd - abe) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^3 \right)}{6a^2} \\
 &= \frac{d}{3ax^3} - \frac{(b^2d - 2acd - abe) \tanh^{-1} \left(\frac{b + 2cx^3}{\sqrt{b^2 - 4ac}} \right)}{3a^2 \sqrt{b^2 - 4ac}} - \frac{(bd - ae) \log(x)}{a^2} + \frac{(bd - ae) \log(a + bx^3 + cx^6)}{6a^2}
 \end{aligned}$$

Mathematica [C] time = 0.05, size = 130, normalized size = 1.16

$$\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{-\#1^3ace \log(x-\#1) + \#1^3bcd \log(x-\#1) - abe \log(x-\#1) - acd \log(x-\#1) + b^2d \log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2} + \frac{\log(x)(ae - l)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)), x]

[Out] $-1/3*d/(a*x^3) + ((-(b*d) + a*e)*\text{Log}[x])/a^2 + \text{RootSum}[a + b*\#1^3 + c*\#1^6 \&, (b^2*d*\text{Log}[x - \#1] - a*c*d*\text{Log}[x - \#1] - a*b*e*\text{Log}[x - \#1] + b*c*d*\text{Log}[x - \#1]*\#1^3 - a*c*e*\text{Log}[x - \#1]*\#1^3)/(b + 2*c*\#1^3) \&]/(3*a^2)$

fricas [A] time = 2.32, size = 385, normalized size = 3.44

$$\left[\frac{(abe - (b^2 - 2ac)d)\sqrt{b^2 - 4ac} x^3 \log\left(\frac{2c^2x^6 + 2bcx^3 + b^2 - 2ac + (2cx^3 + b)\sqrt{b^2 - 4ac}}{cx^6 + bx^3 + a}\right) + ((b^3 - 4abc)d - (ab^2 - 4a^2c)e)x^3}{6(a^2b^2 - 4a^3c)x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] $[1/6*((a*b*e - (b^2 - 2*a*c)*d)*\text{sqrt}(b^2 - 4*a*c)*x^3*\log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*\log(c*x^6 + b*x^3 + a) - 6*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*\log(x) - 2*(a*b^2 - 4*a^2*c)*d]/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*\text{sqrt}(-b^2 + 4*a*c)*x^3*\arctan(-(2*c*x^3 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*\log(c*x^6 + b*x^3 + a) - 6*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*\log(x) - 2*(a*b^2 - 4*a^2*c)*d]/((a^2*b^2 - 4*a^3*c)*x^3)]$

giac [A] time = 1.07, size = 128, normalized size = 1.14

$$\frac{(bd - ae) \log(cx^6 + bx^3 + a)}{6a^2} - \frac{(bd - ae) \log(|x|)}{a^2} + \frac{(b^2d - 2acd - abe) \arctan\left(\frac{2cx^3 + b}{\sqrt{-b^2 + 4ac}}\right)}{3\sqrt{-b^2 + 4ac}a^2} + \frac{bdx^3 - ax^3e - ad}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] $1/6*(b*d - a*e)*\log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*\log(\text{abs}(x))/a^2 + 1/3*(b^2*d - 2*a*c*d - a*b*e)*\arctan((2*c*x^3 + b)/\text{sqrt}(-b^2 + 4*a*c))/(\text{sqrt}(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e - a*d)/(a^2*x^3)$

maple [A] time = 0.01, size = 191, normalized size = 1.71

$$\frac{be \arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}a} - \frac{2cd \arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}a} + \frac{b^2d \arctan\left(\frac{2cx^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}a^2} + \frac{e \ln(x)}{a} - \frac{e \ln(cx^6+bx^3+a)}{6a} - \frac{bd \ln(x)}{a^2} + \frac{bd}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)

[Out] $-1/3/a*d/x^3+1/a*\ln(x)*e-1/a^2*\ln(x)*b*d-1/6/a*\ln(c*x^6+b*x^3+a)*e+1/6/a^2*\ln(c*x^6+b*x^3+a)*b*d-1/3/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})^2)*b*e-2/3/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*c*d+1/3/a^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^3+b)/(4*a*c-b^2)^{(1/2)})*b^2*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 9.57, size = 7282, normalized size = 65.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)

[Out] $(\log(x)*(a*e - b*d))/a^2 - (\log((((((((a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^{(1/2)}*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^{(1/2)}))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^{(1/2)})/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^{(1/2)})/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4*((((((b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d))^2/(a^4*(4*a*c - b^2)))^{(1/2)}*((27*$

$$\begin{aligned}
& (2*c*e - 12*a*b*c*d)/(2*a^4*(36*a^3*c - 9*a^2*b^2))*(a*b*e - b^2*d + 2*a*c*d) \\
&)/(6*a^2*(4*a*c - b^2)^{(1/2)}) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^{(1/2)}*(36*a^3*c - 9*a^2*b^2))*(a*b*e - b^2*d + 2*a*c*d) \\
&)/(6*a^2*(4*a*c - b^2)^{(1/2)}) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)^2 \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d)/(2*(36*a^3*c - 9*a^2*b^2)) - (((7*a^2*c^6*d^2*e - 12*a*b*c^6*d^3)/a^4 \\
& - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 \\
& + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) + (c^7*d^4)/a^4 - ((108*a^4*b^4*c^3 \\
& - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)^4)/(1296*a^12*(4*a*c - b^2)^2) + ((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 \\
& + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)) \\
& *(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^{(1/2)}) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^{(1/2)}*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 \\
& - 42*a^3*b*c^5*d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& *(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^{(1/2)))*((8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b^2*c^2*d \\
& + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(64*a^4*c^3*(4*a*c - b^2)^{(1/2)} \\
& *(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e)) \\
& *(4*a*c - b^2)^2)/(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^3*e^3 \\
& + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 \\
& + 6*a^3*b^2*c^4*d*e^2) + (3*a^4*(4*a*c - b^2)^2*((((((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d \\
& + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& *(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^{(1/2)}) + (9*b^3*c^3*(a*b*e - b^2*d + 2*a*c*d) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(4*a*(4*a*c - b^2)^{(1/2)} \\
& *(36*a^3*c - 9*a^2*b^2))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& + (((9*a^3*b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + ((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d \\
& + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& *(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) \\
& *(a*b*e - b^2*d + 2*a*c*d)))/
\end{aligned}$$

$$\begin{aligned}
& ((6a^2(4ac - b^2)^{1/2})) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d) \\
&) / (2(36a^3c - 9a^2b^2)) + (((a^2c^6d^3 - 9ab^2c^5d^3 + 9a^2b*c^5d^2e) / a^4 + (((9a^3b*c^5d^2 - 27a^2b^3c^4d^2 + 27a^3b^2c^4d * e) / a^4 + (((27a^4b^2c^4d - 27a^3b^4c^3d + 27a^4b^3c^3e) / a^4 + \\
& (27ab^3c^3(3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2(4ac - b^2)^{1/2}) \\
& - (((((((27a^4b^2c^4d - 27a^3b^4c^3d + 27a^4b^3c^3e) / a^4 + (27 * ab^3c^3(3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9 * a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2(4ac - b^2)^{1/2}) + (9b^3 * c^3(ab^2e - b^2d + 2ac*d) * (3b^3d - 3ab^2e + 12a^2ce - 12abc * d)) / (4a(4ac - b^2)^{1/2} * (36a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2a * c*d)) / (6a^2(4ac - b^2)^{1/2}) + (3b^3c^3(ab^2e - b^2d + 2ac*d)^2 * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (8a^3(4ac - b^2) * (36 * a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2(4ac - b^2)^{1/2}) \\
& - (b^3c^3(ab^2e - b^2d + 2ac*d)^3 * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (16a^5(4ac - b^2)^{3/2} * (36a^3c - 9a^2b^2))) * (4b^5d - 7a^3c^2e - 4ab^4e - 16ab^3c*d + 9a^2b*c^2d + 15a^2b^2c*e) \\
&) / (c^3(49a^3c^2e^2 - 12b^4d^2 - 12a^2b^2e^2 + a^2c^2d^2 + 24ab^3 * d^2e + 48ab^2c^2d^2 - 97a^2b*c^2d^2e) * (8a^3c^6d^3 - b^6c^3d^3 + 6a * b^4c^4d^3 - 12a^2b^2c^5d^3 + a^3b^3c^3e^3 + 3ab^5c^3d^2e + 12 * a^3b*c^5d^2e - 12a^2b^3c^4d^2e - 3a^2b^4c^3d^2e^2 + 6a^3b^2c^4 * d^2e^2)) - (3a^4(4ac - b^2)^{3/2} * ((b^6c^4d^4 - a^6c^3d^3e) / a^4 - ((a^2c^6d^3 - 9ab^2c^5d^3 + 9a^2b*c^5d^2e) / a^4 + (((9a^3b*c^5d^2 - 27a^2b^3c^4d^2 + 27a^3b^2c^4d * e) / a^4 + (((27a^4b^2c^4d - 27 * a^3b^4c^3d + 27a^4b^3c^3e) / a^4 + (27ab^3c^3(3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2 * e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2 * e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2 * e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2)) + (((((((27a^4 * b^2c^4d - 27a^3b^4c^3d + 27a^4b^3c^3e) / a^4 + (27ab^3c^3(3b^3d - 3ab^2 * e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2(4ac - b^2)^{1/2}) + (9b^3c^3(ab^2e - b^2d + 2ac*d) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (4a(4 * ac - b^2)^{1/2} * (36a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2 * (4ac - b^2)^{1/2}) + (3b^3c^3(ab^2e - b^2d + 2ac*d)^2 * (3b^3d - 3 * ab^2e + 12a^2ce - 12abc*d)) / (8a^3(4ac - b^2) * (36a^3c - 9a^2 * b^2))) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2 * b^2)) + (((((((27a^4b^2c^4d - 27a^3b^4c^3d + 27a^4b^3c^3e) / a^4 + (27ab^3c^3(3b^3d - 3ab^2 * e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2))) * (ab^2e - b^2d + 2ac*d)) / (6a^2(4ac - b^2)^{1/2}) + (9b^3c^3(ab^2e - b^2d + 2ac*d) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (4a(4 * ac - b^2)^{1/2} * (36a^3c - 9a^2b^2))) * (3b^3d - 3ab^2e + 12a^2ce - 12abc*d)) / (2(36a^3c - 9a^2b^2)) + (((9a^3
\end{aligned}$$

$$\begin{aligned} & *b*c^5*d^2 - 27*a^2*b^3*c^4*d^2 + 27*a^3*b^2*c^4*d*e)/a^4 + (((27*a^4*b^2*c^4*d - 27*a^3*b^4*c^3*d + 27*a^4*b^3*c^3*e)/a^4 + (27*a*b^3*c^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(a*b*e - b^2*d + 2*a*c*d)^4)/(48*a^7*(4*a*c - b^2)^2))*(8*a^3*c^3*d - 16*b^6*d + 16*a*b^5*e - 132*a^2*b^2*c^2*d + 96*a*b^4*c*d - 92*a^2*b^3*c*e + 116*a^3*b*c^2*e))/(4*c^3*(49*a^3*c*e^2 - 12*b^4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a^2*b*c*d*e))*(8*a^3*c^6*d^3 - b^6*c^3*d^3 + 6*a*b^4*c^4*d^3 - 12*a^2*b^2*c^5*d^3 + a^3*b^3*c^3*e^3 + 3*a*b^5*c^3*d^2*e + 12*a^3*b*c^5*d^2*e - 12*a^2*b^3*c^4*d^2*e - 3*a^2*b^4*c^3*d*e^2 + 6*a^3*b^2*c^4*d*e^2)))*(a*b*e - b^2*d + 2*a*c*d))/(3*a^2*(4*a*c - b^2)^(1/2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.14 \quad \int \frac{x^4(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=723

$$\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) + \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right)}{6 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}}$$

[Out] $\frac{1}{2} e x^2 / c - 1/6 \ln(2^{1/3} c^{1/3} x + (b - (-4ac + b^2)^{1/2})^{1/3}) * (cd - b e + (-2ac e + b^2 e - b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} / (b - (-4ac + b^2)^{1/2})^{1/3} + 1/12 \ln(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x * (b - (-4ac + b^2)^{1/2})^{1/3} + (b - (-4ac + b^2)^{1/2})^{2/3}) * (cd - b e + (-2ac e + b^2 e - b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} / (b - (-4ac + b^2)^{1/2})^{1/3} - 1/6 \arctan(1/3 * (1 - 2 * 2^{1/3} c^{1/3} x) / (b - (-4ac + b^2)^{1/2})^{1/3}) * 3^{1/2} * (cd - b e + (-2ac e + b^2 e - b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} * 3^{1/2} / (b - (-4ac + b^2)^{1/2})^{1/3} - 1/6 \ln(2^{1/3} c^{1/3} x + (b + (-4ac + b^2)^{1/2})^{1/3}) * (cd - b e + (2ac e - b^2 e + b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} / (b + (-4ac + b^2)^{1/2})^{1/3} + 1/12 \ln(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x * (b + (-4ac + b^2)^{1/2})^{1/3} + (b + (-4ac + b^2)^{1/2})^{2/3}) * (cd - b e + (2ac e - b^2 e + b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} / (b + (-4ac + b^2)^{1/2})^{1/3} - 1/6 \arctan(1/3 * (1 - 2 * 2^{1/3} c^{1/3} x) / (b + (-4ac + b^2)^{1/2})^{1/3}) * 3^{1/2} * (cd - b e + (2ac e - b^2 e + b c d) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{5/3} * 3^{1/2} / (b + (-4ac + b^2)^{1/2})^{1/3})$

Rubi [A] time = 1.81, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1502, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) + \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right)}{6 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] $(e x^2) / (2 c) - ((c d - b e - (b c d - b^2 e + 2 a c e) / \text{Sqrt}[b^2 - 4 a c]) * \text{ArcTan}[(1 - (2 * 2^{1/3} c^{1/3} x) / (b - \text{Sqrt}[b^2 - 4 a c]))^{1/3}] / \text{Sqrt}[3]]) /$

$$\begin{aligned} & (2^{(2/3)}\sqrt{3}c^{(5/3)}(b - \sqrt{b^2 - 4ac})^{(1/3)}) - ((cd - be + (b* \\ & cd - b^2e + 2ac*e)/\sqrt{b^2 - 4ac})\text{ArcTan}[(1 - (2^{(1/3)}c^{(1/3)}x) \\ & / (b + \sqrt{b^2 - 4ac})^{(1/3)})/\sqrt{3}]) / (2^{(2/3)}\sqrt{3}c^{(5/3)}(b + \sqrt{ \\ & b^2 - 4ac})^{(1/3)}) - ((cd - be - (b*cd - b^2e + 2ac*e)/\sqrt{b^2 - \\ & 4ac})\text{Log}[(b - \sqrt{b^2 - 4ac})^{(1/3)} + 2^{(1/3)}c^{(1/3)}x]) / (3*2^{(2/3)} \\ & *c^{(5/3)}(b - \sqrt{b^2 - 4ac})^{(1/3)}) - ((cd - be + (b*cd - b^2e + 2* \\ & ac*e)/\sqrt{b^2 - 4ac})\text{Log}[(b + \sqrt{b^2 - 4ac})^{(1/3)} + 2^{(1/3)}c^{(1/ \\ & 3)}x]) / (3*2^{(2/3)}c^{(5/3)}(b + \sqrt{b^2 - 4ac})^{(1/3)}) + ((cd - be - (b \\ & *cd - b^2e + 2ac*e)/\sqrt{b^2 - 4ac})\text{Log}[(b - \sqrt{b^2 - 4ac})^{(2/3)} \\ &) - 2^{(1/3)}c^{(1/3)}(b - \sqrt{b^2 - 4ac})^{(1/3)}x + 2^{(2/3)}c^{(2/3)}x^2]) \\ & / (6*2^{(2/3)}c^{(5/3)}(b - \sqrt{b^2 - 4ac})^{(1/3)}) + ((cd - be + (b*cd - \\ & b^2e + 2ac*e)/\sqrt{b^2 - 4ac})\text{Log}[(b + \sqrt{b^2 - 4ac})^{(2/3)} - 2^{ \\ & (1/3)}c^{(1/3)}(b + \sqrt{b^2 - 4ac})^{(1/3)}x + 2^{(2/3)}c^{(2/3)}x^2]) / (6*2^{ \\ & (2/3)}c^{(5/3)}(b + \sqrt{b^2 - 4ac})^{(1/3)}) \end{aligned}$$

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_.))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6} dx &= \frac{ex^2}{2c} - \frac{\int \frac{x(2ae-2(cd-be)x^3)}{a+bx^3+cx^6} dx}{2c} \\
&= \frac{ex^2}{2c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3} dx}{2c} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}} dx}{2c} \\
&= \frac{ex^2}{2c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} c^{4/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} c^{4/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&= \frac{ex^2}{2c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&= \frac{ex^2}{2c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2} \sqrt[3]{c}x\right)}{3 \cdot 2^{2/3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} \\
&= \frac{ex^2}{2c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} c^{5/3} \sqrt[3]{b-\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{3ex^2 - 2\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3be \log(x-\#1) + \#1^3(-c)d \log(x-\#1) + ae \log(x-\#1)}{2\#1^4c + \#1b}\right]}{6c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 &, (a*e*Log[x - #1] - c*d*Log[x - #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) &])/(6*c)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.01, size = 70, normalized size = 0.10

$$\frac{e x^2}{2c} \frac{\left((be - cd) \operatorname{RootOf}(_Z^6 c + _Z^3 b + a)^4 + \operatorname{RootOf}(_Z^6 c + _Z^3 b + a) a e \right) \ln\left(-\operatorname{RootOf}(_Z^6 c + _Z^3 b + a)\right)}{3c \left(2 \operatorname{RootOf}(_Z^6 c + _Z^3 b + a)^5 c + \operatorname{RootOf}(_Z^6 c + _Z^3 b + a)^2 b \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)

[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+_R*a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e x^2}{2c} - \frac{\int \frac{(cd-be)x^4 - aex}{cx^6 + bx^3 + a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)/c

mupad [B] time = 42.01, size = 13112, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^4(d + e*x^3))/(a + b*x^3 + c*x^6), x)$

[Out] $\log\left(\frac{\left(2^{1/3}\right)\left(2^{2/3}\right)\left(27a^2c^2x(4ac - b^2)(b^2e^2 + 2c^2d^2 - 2ac^2e^2 - 2b^2c^2de) - (27 \cdot 2^{1/3})ab^3c^3(4ac - b^2)^2(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^2e^3 - 3b^7c^2de^2 - 5ab^3c^2e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 - 3b^4c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2c^3d^2e(-4ac - b^2)^3)^{1/2}}{(c^5(4ac - b^2)^3)^{2/3}}\right)^{1/2} \cdot \frac{(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^2e^3 - 3b^7c^2de^2 - 5ab^3c^2e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 - 3b^4c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2c^3d^2e(-4ac - b^2)^3)^{1/2}}{(c^5(4ac - b^2)^3)^{1/3}}\right)^{1/6} - \frac{9a(4ac - b^2)(be - cd)(b^4e^2 - ac^3d^2 + 3a^2c^2e^2 + b^2c^2d^2 - 2b^3c^2de - 4ab^2c^2e^2 + 5ab^2c^2de)}{c^2} \cdot \frac{(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^2e^3 - 3b^7c^2de^2 - 5ab^3c^2e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 - 3b^4c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2c^3d^2e(-4ac - b^2)^3)^{1/2}}{(c^5(4ac - b^2)^3)^{2/3}}\right)^{1/18} - \frac{(a^2x^2(ae^2 + cd^2 - bde)^2(ae - b^2e + bcd))}{c^2} \cdot \frac{(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^2e^3 - 3b^7c^2de^2 - 5ab^3c^2e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 - 3b^4c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2c^3d^2e(-4ac - b^2)^3)^{1/2}}{(c^5(4ac - b^2)^3)^{1/3}}\right)^{1/6}$

$$\begin{aligned}
& b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} + 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^3c^3d^2e(-4ac - b^2)^3)^{1/2})/(54(64a^3c^8 - b^6c^5 + 12ab^4c^6 - 48a^2b^2c^7)))^{1/3} + \log((2^{1/3})(2^{2/3})(27a^2c^2x(4ac - b^2)(b^2e^2 + 2c^2d^2 - 2ace^2 - 2bcd^2e) - (272^{1/3})ab^3c^3(4ac - b^2)^2(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^3c^5d^3 - 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^3e^3 - 3b^7c^2d^2e + 5ab^3c^3e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e + 96a^3b^3c^4d^2e + 3b^4c^2d^2e(-4ac - b^2)^3)^{1/2} - 5a^2b^3c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e + 6a^2c^3d^2e(-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 9ab^3c^3d^2e(-4ac - b^2)^3)^{1/2})/(c^5(4ac - b^2)^3)^{2/3})/2)*(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^3c^5d^3 - 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^3e^3 - 3b^7c^2d^2e + 5ab^3c^3e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e + 96a^3b^3c^4d^2e + 3b^4c^2d^2e(-4ac - b^2)^3)^{1/2} - 5a^2b^3c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e + 6a^2c^3d^2e(-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 9ab^3c^3d^2e(-4ac - b^2)^3)^{1/2})/(c^5(4ac - b^2)^3)^{1/3})/6 - (9a(4ac - b^2)(b^4e^2 - ac^3d^2 + 3a^2c^2e^2 + b^2c^2d^2 - 2b^3c^2d^2e - 4ab^2c^2e^2 + 5ab^3c^2d^2e))/c^2)*(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^3c^5d^3 - 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^3e^3 - 3b^7c^2d^2e + 5ab^3c^3e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e + 96a^3b^3c^4d^2e + 3b^4c^2d^2e(-4ac - b^2)^3)^{1/2} - 5a^2b^3c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e + 6a^2c^3d^2e(-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e(-4ac - b^2)^3)^{1/2} - 12ab^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 9ab^3c^3d^2e(-4ac - b^2)^3)^{1/2})/(c^5(4ac - b^2)^3)^{2/3})/18 - (a^2x(ae^2 + cd^2 - bde)^2(a^2e - b^2e + bcd))/c^2)*(-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3(-4ac - b^2)^3)^{1/2} + 8ab^3c^4d^3 - 16a^2b^3c^5d^3 - 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3(-4ac - b^2)^3)^{1/2} - 11ab^6c^3e^3 - 3b^7c^2d^2e + 5ab^3c^3e^3(-4ac - b^2)^3)^{1/2} - 27ab^4c^3d^2e + 30ab^5c^2d^2e + 96a^3b^3c^4d^2e + 3b^4c^2d^2e(-4ac - b^2)^3)^{1/2} - 5a^2b^3c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e -
\end{aligned}$$

$$\begin{aligned}
& 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c^2 \\
& *d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(54*(64*a^3*c^8 - b^6*c^5 + \\
& 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^{(1/3)} + (e*x^2)/(2*c) + \log(-(2^{(1/3)}*((2 \\
& ^{(2/3)}*(3^{(1/2)}*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2* \\
& a*c*e^2 - 2*b*c*d*e) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2 \\
& *(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3 \\
& c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d \\
& *e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b \\
& ^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b \\
& b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c^2*d^2*e* \\
& (- (4*a*c - b^2)^3)^{(1/2)} + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9* \\
& a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(c^5*(4*a*c - b^2)^3))^{(2/3)}/4*(- \\
& (b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3 \\
& *e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 \\
& - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5* \\
& c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 5 \\
& *a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3 \\
& *c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c^2*d^2*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b \\
& *c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(c^5*(4*a*c - b^2)^3))^{(1/3)}/12 - (9* \\
& a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2* \\
& d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2*(3^{(1/2)}*1i + 1)* \\
& (- (b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3 \\
& *e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 \\
& ^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5 \\
& *c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3 \\
& *c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c^2*d^2*e*(-(\\
& 4*a*c - b^2)^3)^{(1/2)} + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a* \\
& b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(c^5*(4*a*c - b^2)^3))^{(2/3)}/36 - (a \\
& ^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2*((3^{(1/2)}*1i)/ \\
& 2 - 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56* \\
& a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3 \\
& *b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e \\
& + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)
\end{aligned}$$

$$\begin{aligned}
&)^{(1/2)} + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - \\
&96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^{(1/3)} + \log(- (2^{(1/3)}*((2^{(2/3)}*(3^{(1/2)}*1i - 1)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*e) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(c^5*(4*a*c - b^2)^3))^{(2/3)})/4)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(c^5*(4*a*c - b^2)^3))^{(1/3)})/12 - (9*a*(4*a*c - b^2)*(b*e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e - 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2*(3^{(1/2)}*1i + 1)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(c^5*(4*a*c - b^2)^3))^{(2/3)})/36 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*e - b^2*e + b*c*d))/c^2*((3^{(1/2)}*1i)/2 - 1/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*
\end{aligned}$$

$$\begin{aligned}
& d^2e^2 + 96a^3b^2c^4d^2e^2 + 3b^4c^4d^2e^2(-4ac - b^2)^3)^{1/2} - 5a^2 \\
& *b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3 \\
& *d^2e^2 + 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e^2(-4ac \\
& c - b^2)^3)^{1/2} - 12a^2b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 9a^2b^2c^3 \\
& *d^2e^2(-4ac - b^2)^3)^{1/2})/(54(64a^3c^8 - b^6c^5 + 12a^2b^4c^6 - \\
& 48a^2b^2c^7)))^{1/3} - \log(- (2^{1/3})((2^{2/3})(3^{1/2})1i + 1)(27a^2 \\
& 2c^2x(4ac - b^2)(b^2e^2 + 2c^2d^2 - 2ac^2e^2 - 2b^2c^2d^2e) - (27a^2 \\
& 1/3)a^2b^2c^3(3^{1/2})1i - 1)(4ac - b^2)^2(-b^8e^3 + 16a^4c^4e^3 - \\
& b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2 \\
& b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6 \\
& c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac \\
& - b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 - 5a^2b^3c^2e^3(-4ac \\
& - b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2 \\
& e^2 - 3b^4c^4d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b \\
& ^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2 \\
& 2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 12a \\
& a^2b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9a^2b^2c^3d^2e^2(-4ac - b^2)^ \\
& 3)^{1/2})/(c^5(4ac - b^2)^3)^{2/3})/4*(-(b^8e^3 + 16a^4c^4e^3 - b^ \\
& 5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2 \\
& c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2 \\
& 2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - \\
& b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 - 5a^2b^3c^2e^3(-4ac - b \\
& ^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2 \\
& e^2 - 3b^4c^4d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2) \\
& ^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(- \\
& -4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9a^2b^2c^3d^2e^2(-4ac - b^2)^3) \\
& ^{1/2})/(c^5(4ac - b^2)^3)^{1/3})/12 + (9a^2(4ac - b^2)(b^4e^2 - a^2c^3d^2 \\
& + 3a^2c^2e^2 + b^2c^2d^2 - 2b^3c^2d^2e - 4a^2b^2c^2 \\
& e^2 + 5a^2b^2c^2d^2e))/c^2*(3^{1/2})1i - 1)*(-(b^8e^3 + 16a^4c^4e^3 - b \\
& ^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2 \\
& c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2 \\
& 2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac - \\
& b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 - 5a^2b^3c^2e^3(-4ac - \\
& b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2 \\
& e^2 - 3b^4c^4d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2) \\
& ^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(- \\
& -4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 12a^2 \\
& b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9a^2b^2c^3d^2e^2(-4ac - b^2)^3) \\
& ^{1/2})/(c^5(4ac - b^2)^3)^{2/3})/36 - (a^2x(a^2e^2 + c^2d^2 - b^2d^2e) \\
& ^2*(ac^2e - b^2e + b^2cd))/c^2*((3^{1/2})1i)/2 + 1/2)*(-(b^8e^3 + 16a^4c^4 \\
& e^3 - b^5c^3d^3 + b^5e^3(-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - \\
& 16a^2b^2c^5d^3 + 2ac^4d^3(-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e \\
& + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 - b^2c^3d^3(-4ac \\
& - b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 - 5a^2b^3c^2e^3(-4ac -
\end{aligned}$$

$$\begin{aligned}
& -(4ac - b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 - 3b^4c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 5a^2b^2c^2e^3(-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 - 6a^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3c^2d^2e^2(-4ac - b^2)^3)^{1/2} + 12a^2b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9a^2b^2c^3d^2e^2(-4ac - b^2)^3)^{1/2} / (54(64a^3c^8 - b^6c^5 + 12a^2b^4c^6 - 48a^2b^2c^7))^{1/3} - \log(- (2^{1/3}) * ((2^{2/3}) * (3^{1/2}) * i + 1) * (27a^2c^2 * x * (4ac - b^2) * (b^2e^2 + 2c^2d^2 - 2a^2c^2e^2 - 2b^2c^2d^2e) - (27 * 2^{1/3}) * a^2b^2c^3 * (3^{1/2}) * i - 1) * (4ac - b^2)^2 * (-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3 * (-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2c^5d^3 - 2a^2c^4d^3 * (-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3 * (-4ac - b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 + 5a^2b^3c^2e^3 * (-4ac - b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 + 3b^4c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 5a^2b^2c^2e^3 * (-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 + 6a^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 12a^2b^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} + 9a^2b^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} / (c^5 * (4ac - b^2)^3)^{2/3} / 4) * (-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3 * (-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2c^5d^3 - 2a^2c^4d^3 * (-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3 * (-4ac - b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 + 5a^2b^3c^2e^3 * (-4ac - b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 + 3b^4c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 5a^2b^2c^2e^3 * (-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 + 6a^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 12a^2b^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} + 9a^2b^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} / (c^5 * (4ac - b^2)^3)^{1/3} / 12 + (9a^2 * (4ac - b^2) * (b^2e - cd) * (b^4e^2 - a^2c^3d^2 + 3a^2c^2e^2 + b^2c^2d^2 - 2b^3c^2d^2e - 4a^2b^2c^2e^2 + 5a^2b^2c^2d^2e)) / c^2) * (3^{1/2}) * i - 1) * (-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3 * (-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2c^5d^3 - 2a^2c^4d^3 * (-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3 * (-4ac - b^2)^3)^{1/2} - 11a^2b^6c^2e^3 - 3b^7c^2d^2e^2 + 5a^2b^3c^2e^3 * (-4ac - b^2)^3)^{1/2} - 27a^2b^4c^3d^2e + 30a^2b^5c^2d^2e^2 + 96a^3b^2c^4d^2e^2 + 3b^4c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 5a^2b^2c^2e^3 * (-4ac - b^2)^3)^{1/2} + 72a^2b^2c^4d^2e - 96a^2b^3c^3d^2e^2 + 6a^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} - 3b^3c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} - 12a^2b^2c^2d^2e^2 * (-4ac - b^2)^3)^{1/2} + 9a^2b^2c^3d^2e^2 * (-4ac - b^2)^3)^{1/2} / (c^5 * (4ac - b^2)^3)^{2/3} / 36 - (a^2 * x * (a^2e^2 + c^2d^2 - b^2d^2e) * (a^2c^2e - b^2e + b^2cd)) / c^2) * ((3^{1/2}) * i) / 2 + 1/2) * (-b^8e^3 + 16a^4c^4e^3 - b^5c^3d^3 - b^5e^3 * (-4ac - b^2)^3)^{1/2} + 8a^2b^3c^4d^3 - 16a^2b^2c^5d^3 - 2a^2c^4d^3 * (-4ac - b^2)^3)^{1/2} - 48a^3c^5d^2e + 3b^6c^2d^2e + 41a^2b^4c^2e^3 - 56a^3b^2c^3e^3 + b^2c^3d^3 * (-4ac - b^2)^3)^{1/2}
\end{aligned}$$

$$\begin{aligned} &^3)^{(1/2)} - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3 \\ &*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4* \\ &a*c - b^2)^3)^{(1/2)} - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\ &))/ (54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^{(1/3)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.15 \quad \int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=718

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}}$$

[Out] $e*x/c + 1/6*\ln(2^{(1/3)*c^{(1/3)*x} + (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * (c*d - b*e + (-2*a*c*e + b^2*e - b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} / (b - (-4*a*c + b^2)^{(1/2)})^{(2/3)} - 1/12*\ln(2^{(2/3)*c^{(2/3)*x^2 - 2^{(1/3)*c^{(1/3)*x} * (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)} + (b - (-4*a*c + b^2)^{(1/2)})^{(2/3)}) * (c*d - b*e + (-2*a*c*e + b^2*e - b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} / (b - (-4*a*c + b^2)^{(1/2)})^{(2/3)} - 1/6*\arctan(1/3*(1 - 2*2^{(1/3)*c^{(1/3)*x} / (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * 3^{(1/2)}) * (c*d - b*e + (-2*a*c*e + b^2*e - b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} * 3^{(1/2)} / (b - (-4*a*c + b^2)^{(1/2)})^{(2/3)} + 1/6*\ln(2^{(1/3)*c^{(1/3)*x} + (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * (c*d - b*e + (2*a*c*e - b^2*e + b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} / (b + (-4*a*c + b^2)^{(1/2)})^{(2/3)} - 1/12*\ln(2^{(2/3)*c^{(2/3)*x^2 - 2^{(1/3)*c^{(1/3)*x} * (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)} + (b + (-4*a*c + b^2)^{(1/2)})^{(2/3)}) * (c*d - b*e + (2*a*c*e - b^2*e + b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} / (b + (-4*a*c + b^2)^{(1/2)})^{(2/3)} - 1/6*\arctan(1/3*(1 - 2*2^{(1/3)*c^{(1/3)*x} / (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * 3^{(1/2)}) * (c*d - b*e + (2*a*c*e - b^2*e + b*c*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(2/3)} / c^{(4/3)} * 3^{(1/2)} / (b + (-4*a*c + b^2)^{(1/2)})^{(2/3)})$

Rubi [A] time = 1.46, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1502, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}\right)}{6\sqrt[3]{2} c^{4/3} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] $(e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(1 - (2*2^{(1/3)*c^{(1/3)*x})/(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)}$

$$3) \sqrt[3]{3} c^{4/3} (b - \sqrt{b^2 - 4ac})^{2/3} - ((cd - b^2e + (bcd - b^2e + 2ace)/\sqrt{b^2 - 4ac}) \operatorname{ArcTan}[(1 - (2^{1/3}c^{1/3}x)/(b + \sqrt{b^2 - 4ac})^{1/3})/\sqrt[3]{3}]) / (2^{1/3} \sqrt[3]{3} c^{4/3} (b + \sqrt{b^2 - 4ac})^{2/3}) + ((cd - b^2e - (bcd - b^2e + 2ace)/\sqrt{b^2 - 4ac}) \operatorname{Log}[(b - \sqrt{b^2 - 4ac})^{1/3} + 2^{1/3}c^{1/3}x] / (3^{2/3}c^{4/3} (b - \sqrt{b^2 - 4ac})^{2/3})) + ((cd - b^2e + (bcd - b^2e + 2ace)/\sqrt{b^2 - 4ac}) \operatorname{Log}[(b + \sqrt{b^2 - 4ac})^{1/3} + 2^{1/3}c^{1/3}x] / (3^{2/3}c^{4/3} (b + \sqrt{b^2 - 4ac})^{2/3})) - ((cd - b^2e - (bcd - b^2e + 2ace)/\sqrt{b^2 - 4ac}) \operatorname{Log}[(b - \sqrt{b^2 - 4ac})^{2/3} - 2^{1/3}c^{1/3}(b - \sqrt{b^2 - 4ac})^{1/3}x + 2^{2/3}c^{2/3}x^2]) / (6^{2/3}c^{4/3} (b - \sqrt{b^2 - 4ac})^{2/3})) - ((cd - b^2e + (bcd - b^2e + 2ace)/\sqrt{b^2 - 4ac}) \operatorname{Log}[(b + \sqrt{b^2 - 4ac})^{2/3} - 2^{1/3}c^{1/3}(b + \sqrt{b^2 - 4ac})^{1/3}x + 2^{2/3}c^{2/3}x^2]) / (6^{2/3}c^{4/3} (b + \sqrt{b^2 - 4ac})^{2/3}))$$

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b^2e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rule 1502

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6} dx &= \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6} dx}{c} \\
&= \frac{ex}{c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3} dx}{2c} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3} dx}{2c} \\
&= \frac{ex}{c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{2^{2/3}\sqrt[3]{b}}{\left(b-\sqrt{b^2-4ac}\right)^{2/3}} dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} \\
&= \frac{ex}{c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&= \frac{ex}{c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b-\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b+\sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \\
&= \frac{ex}{c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt[3]{3}}\right)}{\sqrt[3]{2}\sqrt[3]{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt[3]{3}}\right)}{\sqrt[3]{2}\sqrt[3]{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3be \log(x-\#1) + \#1^3(-c)d \log(x-\#1) + ae \log(x-\#1)}{2\#1^5c + \#1^2b}\& \right]}{3c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 &, (a*e*Log[x - #1] - c*d*Log[x - #1] + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) &]/(3*c)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^3 + d)x^3}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)*x^3/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 67, normalized size = 0.09

$$\frac{ex}{c} + \frac{\left((-be + cd) \operatorname{RootOf}(_Z^6c + _Z^3b + a)^3 - ae\right) \ln\left(-\operatorname{RootOf}(_Z^6c + _Z^3b + a) + x\right)}{3c \left(2 \operatorname{RootOf}(_Z^6c + _Z^3b + a)^5 c + \operatorname{RootOf}(_Z^6c + _Z^3b + a)^2 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)

[Out] 1/c*e*x+1/3/c*sum(((b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c} - \int \frac{(cd-be)x^3-ae}{cx^6+bx^3+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c

mupad [B] time = 30.15, size = 11453, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^3(d + ex^3))/(a + bx^3 + cx^6), x)$

[Out] $\log((3axx(ab^4e^4 - 2ac^4d^4 - b^5d^3e + 2a^3c^2e^4 + b^2c^3d^4 - 4a^2b^2c^3e^4 - 3b^3c^2d^3e + 3b^4c^2d^2e^2 + 8ab^3c^3d^3e + 2ab^3c^3d^3e + 4a^2b^2c^2d^2e^3 - 9ab^2c^2d^2e^2)))/c - (2^{(2/3)}) \cdot ((2^{(1/3)}) \cdot (81ac^3d^3x(4ac - b^2)^2 - (81 \cdot 2^{(2/3)}) \cdot ab^3c^3(4ac - b^2)^2 \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e^2 - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(c^4(4ac - b^2)^3)^{(1/3)})/2) \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(c^4(4ac - b^2)^3)^{(2/3)})/18 + (9a(4ac - b^2)(b^4e^3 - b^3c^3d^3 + a^2c^2e^3 + 3b^2c^2d^2e - 3ab^2c^3e^3 - 3ac^3d^2e - 3b^3c^3d^2e + 6ab^3c^2d^2e))/c) \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(c^4(4ac - b^2)^3)^{(1/3)})/6) \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6)))^{(1/3)} + \log((3axx(ab^4e^4 - 2ac^4d^4 - b^5d^3e + 2a^3c^2e^4 + b^2c^3d^4 - 4a^2b^2c^3e^4 - 3b^3c^2d^3e + 3b^4c^2d^2e^2 + 8ab^3c^3d^3e - 3b^3c^3d^3e + 4a^2b^2c^2d^2e^3 - 9ab^2c^2d^2e^2)))/c - (2^{(2/3)}) \cdot ((2^{(1/3)}) \cdot (81ac^3d^3x(4ac - b^2)^2 - (81 \cdot 2^{(2/3)}) \cdot ab^3c^3(4ac - b^2)^2 \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e^2 - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(c^4(4ac - b^2)^3)^{(1/3)})/6) \cdot ((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e + 3b^5c^2d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2e - 4ab^2c^3e^3 \cdot (-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4c^2d^2e + 48a^2b^3c^4d^2e - 6ac^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e \cdot (-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e + 3b^2c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)} + 9ab^3c^2d^2e \cdot (-4ac - b^2)^3)^{(1/2)})/(54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6)))^{(1/3)}$

$$\begin{aligned}
& *e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2)/c - (2^{(2/3)} \\
& / 3) * ((2^{(1/3)} * (81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^{(2/3)}*a*b*c^3*(4*a*c - b \\
& ^2)^2 * ((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3 * (-4*a*c - b^2)^3) \\
& ^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3 * (-4*a*c - b^2)^3) \\
& ^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2 \\
& *e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c* \\
& e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48 \\
& *a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e * (-4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c*d*e^2 * (\\
& -4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e * (-4*a*c - \\
& b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)}) / (c^4 * (4*a*c - b^ \\
& 2)^3)^{(1/3)} / 2) * ((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3 * (-4*a* \\
& c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3 * (-4*a*c \\
& - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 \\
& - 2*a^2*c^2*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + \\
& 4*a*b^2*c*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2 \\
& *d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e * (-4*a*c - b^2)^3)^{(1/2)} + 3*b^ \\
& 3*c*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e \\
& * (-4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)}) / (c^4 * \\
& (4*a*c - b^2)^3)^{(2/3)} / 18 + (9*a * (4*a*c - b^2) * (b^4*e^3 - b*c^3*d^3 + a^2 \\
& *c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 \\
& + 6*a*b*c^2*d*e^2)) / c) * ((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3 * (\\
& -4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3 * (\\
& -4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^ \\
& 2*e^3 - 2*a^2*c^2*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d \\
& *e^2 + 4*a*b^2*c*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b \\
& ^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e * (-4*a*c - b^2)^3)^{(1/2)} \\
& + 3*b^3*c*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2 \\
& *d^2*e * (-4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)}) \\
& / (c^4 * (4*a*c - b^2)^3)^{(1/3)} / 6) * ((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 \\
& - b^4*e^3 * (-4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b \\
& *c^3*d^3 * (-4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32 \\
& *a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 \\
& - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3 * (-4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2 \\
& *e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e * (-4*a*c - b^2 \\
&)^3)^{(1/2)} + 3*b^3*c*d*e^2 * (-4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 \\
& - 3*b^2*c^2*d^2*e * (-4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2 * (-4*a*c - b^2 \\
&)^3)^{(1/2)}) / (54 * (64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^{(1 \\
& / 3)} + \log((2^{(2/3)} * (3^{(1/2)} * 1i - 1) * ((2^{(1/3)} * (3^{(1/2)} * 1i + 1) * (81*a*c^3*d* \\
& x*(4*a*c - b^2)^2 - (81*2^{(2/3)}*a*b*c^3*(3^{(1/2)} * 1i - 1) * (4*a*c - b^2)^2 * ((\\
& b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3 * (-4*a*c - b^2)^3)^{(1/2)} + \\
& 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3 * (-4*a*c - b^2)^3)^{(1/2)} + \\
& 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3 * (\\
& -4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3 * (\\
& -4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c \\
& ^4*d^2*e - 6*a*c^3*d^2*e * (-4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c*d*e^2 * (-4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))/(c^4*(4*a*c - b^2)^3)^{(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(c^4*(4*a*c - b^2)^3)^{(2/3))/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(c^4*(4*a*c - b^2)^3)^{(1/3))/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c)*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((2^(1/3)*(3^(1/2)*1i + 1)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(c^4*(4*a*c - b^2)^3)^{(1/3))/4)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a
\end{aligned}$$

$$\begin{aligned}
& ^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a* \\
& b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^ \\
& 2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b^3*c*d \\
& *e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))}/(c^4*(4*a* \\
& c - b^2)^3))^{(2/3)}/36 - (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^2* \\
& e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6*a \\
& *b*c^2*d*e^2))/c*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 \\
& - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 \\
& + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^ \\
& 2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 3*b \\
& ^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2* \\
& e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))}/(c^4 \\
& *(4*a*c - b^2)^3))^{(1/3)}/12 + (3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 \\
& + 2*a^3*c^2*e^4 + b^2*c^3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c \\
& *d^2*e^2 + 8*a*b*c^3*d^3*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2* \\
& c^2*d^2*e^2))/c*((3^(1/2)*1i)/2 - 1/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^ \\
& 3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e \\
& ^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2* \\
& e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5* \\
& c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c \\
& ^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3* \\
& d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a*b*c^2*d*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2))}/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6 \\
&)))^{(1/3)} - \log(- (2^(2/3)*(3^(1/2)*1i + 1)*((2^(1/3)*(3^(1/2)*1i - 1)*(81* \\
& a*c^3*d*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b \\
& ^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^ \\
& ^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2 \\
& *e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c* \\
& e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48 \\
& *a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^3*c*d*e^2*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))}/(c^4*(4*a*c - b^ \\
& 2)^3))^{(1/3)}/4*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 \\
& + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - \\
& 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2 \\
& *d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 3*b^ \\
& 3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))}/(c^4*
\end{aligned}$$

$$\begin{aligned}
& ((4ac - b^2)^3)^{(2/3)}/36 + (9a(4ac - b^2)(b^4e^3 - b^3c^3d^3 + a^2 \\
& *c^2e^3 + 3b^2c^2d^2e - 3ab^2c^2e^3 - 3ac^3d^2e - 3b^3c^3d^2e^2 \\
& + 6ab^2c^2d^2e^2)/c)((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 + b^4e^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 - b^3c^3d^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2 \\
& *e^3 + 2a^2c^2e^3(-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d \\
& *e^2 - 4ab^2c^3e^3(-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4 \\
& *c^2d^2e^2 + 48a^2b^3c^4d^2e - 6ac^3d^2e(-4ac - b^2)^3)^{(1/2)} \\
& - 3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e^2 + 3b^2c^2 \\
& *d^2e^2(-4ac - b^2)^3)^{(1/2)} + 9ab^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)}) \\
& / (c^4(4ac - b^2)^3)^{(1/3)}/12 - (3axx(ab^4e^4 - 2ac^4d^4 - b^5d \\
& *e^3 + 2a^3c^2e^4 + b^2c^3d^4 - 4a^2b^2c^4e - 3b^3c^2d^3e + 3b^4 \\
& *c^2d^2e^2 + 8ab^3c^3d^3e + 2ab^3c^3d^3e^3 + 4a^2b^3c^2d^3e^3 - 9a \\
& *b^2c^2d^2e^2)/c)((3^{(1/2)}*i)/2 + 1/2)((b^7e^3 - 16a^2c^5d^3 - b^4 \\
& *c^3d^3 + b^4e^3(-4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3 \\
& *e^3 - b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2 \\
& *d^2e + 32a^2b^3c^2e^3 + 2a^2c^2e^3(-4ac - b^2)^3)^{(1/2)} - 10a \\
& *b^5c^3e^3 - 3b^6c^3d^2e^2 - 4ab^2c^3e^3(-4ac - b^2)^3)^{(1/2)} - 24a \\
& *b^3c^3d^2e + 27ab^4c^2d^2e^2 + 48a^2b^3c^4d^2e - 6ac^3d^2e(-4 \\
& *ac - b^2)^3)^{(1/2)} - 3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 72a^2b^2 \\
& *c^3d^2e^2 + 3b^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} + 9ab^2c^2d^2e^2(- \\
& (4ac - b^2)^3)^{(1/2)})/(54(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2 \\
& *c^6)))^{(1/3)} - \log(-(2^{(2/3)}(3^{(1/2)}*i + 1)((2^{(1/3)}(3^{(1/2)}*i - 1) \\
& *(81ac^3d^2x(4ac - b^2)^2 + (81*2^{(2/3)})*ab^3c^3(3^{(1/2)}*i + 1)*(4ac \\
& - b^2)^2((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 - b^4e^3(-4ac - b^2)^3)^{(1/2)} \\
& + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 + b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} \\
& + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2e^3 - 2a^2c^2e^3(-4ac - b^2)^3)^{(1/2)} \\
& - 10ab^5c^3e^3 - 3b^6c^3d^2e^2 + 4ab^2c^3e^3(-4ac - b^2)^3)^{(1/2)} - 24ab^3 \\
& *c^3d^2e + 27ab^4c^2d^2e^2 + 48a^2b^3c^4d^2e - 6ac^3d^2e(-4ac - b^2)^3)^{(1/2)} \\
& + 3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e^2 - 3b^2c^2d^2e^2(-4 \\
& *ac - b^2)^3)^{(1/2)} - 9ab^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)})/(c^4(4ac \\
& - b^2)^3)^{(1/3)}/4)((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 - b^4e^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 + b^3c^3d^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2 \\
& *e^3 - 2a^2c^2e^3(-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2 \\
& *e^2 + 4ab^2c^3e^3(-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4 \\
& *c^2d^2e^2 + 48a^2b^3c^4d^2e + 6ac^3d^2e(-4ac - b^2)^3)^{(1/2)} + 3b^3 \\
& *c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e^2 - 3b^2c^2d^2e^2(-4 \\
& *ac - b^2)^3)^{(1/2)} - 9ab^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)})/ \\
& (c^4(4ac - b^2)^3)^{(2/3)}/36 + (9a(4ac - b^2)(b^4e^3 - b^3c^3d^3 \\
& + a^2c^2e^3 + 3b^2c^2d^2e - 3ab^2c^2e^3 - 3ac^3d^2e - 3b^3c^3d^2 \\
& *e^2 + 6ab^2c^2d^2e^2)/c)((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 - b^4e^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 + b^3c^3d^3(- \\
& (4ac - b^2)^3)^{(1/2)} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e + 32a^2b^3c^2 \\
& *e^3 - 2a^2c^2e^3(-4ac - b^2)^3)^{(1/2)} - 10ab^5c^3e^3 - 3b^6c^3d^2 \\
& *e^2 + 4ab^2c^3e^3(-4ac - b^2)^3)^{(1/2)} - 24ab^3c^3d^2e + 27ab^4 \\
& *c^2d^2e^2 + 48a^2b^3c^4d^2e + 6ac^3d^2e(-4ac - b^2)^3)^{(1/2)} + 3b^3 \\
& *c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 72a^2b^2c^3d^2e^2 - 3b^2c^2d^2e^2(-4 \\
& *ac - b^2)^3)^{(1/2)} - 9ab^2c^2d^2e^2(-4ac - b^2)^3)^{(1/2)})/
\end{aligned}$$

$$\begin{aligned} &^3c^2e^3 - 2a^2c^2e^3(-4ac - b^2)^3)^{1/2} - 10ab^5c^3e^3 - 3b^6cd^2e^2 + 4ab^2c^2e^3(-4ac - b^2)^3)^{1/2} - 24ab^3c^3d^2e^2 + 27ab^4c^2d^2e^2 + 48a^2b^4c^4d^2e^2 + 6ac^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3cd^2e^2(-4ac - b^2)^3)^{1/2} - 72a^2b^2c^3d^2e^2 - 3b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2cd^2e^2(-4ac - b^2)^3)^{1/2})/(c^4(4ac - b^2)^3)^{1/3})/12 - (3ax*(ab^4e^4 - 2ac^4d^4 - b^5d^3e^3 + 2a^3c^2e^4 + b^2c^3d^4 - 4a^2b^2c^4e^4 - 3b^3c^2d^3e^3 + 3b^4cd^2e^2 + 8abc^3d^3e^3 + 2ab^3cd^3e^3 + 4a^2b^2c^2d^3e^3 - 9ab^2c^2d^2e^2))/c*((3^{1/2}*i)/2 + 1/2)*((b^7e^3 - 16a^2c^5d^3 - b^4c^3d^3 - b^4e^3(-4ac - b^2)^3)^{1/2} + 8ab^2c^4d^3 - 32a^3b^3c^3e^3 + b^3cd^3(-4ac - b^2)^3)^{1/2} + 48a^3c^4d^2e^2 + 3b^5c^2d^2e^2 + 32a^2b^3c^2e^3 - 2a^2c^2e^3(-4ac - b^2)^3)^{1/2} - 10ab^5c^3e^3 - 3b^6cd^2e^2 + 4ab^2c^2e^3(-4ac - b^2)^3)^{1/2} - 24ab^3c^3d^2e^2 + 27ab^4c^2d^2e^2 + 48a^2b^4c^4d^2e^2 + 6ac^3d^2e^2(-4ac - b^2)^3)^{1/2} + 3b^3cd^2e^2(-4ac - b^2)^3)^{1/2} - 72a^2b^2c^3d^2e^2 - 3b^2c^2d^2e^2(-4ac - b^2)^3)^{1/2} - 9ab^2cd^2e^2(-4ac - b^2)^3)^{1/2})/(54*(64a^3c^7 - b^6c^4 + 12ab^4c^5 - 48a^2b^2c^6)))^{1/3} + (ex)/c \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.16 \quad \int \frac{x(d+ex^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b-\sqrt{b^2-4ac}} + \left(b-\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b+\sqrt{b^2-4ac}} + \left(b+\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

[Out] $-\frac{1}{6} \ln(2^{1/3} c^{1/3} x + (b - (-4ac + b^2)^{1/2})^{1/3}) * (e + (-b^2 + 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} / (b - (-4ac + b^2)^{1/2})^{1/3} + \frac{1}{12} \ln(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x * (b - (-4ac + b^2)^{1/2})^{1/3} + (b - (-4ac + b^2)^{1/2})^{2/3}) * (e + (-b^2 + 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} / (b - (-4ac + b^2)^{1/2})^{1/3} - \frac{1}{6} \arctan(1/3 * (1 - 2^{1/3} c^{1/3} x) / (b - (-4ac + b^2)^{1/2})^{1/3}) * 3^{1/2}) * (e + (-b^2 + 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} / (b - (-4ac + b^2)^{1/2})^{1/3} - \frac{1}{6} \ln(2^{1/3} c^{1/3} x + (b + (-4ac + b^2)^{1/2})^{1/3}) * (e + (b^2 - 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} / (b + (-4ac + b^2)^{1/2})^{1/3} + \frac{1}{12} \ln(2^{2/3} c^{2/3} x^2 - 2^{1/3} c^{1/3} x * (b + (-4ac + b^2)^{1/2})^{1/3} + (b + (-4ac + b^2)^{1/2})^{2/3}) * (e + (b^2 - 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} / (b + (-4ac + b^2)^{1/2})^{1/3} - \frac{1}{6} \arctan(1/3 * (1 - 2^{1/3} c^{1/3} x) / (b + (-4ac + b^2)^{1/2})^{1/3}) * 3^{1/2}) * (e + (b^2 - 2cd) / (-4ac + b^2)^{1/2}) * 2^{1/3} / c^{2/3} * 3^{1/2} / (b + (-4ac + b^2)^{1/2})^{1/3}$

Rubi [A] time = 0.73, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b-\sqrt{b^2-4ac}} + \left(b-\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b+\sqrt{b^2-4ac}} + \left(b+\sqrt{b^2-4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] $-\frac{((e + (2cd - b^2) / \sqrt{b^2 - 4ac}) / \sqrt{b^2 - 4ac}) * \text{ArcTan}[(1 - (2^{1/3} c^{1/3} x) / (b - \sqrt{b^2 - 4ac})^{1/3}) / \sqrt{3}]}{(2^{2/3} \sqrt{3} c^{2/3} * (b - \sqrt{b^2 - 4ac})^{1/3})} - \frac{((e - (2cd - b^2) / \sqrt{b^2 - 4ac}) / \sqrt{b^2 - 4ac}) * \text{ArcTan}[(1 - (2^{1/3} c^{1/3} x) / (b + \sqrt{b^2 - 4ac})^{1/3}) / \sqrt{3}]}{(2^{2/3} \sqrt{3} c^{2/3} * (b + \sqrt{b^2 - 4ac})^{1/3})}$

$$\begin{aligned} & \text{rt}[3]*c^{(2/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 \\ & - 4*a*c])* \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)*x})/(3*2^{(2/3)} \\ & *c^{(2/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - \\ & 4*a*c])* \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)*x})/(3*2^{(2/3)}* \\ & c^{(2/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a \\ & *c])* \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4* \\ & a*c])^{(1/3)*x} + 2^{(2/3)}*c^{(2/3)*x^2}]/(6*2^{(2/3)}*c^{(2/3)}*(b - \text{Sqrt}[b^2 - 4* \\ & a*c])^{(1/3)}) + ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])* \text{Log}[(b + \text{Sqrt}[b^2 - 4 \\ & *a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)*x} + 2^{(2/3)}*c^{ \\ & (2/3)*x^2}]/(6*2^{(2/3)}*c^{(2/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) \end{aligned}$$
Rule 31

$$\text{Int}[\{(a_ + (b_)*(x_))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ /; FreeQ}\{a, b\}, x]$$
Rule 204

$$\text{Int}[\{(a_ + (b_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \text{ /; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$$
Rule 292

$$\text{Int}[(x_)/\{(a_ + (b_)*(x_)^3), x_Symbol] \rightarrow -\text{Dist}[(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{(-1)}, \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] \text{ /; FreeQ}\{a, b\}, x]$$
Rule 617

$$\text{Int}[\{(a_ + (b_)*(x_ + (c_)*(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S \text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] \text{ /; RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) \ \&\& \ \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$
Rule 628

$$\text{Int}[\{(d_ + (e_)*(x_))/\{(a_ + (b_)*(x_ + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$
Rule 634

$$\text{Int}[\{(d_ + (e_)*(x_))/\{(a_ + (b_)*(x_ + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x]$$

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1510

$\text{Int}[(((f_)*(x_))^{(m_)}*((d_)+(e_)*(x_)^{(n_)}))/((a_)+(b_)*(x_)^{(n_)}+(c_)*(x_)^{(n2_)}), x_Symbol] :> \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\int \frac{x(d + ex^3)}{a + bx^3 + cx^6} dx = \frac{1}{2} \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx + \frac{1}{2} \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx$$

$$= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} \sqrt[3]{c} \sqrt[3]{b + \sqrt{b^2 - 4ac}}} + \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} - \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} \sqrt[3]{c} \sqrt[3]{b + \sqrt{b^2 - 4ac}}}$$

$$= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x \right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} - \sqrt[3]{2} \sqrt[3]{c} x \right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}}$$

$$= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x \right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} - \sqrt[3]{2} \sqrt[3]{c} x \right)}{3 \cdot 2^{2/3} c^{2/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}}$$

$$= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3} c^{2/3} \sqrt[3]{b - \sqrt{b^2 - 4ac}}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{2} \sqrt[3]{c} x}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}} \right)}{2^{2/3} \sqrt{3} c^{2/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}}} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right)}{3}$$

Mathematica [C] time = 0.03, size = 59, normalized size = 0.09

$$\frac{1}{3} \text{RootSum} \left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^3 e \log(x - \#1) + d \log(x - \#1)}{2\#1^4 c + \#1 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6), x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) &]/3

fricas [B] time = 93.49, size = 13607, normalized size = 21.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] $\frac{2}{3} \sqrt{3} \left(\frac{1}{2} \right)^{1/3} \left(- (c^2 d^3 - 3 a c d e^2 + a b e^3 + (a b^2 c^2 - 4 a^2 c^3) \sqrt{(b^2 c^4 d^6 - 12 a b c^4 d^5 e + 6 (a b^2 c^3 + 6 a^2 c^4) d^4 e^2 - 2 (a b^3 c^2 + 16 a^2 b c^3) d^3 e^3 + 3 (7 a^2 b^2 c^2 - 8 a^3 c^3) d^2 e^4 - 6 (a^2 b^3 c - 2 a^3 b c^2) d e^5 + (a^2 b^4 - 4 a^3 b^2 c + 4 a^4 c^2) e^6}) / (a^2 b^6 c^4 - 12 a^3 b^4 c^5 + 48 a^4 b^2 c^6 - 64 a^5 c^7) \right) \arctan \left(- \frac{1}{3} \left(\frac{1}{2} \right)^{5/6} \sqrt{3} \left(2 (a b^4 c^3 - 8 a^2 b^2 c^4 + 16 a^3 c^5) d - (a b^5 c^2 - 8 a^2 b^3 c^3 + 16 a^3 b c^4) e \right) \sqrt{(b^2 c^4 d^6 - 12 a b c^4 d^5 e + 6 (a b^2 c^3 + 6 a^2 c^4) d^4 e^2 - 2 (a b^3 c^2 + 16 a^2 b c^3) d^3 e^3 + 3 (7 a^2 b^2 c^2 - 8 a^3 c^3) d^2 e^4 - 6 (a^2 b^3 c - 2 a^3 b c^2) d e^5 + (a^2 b^4 - 4 a^3 b^2 c + 4 a^4 c^2) e^6} / (a^2 b^6 c^4 - 12 a^3 b^4 c^5 + 48 a^4 b^2 c^6 - 64 a^5 c^7) \right) - \sqrt{3} \left((b^3 c^2 - 4 a b c^3) d^3 e - 6 (a b^2 c^2 - 4 a^2 c^3) d^2 e^2 + 3 (a b^3 c - 4 a^2 b c^2) d e^3 - (a b^4 - 6 a^2 b^2 c + 8 a^3 c^2) e^4 \right) \left(- (c^2 d^3 - 3 a c d e^2 + a b e^3 + (a b^2 c^2 - 4 a^2 c^3) \sqrt{(b^2 c^4 d^6 - 12 a b c^4 d^5 e + 6 (a b^2 c^3 + 6 a^2 c^4) d^4 e^2 - 2 (a b^3 c^2 + 16 a^2 b c^3) d^3 e^3 + 3 (7 a^2 b^2 c^2 - 8 a^3 c^3) d^2 e^4 - 6 (a^2 b^3 c - 2 a^3 b c^2) d e^5 + (a^2 b^4 - 4 a^3 b^2 c + 4 a^4 c^2) e^6} / (a^2 b^6 c^4 - 12 a^3 b^4 c^5 + 48 a^4 b^2 c^6 - 64 a^5 c^7) \right) \right) / (a b^2 c^2 - 4 a^2 c^3) \right)^{1/3} \sqrt{(2 (b c^4 d^7 - 2 (b^2 c^3 + 3 a c^4) d^6 e + (b^3 c^2 + 17 a b c^3) d^5 e^2 - 5 (3 a b^2 c^2 + 2 a^2 c^3) d^4 e^3 + 5 (a b^3 c + 3 a^2 b c^2) d^3 e^4 - (a b^4 + 6 a^2 b^2 c + 2 a^3 c^2) d^2 e^5 + (2 a^2 b^3 - a^3 b c) d e^6 - (a^3 b^2 - 2 a^4 c) e^7) x^2 + \left(\frac{1}{2} \right)^{2/3} \left((a b^6 c^3 - 12 a^2 b^4 c^4 + 48 a^3 b^2 c^5 - 64 a^4 c^6) d^2 - (a^2 b^6 c^2 - 12 a^3 b^4 c^3 + 48 a^4 b^2 c^4 - 64 a^5 c^5) e^2 \right) x \sqrt{(b^2 c^4 d^6 - 12 a b c^4 d^5 e + 6 (a b^2 c^3 + 6 a^2 c^4) d^4 e^2 - 2 (a b^3 c^2 + 16 a^2 b c^3) d^3 e^3 + 3 (7 a^2 b^2 c^2 - 8 a^3 c^3) d^2 e^4 - 6 (a^2 b^3 c - 2 a^3 b c^2) d e^5 + (a^2 b^4 - 4 a^3 b^2 c + 4 a^4 c^2) e^6} / (a^2 b^6 c^4 - 12 a^3 b^4 c^5 + 48 a^4 b^2 c^6 - 64 a^5 c^7) \right)}$

$$\begin{aligned}
& ^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 1 \\
& 2*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^ \\
& 5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24* \\
& a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a \\
& ^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8* \\
& a^4*b*c^2)*e^5)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2* \\
& c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e \\
& ^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d \\
& ^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4 \\
& *c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(\\
& a*b^2*c^2 - 4*a^2*c^3))^(2/3) + (1/2)^(1/3)*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b \\
& ^4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4 \\
& *e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3 \\
& *c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a \\
& *b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + \\
& 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - \\
& 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*sqrt((b^2*c^4*d^6 - 12*a \\
& *b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b* \\
& c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3 \\
& *b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12* \\
& a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 + a*b \\
& *e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a* \\
& b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7* \\
& a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2 \\
& *b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4 \\
& *b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^ \\
& 2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + \\
& 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2 \\
& *c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c) \\
& *e^7)) - (1/2)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d \\
& - (a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a \\
& *b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b* \\
& c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3 \\
& *b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12* \\
& a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) - sqrt(3)*((b^3*c^2 - 4*a*b*c^3) \\
&)*d^3*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e \\
& ^3 - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-(c^2*d^3 - 3*a*c*d*e^2 + a \\
& *b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(\\
& a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(\\
& 7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a \\
& ^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a \\
& ^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) - sqrt(3)*(b*c^3* \\
& d^5 + 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2) \\
&)*d^2*e^3 + (a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5 \\
& + 10*a*b*c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^
\end{aligned}$$

$$\begin{aligned}
& 2e^3 + (ab^3 + a^2bc)d^4 - (a^2b^2 - 2a^3c)e^5) - \frac{2}{3}\sqrt{3} \cdot \left(\frac{1}{2}\right)^{\frac{1}{3}} \cdot \left(-c^2d^3 - 3acde^2 + abe^3 - (ab^2c^2 - 4a^2c^3)\sqrt{\right. \\
& \left.(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3 + 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - \right. \\
& \left. 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7) \Big) / (ab^2c^2 - 4a^2c^3)^{\frac{1}{3}} \cdot \arctan\left(-\frac{1}{3} \cdot \left(\frac{1}{2}\right)^{\frac{5}{6}} \cdot \left(\sqrt{3} \cdot (2(a^2b^4c^3 - 8a^2b^2c^4 + 16a^3c^5)d - (ab^5c^2 - 8a^2b^3c^3 + 16a^3b^3c^4)e)\right) \cdot \sqrt{\right.} \\
& \left.\left.(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3 + 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7)\right) + \sqrt{3} \cdot \left(\frac{1}{2}\right)^{\frac{1}{3}} \cdot \left((b^3c^2 - 4ab^3c^3)d^3e - 6(a^2b^2c^2 - 4a^2c^3)d^2e^2 + 3(ab^3c - 4a^2b^3c^2)d^2e^3 - (ab^4 - 6a^2b^2c + 8a^3c^2)e^4\right) \cdot \left(-c^2d^3 - 3acde^2 + abe^3 - (ab^2c^2 - 4a^2c^3)\sqrt{\right.} \\
& \left.\left.(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3 + 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7)\right) \Big) / (ab^2c^2 - 4a^2c^3)^{\frac{1}{3}} \cdot \sqrt{\left(2(b^3c^4d^7 - 2(b^2c^3 + 3ac^4)d^6e + (b^3c^2 + 17ab^3c^3)d^5e^2 - 5(3ab^2c^2 + 2a^2c^3)d^4e^3 + 5(ab^3c + 3a^2b^3c^2)d^3e^4 - (ab^4 + 6a^2b^2c + 2a^3c^2)d^2e^5 + (2a^2b^3 - a^3b^3c)d^2e^6 - (a^3b^2 - 2a^4c)e^7\right) \cdot x^2 - \left(\frac{1}{2}\right)^{\frac{2}{3}} \cdot \left(\left(a^2b^6c^3 - 12a^2b^4c^4 + 48a^3b^2c^5 - 64a^4c^6\right)d^2 - (a^2b^6c^2 - 12a^3b^4c^3 + 48a^4b^2c^4 - 64a^5c^5)e^2\right) \cdot x \cdot \sqrt{\left(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3 + 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7)}\right) + \left((b^4c^3 - 4ab^2c^4)d^5 - 10(ab^3c^3 - 4a^2b^3c^4)d^4e + 4(ab^4c^2 + 2a^2b^2c^3 - 24a^3c^4)d^3e^2 - (ab^5c + 12a^2b^3c^2 - 64a^3b^3c^3)d^2e^3 + (7a^2b^4c - 36a^3b^2c^2 + 32a^4c^3)d^2e^4 - (a^2b^5 - 6a^3b^3c + 8a^4b^3c^2)e^5\right) \cdot x \cdot \left(-c^2d^3 - 3acde^2 + abe^3 - (ab^2c^2 - 4a^2c^3)\sqrt{\right.} \\
& \left.\left.(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3 + 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7)\right) \Big) / (ab^2c^2 - 4a^2c^3)^{\frac{2}{3}} + \left(\frac{1}{2}\right)^{\frac{1}{3}} \cdot \left((b^3c^3 - 4ab^3c^4)d^6 - (b^4c^2 + 2ab^2c^3 - 24a^2c^4)d^5e + 10(ab^3c^2 - 4a^2b^3c^3)d^4e^2 - 4(ab^4c - 3a^2b^2c^2 - 4a^3c^3)d^3e^3 + (ab^5 - 3a^2b^3c - 4a^3b^3c^2)d^2e^4 - (a^2b^4 - 6a^3b^2c + 8a^4c^2)d^2e^5 + ((ab^5c^3 - 8a^2b^3c^4 + 16a^3b^3c^5)d^3 - (ab^6c^2 - 6a^2b^4c^3 + 32a^4c^5)d^2e + 3(a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^3c^4)d^2e^2 - 2(a^3b^4c^2 - 8a^4b^2c^3 + 16a^5c^4)e^3\right) \cdot \sqrt{\left(b^2c^4d^6 - 12ab^3c^4d^5e + 6(a^2b^2c^3 + 6a^2c^4)d^4e^2 - 2(a^2b^3c^2 + 16a^2b^3c^3)d^3e^3\right.} \\
& \left.\left.+ 3(7a^2b^2c^2 - 8a^3c^3)d^2e^4 - 6(a^2b^3c - 2a^3b^2c^2)d^2e^5 + (a^2b^4 - 4a^3b^2c + 4a^4c^2)e^6\right) / (a^2b^6c^4 - 12a^3b^4c^5 + 48a^4b^2c^6 - 64a^5c^7)\right) \Big) / (ab^2c^2 - 4a^2c^3)^{\frac{1}{3}}
\end{aligned}$$

$$\begin{aligned}
& + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 \\
& + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + \\
& 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2 \\
& *c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a \\
& ^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - \\
& 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3* \\
& b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64 \\
& *a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3))/(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c \\
& ^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^ \\
& 4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^ \\
& 2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)) - (1/2 \\
&)^(1/3)*(sqrt(3)*(2*(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*d - (a*b^5*c^2 \\
& - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e \\
& + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 \\
& + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 \\
& + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + \\
& 48*a^4*b^2*c^6 - 64*a^5*c^7)) + sqrt(3)*((b^3*c^2 - 4*a*b*c^3)*d^3*e - 6*(\\
& a*b^2*c^2 - 4*a^2*c^3)*d^2*e^2 + 3*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 - (a*b^4 - \\
& 6*a^2*b^2*c + 8*a^3*c^2)*e^4)*x)*(-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b \\
& ^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6 \\
& *a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 \\
& - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^ \\
& 3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - \\
& 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) + sqrt(3)*(b*c^3*d^5 + 10*a*b* \\
& c^2*d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (\\
& a*b^3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5))/(b*c^3*d^5 + 10*a*b*c^2* \\
& d^3*e^2 - (b^2*c^2 + 6*a*c^3)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^2*e^3 + (a*b^ \\
& 3 + a^2*b*c)*d*e^4 - (a^2*b^2 - 2*a^3*c)*e^5)) - 1/6*(1/2)^(1/3)*(-c^2*d^3 \\
& - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a \\
& *b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b* \\
& c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3 \\
& *b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12* \\
& a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(1/3) \\
& *log(2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^ \\
& 5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3 \\
& *e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d* \\
& e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 + (1/2)^(2/3)*(((a*b^6*c^3 - 12*a^2*b^4* \\
& c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48 \\
& *a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6* \\
& (a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3* \\
& (7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (\\
& a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48* \\
& a^4*b^2*c^6 - 64*a^5*c^7)) - ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - \\
& 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - \\
& (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b
\end{aligned}$$

$$\begin{aligned}
& ^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x) * \\
& (-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^{(2/3)} + (1/2)^{(1/3)}*((b^3*c^3 - 4*a*b*c^4)*d^6 - (b^4*c^2 + 2*a*b^2*c^3 - 24*a^2*c^4)*d^5*e + 10*(a*b^3*c^2 - 4*a^2*b*c^3)*d^4*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^3 + (a*b^5 - 3*a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^4 - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d*e^5 - ((a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^3 - (a*b^6*c^2 - 6*a^2*b^4*c^3 + 32*a^4*c^5)*d^2*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d*e^2 - 2*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e^3)*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/((a*b^2*c^2 - 4*a^2*c^3))^{(1/3)} - 1/6*(1/2)^{(1/3)}*(-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/((a*b^2*c^2 - 4*a^2*c^3))^{(1/3)}*\log(2*(b*c^4*d^7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - 2*a^4*c)*e^7)*x^2 - (1/2)^{(2/3)}*((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*x*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)) + ((b^4*c^3 - 4*a*b^2*c^4)*d^5 - 10*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5)*x)*(-c^2*d^3 - 3*a*c*d*e^2 + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*\sqrt{(b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6})/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))/((a*b^2*c^2 - 4*a^2*c^3))^{(2/3)}
\end{aligned}$$

$$\begin{aligned}
& - 4*a^2*b*c^4)*d^4*e + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^2 \\
& - (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^2*e^3 + (7*a^2*b^4*c - 36*a^3 \\
& *b^2*c^2 + 32*a^4*c^3)*d*e^4 - (a^2*b^5 - 6*a^3*b^3*c + 8*a^4*b*c^2)*e^5 + \\
& ((a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*d^2 - (a^2*b^6* \\
& c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*e^2)*sqrt((b^2*c^4*d^6 \\
& - 12*a*b*c^4*d^5*e + 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16* \\
& a^2*b*c^3)*d^3*e^3 + 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - \\
& 2*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 \\
& - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7)))*(-(c^2*d^3 - 3*a*c*d*e^2 \\
& + a*b*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((b^2*c^4*d^6 - 12*a*b*c^4*d^5*e + \\
& 6*(a*b^2*c^3 + 6*a^2*c^4)*d^4*e^2 - 2*(a*b^3*c^2 + 16*a^2*b*c^3)*d^3*e^3 + \\
& 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^2*e^4 - 6*(a^2*b^3*c - 2*a^3*b*c^2)*d*e^5 \\
& + (a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*e^6)/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + \\
& 48*a^4*b^2*c^6 - 64*a^5*c^7)))/(a*b^2*c^2 - 4*a^2*c^3))^(2/3) + 2*(b*c^4*d^ \\
& 7 - 2*(b^2*c^3 + 3*a*c^4)*d^6*e + (b^3*c^2 + 17*a*b*c^3)*d^5*e^2 - 5*(3*a*b \\
& ^2*c^2 + 2*a^2*c^3)*d^4*e^3 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^4 - (a*b^4 + \\
& 6*a^2*b^2*c + 2*a^3*c^2)*d^2*e^5 + (2*a^2*b^3 - a^3*b*c)*d*e^6 - (a^3*b^2 - \\
& 2*a^4*c)*e^7)*x)
\end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^3 + d)x}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.00, size = 49, normalized size = 0.08

$$\frac{\left(\text{RootOf}\left(_Z^6c + _Z^3b + a\right)^4 e + \text{RootOf}\left(_Z^6c + _Z^3b + a\right)d\right) \ln\left(-\text{RootOf}\left(_Z^6c + _Z^3b + a\right) + x\right)}{6 \text{RootOf}\left(_Z^6c + _Z^3b + a\right)^5 c + 3 \text{RootOf}\left(_Z^6c + _Z^3b + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum((_R^4*e + _R*d)/(2*_R^5*c + _R^2*b)*ln(-_R+x), _R=RootOf(_Z^6*c + _Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^3 + d)x}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

mupad [B] time = 24.56, size = 7457, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2)))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*
```

$$\begin{aligned}
& e) - (27*2^{(1/3)}*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4 \\
& *c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3 \\
& *c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3 \\
& *(-4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d \\
& ^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a \\
& *c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3))^{(2/3)}/2*((a*b^5*e^3 + 16*a^2 \\
& *c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e \\
& ^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 + 3*a*b* \\
& c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3))^{(1/3)}/6 - 108*a \\
& ^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27 \\
& *a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^ \\
& 2 - b*d*e)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 \\
& - a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + \\
& b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(64 \\
& *a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^{(1/3)} - \log(c*x*(\\
& b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^{(1/3)}*(3^{(1/2)}*1i - 1))*((a*b^5*e^ \\
& 3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b \\
& ^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a \\
& *b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^ \\
& 2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3))^{(2/3)}* \\
& (36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^{(2/3)}*(3^{(1/2)}*1i + 1))*(27*c^3*x*(\\
& 4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^{(1/3)}*a* \\
& b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c \\
& ^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c \\
& *e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^2*c*e^3* \\
& (-4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2 \\
& *e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3))^{(2/3)}/4*((a*b^5*e^3 + 16*a^2*c \\
& ^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 \\
& + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c* \\
& d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3))^{(1/3)}/12 - 45*a^2 \\
& *b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 10 \\
& 8*a^2*b*c^3*d*e^2))/36*((3^{(1/2)}*1i)/2 + 1/2))*((a*b^5*e^3 + 16*a^2*c^4*d^3 \\
& + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a \\
& ^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^ \\
& 2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a \\
& *c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2* \\
& (-4*a*c - b^2)^3)^{(1/2)})/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48
\end{aligned}$$

$$\begin{aligned}
& *a^3*b^2*c^4))^{(1/3)} - \log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^ \\
& (1/3)*(3^{(1/2)}*1i - 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2 \\
& *c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b* \\
& c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&))/(a*c^2*(4*a*c - b^2)^3)^{(2/3)}*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^ \\
& (2/3)*(3^{(1/2)}*1i + 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c* \\
& d^2 - 2*a*b*d*e) - (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i - 1)*(4*a*c - b^2)^2*((a \\
& *b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4* \\
& a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^ \\
& 2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c \\
& ^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a*c^2*(4*a*c - b^2)^3)) \\
& ^{(2/3)}/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a \\
& *b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b* \\
& c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 4 \\
& 8*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a*c^2*(4* \\
& a*c - b^2)^3))^{(1/3)}/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^ \\
& 3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/36)*((3^{(1/2)}*1i)/2 + \\
& 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e \\
& ^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^ \\
& 3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3* \\
& c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a \\
& ^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(64*a^4*c^5 \\
& - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4))^{(1/3)} + \log(c*x*(b*e - c*d \\
&)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^{(1/3)}*(3^{(1/2)}*1i + 1)*((a*b^5*e^3 + 16*a^ \\
& 2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1 \\
& /2)} - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d* \\
& e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b \\
& *c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a*c^2*(4*a*c - b^2)^3))^{(2/3)}*(36*a^3*c \\
& ^3*e^3 - 108*a^2*c^4*d^2*e - (2^{(2/3)}*(3^{(1/2)}*1i - 1)*(27*c^3*x*(4*a*c - b \\
& ^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) + (27*2^{(1/3)}*a*b*c^3*(3^ \\
& (1/2)*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - \\
& 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16 \\
& *a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^2*c*e^3*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a \\
& *c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)))/(a*c^2*(4*a*c - b^2)^3))^{(2/3)}/4)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + \\
& b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2* \\
& b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^2*c \\
& *e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^ \\
& 2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3)^{(1/3)})/12 - 45*a^2*b^2*c^2* \\
& e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c \\
& ^3*d*e^2)/36)*((3^{(1/2)}*1i)/2 - 1/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2 \\
& ^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c* \\
& e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 2*a^2*c*e^3*(\\
& -(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2* \\
& e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2* \\
& c^4)))^{(1/3)} + \log(c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 - (2^{(1/3)}*(3^{(\\
& 1/2)}*1i + 1)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 \\
& - a*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + \\
& b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/ \\
& 2)} + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2* \\
& (4*a*c - b^2)^3)^{(2/3)}*(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e - (2^{(2/3)}*(3^{(\\
& 1/2)}*1i - 1)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a \\
& *b*d*e) + (27*2^{(1/3)}*a*b*c^3*(3^{(1/2)}*1i + 1)*(4*a*c - b^2)^2*((a*b^5*e^3 \\
& + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 - 3*a*b \\
& ^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d*e^2 \\
& + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2)^3)^{(2/3)})/4 \\
&)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3* \\
& -(4*a*c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(\\
& -(4*a*c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3 \\
& *d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2* \\
& b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(a*c^2*(4*a*c - b^2 \\
&)^3)^{(1/3)})/12 - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - \\
& 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2)/36)*((3^{(1/2)}*1i)/2 - 1/2)*((a* \\
& b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a \\
& *c - b^2)^3)^{(1/2)} - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a* \\
& c - b^2)^3)^{(1/2)} + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d*e^2 \\
& - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^ \\
& 2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(64*a^4*c^5 - a*b^6*c \\
& ^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^{(1/3)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a), x)

[Out] Timed out

$$3.17 \quad \int \frac{d+ex^3}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}}$$

[Out] $1/6*\ln(2^{(1/3)}*c^{(1/3)}*x+(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)})*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}-1/12*\ln(2^{(2/3)})*c^{(2/3)}*x^2-2^{(1/3)}*c^{(1/3)}*x*(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)}+(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)})*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}-1/6*\arctan(1/3*(1-2*2^{(1/3)}*c^{(1/3)}*x/(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)})*3^{(1/2)})*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}*3^{(1/2)}/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}+1/6*\ln(2^{(1/3)}*c^{(1/3)}*x+(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)})*(e+(b*e-2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}-1/12*\ln(2^{(2/3)})*c^{(2/3)}*x^2-2^{(1/3)}*c^{(1/3)}*x*(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)}+(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)})*(e+(b*e-2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}-1/6*\arctan(1/3*(1-2*2^{(1/3)}*c^{(1/3)}*x/(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)})*3^{(1/2)})*(e+(b*e-2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/c^{(1/3)}*3^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}$

Rubi [A] time = 0.65, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right) \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \log\left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac}\right)^{2/3} + 2^{2/3} c^{2/3} x^2\right)}{6\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6), x]

[Out] $-(((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)}*c^{(1/3)}*x)/(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)}*\text{Sqrt}[3]*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)})) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(1 - (2*2^{(1/3)}*c^{(1/3)}*x)/(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)}*\text{Sqrt}[3]*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}))$

$$\begin{aligned} & \text{rt}[3]*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} + ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 \\ & - 4*a*c])*\text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)*x})/(3*2^{(1/3)} \\ & *c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) + ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - \\ & 4*a*c])*\text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)} + 2^{(1/3)}*c^{(1/3)*x})/(3*2^{(1/3)}* \\ & c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a \\ & *c])*\text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4* \\ & a*c])^{(1/3)*x} + 2^{(2/3)}*c^{(2/3)*x^2}]/(6*2^{(1/3)}*c^{(1/3)}*(b - \text{Sqrt}[b^2 - 4* \\ & a*c])^{(2/3)}) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{Log}[(b + \text{Sqrt}[b^2 - 4 \\ & *a*c])^{(2/3)} - 2^{(1/3)}*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(1/3)*x} + 2^{(2/3)}*c^{ \\ & (2/3)*x^2}]/(6*2^{(1/3)}*c^{(1/3)}*(b + \text{Sqrt}[b^2 - 4*a*c])^{(2/3)}) \end{aligned}$$
Rule 31

$$\text{Int}[(a_ + (b_)*x_)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$$
Rule 200

$$\text{Int}[(a_ + (b_)*x_)^{-1}, x_Symbol] \rightarrow \text{Dist}[1/(3*\text{Rt}[a, 3]^2), \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]^2), \text{Int}[(2*\text{Rt}[a, 3] - \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] /; \text{FreeQ}\{a, b\}, x]$$
Rule 204

$$\text{Int}[(a_ + (b_)*x_)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$$
Rule 617

$$\text{Int}[(a_ + (b_)*x_ + (c_)*x_^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$$
Rule 628

$$\text{Int}[(d_ + (e_)*x_)/((a_ + (b_)*x_ + (c_)*x_^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$$
Rule 634

$$\text{Int}[(d_ + (e_)*x_)/((a_ + (b_)*x_ + (c_)*x_^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$$

$\text{t}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2cd - b^2, 0] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 1422

$\text{Int}[(d + (e \cdot x^n)/(a + (b \cdot x + c \cdot x^2))), x_Symbol] \text{:> With}[\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - b^2e)/(2q), \text{Int}[1/(b/2 - q/2 + c \cdot x^n), x], x] + \text{Dist}[e/2 - (2cd - b^2e)/(2q), \text{Int}[1/(b/2 + q/2 + c \cdot x^n), x], x]] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{EqQ}[n, 2] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - b \cdot d \cdot e + a \cdot e^2, 0] \ \&\& \ (\text{PosQ}[b^2 - 4ac] \ || \ \text{!IGtQ}[n/2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{d + ex^3}{a + bx^3 + cx^6} dx &= \frac{1}{2} \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx + \frac{1}{2} \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2} \left(b + \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{2^{2/3} \sqrt[3]{b + \sqrt{b^2 - 4ac}} - \sqrt[3]{c}x}{\frac{(b + \sqrt{b^2 - 4ac})^{2/3}}{2^{2/3}} - \frac{\sqrt[3]{c} \sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + c^{2/3}x^2} dx}{3\sqrt[3]{2} \left(b + \sqrt{b^2 - 4ac} \right)^{2/3}} \\ &= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x \right)}{3\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x \right)}{3\sqrt[3]{2} \sqrt[3]{c} \left(b + \sqrt{b^2 - 4ac} \right)^{2/3}} \\ &= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x \right)}{3\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \log \left(\sqrt[3]{b + \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c}x \right)}{3\sqrt[3]{2} \sqrt[3]{c} \left(b + \sqrt{b^2 - 4ac} \right)^{2/3}} \\ &= -\frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2} \sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}} \right)}{\sqrt[3]{2} \sqrt{3} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{1 - \frac{2\sqrt[3]{2} \sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}} \right)}{\sqrt[3]{2} \sqrt{3} \sqrt[3]{c} \left(b + \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right)}{3\sqrt[3]{2} \sqrt[3]{c} \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} \end{aligned}$$

Mathematica [C] time = 0.03, size = 61, normalized size = 0.10

$$\frac{1}{3} \text{RootSum} \left[\#1^6 c + \#1^3 b + a \&, \frac{\#1^3 e \log(x - \#1) + d \log(x - \#1)}{2\#1^5 c + \#1^2 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6), x]

[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) &]/3

fricas [B] time = 27.69, size = 14094, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out]
$$\begin{aligned} & -2/3*\text{sqrt}(3)*(1/2)^{(1/3)}*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*\text{sqrt}(- (12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))) \\ & / (a^2*b^2*c - 4*a^3*c^2)^{(1/3)}*\text{arctan}(-1/6*(2*(1/2)^{(2/3)}*(\text{sqrt}(3))*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*x*\text{sqrt}(- (12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))) - \text{sqrt}(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5)*x) \\ & * ((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*\text{sqrt}(- (12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))) \\ & / (a^2*b^2*c - 4*a^3*c^2)^{(2/3)} - (1/2)^{(1/6)}*(\text{sqrt}(3))*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*\text{sqrt}(- (12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))) \end{aligned}$$

$$\begin{aligned}
& *b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - \text{sqrt}(3)*((b^5*c^2 - 6*a*b^3*c^3 \\
& + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a \\
& *b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2* \\
& c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - \\
& 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3* \\
& c^2)*\text{sqrt}(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2 \\
& *c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^ \\
& 3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^ \\
& 2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/ \\
& (a^2*b^2*c - 4*a^3*c^2))^(2/3)*\text{sqrt}((2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 \\
& + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 \\
& + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + \\
& (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(\\
& (2/3))*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c \\
& - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 3 \\
& 2*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a \\
& ^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((\\
& a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^ \\
& 6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*\text{sqrt}(-(12*a^4*b*c* \\
& d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^ \\
& 2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3 \\
& *c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^ \\
& 2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e \\
& + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*\text{sqrt}(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 \\
& - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5 \\
& *e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d \\
& ^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + \\
& 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3 \\
&)*((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b \\
& ^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d \\
& *e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*\text{sqrt}(-(12*a^4*b*c* \\
& d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^ \\
& 2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3 \\
& *c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^ \\
& 2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^ \\
& 3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c \\
& - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^ \\
& 3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d* \\
& e^5)*x))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*\text{sqrt}(-(\\
& 12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + \\
& 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + \\
& 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/ \\
& (a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - \\
& 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b \\
& *c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^
\end{aligned}$$

$$\begin{aligned}
& 2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3* \\
& b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6) - 2*\sqrt{3}*(a^4*b*e^7 - (b^2* \\
& c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2 \\
& *a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2* \\
& a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d \\
& *e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - \\
& (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4* \\
& e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 \\
& - 2*(a^3*b^2 + 3*a^4*c)*d*e^6) + 2/3*\sqrt{3}*(1/2)^{(1/3)}*((b*c*d^3 - 3*a*c \\
& *d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^ \\
& 2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^ \\
& 3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b* \\
& c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4 \\
& *c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^{(1/3)}*\arctan \\
& (-1/6*(2*(1/2)^{(2/3)}*(\sqrt{3})*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c \\
& ^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^ \\
& 6*c^4)*e^2)*x*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^ \\
& 3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - \\
& 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + \\
& 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^ \\
& 7*c^5)) + \sqrt{3}*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 \\
& - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3* \\
& b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a \\
& ^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 - \\
& 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e^5 - \\
& a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^ \\
& 2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16* \\
& a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a \\
& ^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^{(2/3)} \\
& - (1/2)^{(1/6)}*(\sqrt{3})*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64 \\
& *a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)* \\
& e^2)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2 \\
& *c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^ \\
& 3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^ \\
& 2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + \\
& \sqrt{3}*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2 \\
& *b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^ \\
& 3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c \\
& - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2* \\
& e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 \\
& - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^ \\
& 5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)* \\
& d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 \\
& + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^{(2/3)}*\sqrt{((2*(a^ \\
& 4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c +
\end{aligned}$$

$$\begin{aligned}
& 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3 \\
& *a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b \\
& ^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c \\
& ^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2* \\
& (7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3 \\
& *c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - \\
& 2*(a^4*b^3 - 4*a^5*b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^ \\
& 3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64 \\
& *a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c \\
& ^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 \\
& - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c \\
& + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a \\
& ^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(\\
& -(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 \\
& + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 \\
& + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4 \\
&))/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c \\
& - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4* \\
& b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c \\
& - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a \\
& ^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c \\
& ^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 \\
& - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c \\
& + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a \\
& ^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 \\
& - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10 \\
& *(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d \\
& ^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 \\
& - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 \\
& - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a \\
& ^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6* \\
& (a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2 \\
& *c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3 \\
& - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2 \\
& *c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3* \\
& c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6 \\
&)) + 2*sqrt(3)*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3) \\
& *d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c \\
& ^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)* \\
& d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^ \\
& 7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 \\
& + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 \\
& + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 1/6*(1/2 \\
&)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(\\
& 12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
\end{aligned}$$

$$\begin{aligned}
& 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + \\
& 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/ \\
& (a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))/ (a^2*b^2*c - \\
& 4*a^3*c^2)^{(1/3)} * \log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 \\
& - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + \\
& 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17 \\
& *a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^{(2/3)}*((b^6*c \\
& - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c \\
& ^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3 \\
& *e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4 \\
& *b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12 \\
& *a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b \\
& ^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^ \\
& 2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^ \\
& 3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b* \\
& c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4 \\
& *c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (\\
& a^2*b^2*c - 4*a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4 \\
& *a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2* \\
& b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^ \\
& 3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^ \\
& 4 - 64*a^7*c^5))/ (a^2*b^2*c - 4*a^3*c^2))^{(2/3)} + (1/2)^{(1/3)}*((a^2*b^5*c \\
& ^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^ \\
& 5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4* \\
& b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^ \\
& 2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^ \\
& 3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b* \\
& c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4 \\
& *c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4) \\
& *d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c \\
& ^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + \\
& 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x*((b*c* \\
& d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e \\
& ^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - \\
& 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c \\
& + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - \\
& 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))/ (a^2*b^2*c - 4*a^3*c^2))^{(\\
& 1/3)} - 1/6*(1/2)^{(1/3)}*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4* \\
& a^3*c^2)*\sqrt{-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4 \\
& *a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^ \\
& 3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^ \\
& 4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5 \\
&)))/(a^2*b^2*c - 4*a^3*c^2))^{(1/3)} * \log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d \\
& ^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^ \\
& 2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
\end{aligned}$$

$$\begin{aligned}
& + (a^2b^3 + 17a^3bc)d^2e^5 - 2(a^3b^2 + 3a^4c)d^2e^6)x^2 - (1/2)^{(2/3)}*((b^6c - 8a^2b^4c^2 + 20a^2b^2c^3 - 16a^3c^4)d^5 - 5(a^2b^5c - 6a^2b^3c^2 + 8a^3bc^3)d^4e + 2(7a^2b^4c - 36a^3b^2c^2 + 32a^4c^3)d^3e^2 - (a^2b^5 + 12a^3b^3c - 64a^4b^2c^2)d^2e^3 + 2(a^3b^4 + 2a^4b^2c - 24a^5c^2)d^2e^4 - 2(a^4b^3 - 4a^5bc)e^5 + ((a^2b^7c - 12a^3b^5c^2 + 48a^4b^3c^3 - 64a^5b^2c^4)d^2 - 2(a^3b^6c - 12a^4b^4c^2 + 48a^5b^2c^3 - 64a^6c^4)d^2e) * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4)/(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5))} * ((b^2cd^3 - 3a^2cd^2e + a^2e^3 - (a^2b^2c - 4a^3c^2) * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4)/(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5))} / (a^2b^2c - 4a^3c^2))^{(2/3)} - (1/2)^{(1/3)} * (((a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^2c^4)d^3 - (a^2b^6c - 6a^3b^4c^2 + 32a^5c^4)d^2e + 3(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^2e^2 - 2(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)e^3) * x * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4)/(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5))} + ((b^4c^2 - 6a^2b^2c^3 + 8a^2c^4)d^6 - (b^5c - 3a^2b^3c^2 - 4a^2b^2c^3)d^5e + 4(a^2b^4c - 3a^2b^2c^2 - 4a^3c^3)d^4e^2 - 10(a^2b^3c - 4a^3b^2c^2)d^3e^3 + (a^2b^4 + 2a^3b^2c - 24a^4c^2)d^2e^4 - (a^3b^3 - 4a^4bc) * d^2e^5) * x) * ((b^2cd^3 - 3a^2cd^2e + a^2e^3 - (a^2b^2c - 4a^3c^2) * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4)/(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5))} / (a^2b^2c - 4a^3c^2))^{(1/3)} + 1/3 * (1/2)^{(1/3)} * ((b^2cd^3 - 3a^2cd^2e + a^2e^3 - (a^2b^2c - 4a^3c^2) * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4)/(a^4b^6c^2 - 12a^5b^4c^3 + 48a^6b^2c^4 - 64a^7c^5))} / (a^2b^2c - 4a^3c^2))^{(1/3)} * \log(2 * (10a^2b^2c^2d^2e^3 + a^3b^2e^5 - (b^2c^2 - 2a^2c^3)d^5 + (b^3c + a^2bc^2)d^4e - 4(a^2b^2c + a^2c^2)d^3e^2 - (a^2b^2 + 6a^3c) * d^2e^4) * x + (1/2)^{(1/3)} * ((b^4c - 6a^2b^2c^2 + 8a^2c^3)d^4 - 3(a^2b^3c - 4a^2b^2c^2)d^3e + 6(a^2b^2c - 4a^3c^2)d^2e^2 - (a^2b^3 - 4a^3bc) * d^2e^3 - ((a^2b^5c - 8a^3b^3c^2 + 16a^4b^2c^3) * d - 2(a^3b^4c - 8a^4b^2c^2 + 16a^5c^3) * e) * \sqrt{-(12a^4b^2c^2d^3e^5 - a^4b^2e^6 - (b^4c^2 - 4a^2b^2c^3 + 4a^2c^4)d^6 + 6(a^2b^3c^2 - 2a^2b^2c^3)d^5e - 3(7a^2b^2c^2 - 8a^3c^3)d^4e^2 + 2(a^2b^3c + 16a^3b^2c^2)d^3e^3 - 6(a^3b^2c + 6a^4c^2)d^2e^4}
\end{aligned}$$

```

*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5))*((
b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c
*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c
^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^
3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c
^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2
))^((1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c
- 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3
+ 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7
*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^((1/3))*log(2*(10*a^2*b*c*d^2*e^3 + a^3*b*e^
5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^
2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c
^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^
3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + ((a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12
*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*
(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2
*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a
^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*
a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4
*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b
*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3
*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*
b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^((1/3))

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)

maple [C] time = 0.01, size = 47, normalized size = 0.07

$$\frac{\left(\text{RootOf}\left(-Z^6c + Z^3b + a\right)^3 e + d\right) \ln\left(-\text{RootOf}\left(-Z^6c + Z^3b + a\right) + x\right)}{6 \text{RootOf}\left(-Z^6c + Z^3b + a\right)^5 c + 3 \text{RootOf}\left(-Z^6c + Z^3b + a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x)

[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)

mupad [B] time = 18.96, size = 7469, normalized size = 11.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)/(a + b*x^3 + c*x^6),x)

[Out] log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)))/(a^2*c*(4*a*c - b^2)^3)^(1/3))*((2^(1/3)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)))/(a^2*c*(4*a*c - b^2)^3)^(1/3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2)))/(a^2*c*(4*a*c - b^2)^3)^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e

$$\begin{aligned}
& e^{(- (4ac - b^2)^3)^{1/2}} / (54(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3))^{1/3} + \log(3c^2x(2c^3d^4 + ab^2e^4 - 2a^2c^2e^4 - b^3d^2e^3 + 3b^2cd^2e^2 - 4b^2c^2d^3e) - (2^{2/3})^{(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 + 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} + a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 - b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3)^{1/3} * ((2^{1/3})^{(81c^3x(4ac - b^2)^2(ae - bd) - (81^{2/3})ab^2c^3(4ac - b^2)^2(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 + 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} + a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 - b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3))^{1/3}) / 2)^{(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 + 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} + a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 - b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3))^{2/3}) / 18 - 36a^5c^5d^3 + 9b^2c^4d^3 + 9ab^3c^2e^3 - 36a^2b^3c^3e^3 + 108a^2c^4d^2e^2 - 27ab^2c^3d^2e^2) / 6)^{(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 + 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} + a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 - b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (54(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3))^{1/3} + \log(3c^2x(2c^3d^4 + ab^2e^4 - 2a^2c^2e^4 - b^3d^2e^3 + 3b^2cd^2e^2 - 4b^2c^2d^3e) + (2^{2/3})^{(3^{1/2})i - 1})^{(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 - 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} - a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 + b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e + 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e - 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3))^{1/3} * (36a^5c^5d^3 - 9b^2c^4d^3 - 9ab^3c^2e^3 + 36a^2b^3c^3e^3 - 108a^2c^4d^2e^2 + (2^{1/3})^{(3^{1/2})i + 1})^{(81c^3x(4ac - b^2)^2(ae - bd) - (81^{2/3})ab^2c^3(3^{1/2})i - 1)(4ac - b^2)^2(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 - 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} - a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 + b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e + 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e - 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3))^{1/3}) / 4)^{(- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^2c^3d^3 - 2a^2c^2d^3(- (4ac - b^2)^3)^{1/2} - a^2b^2e^3(- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^2e^3 + b^2cd^3(- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e + 6a^2cd^2e^2(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e - 3ab^2cd^2e^2(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3))^{1/3}}
\end{aligned}$$

$$\begin{aligned}
&(- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e - 3abc^2d^2e(- (4ac - \\
&b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3)^{2/3} / 36 + 27ab^2c^3d^2e^2) / 12 \\
&)* ((3^{1/2} * i) / 2 - 1/2) * (- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8a^* \\
&b^3c^2d^3 + 16a^2b^3c^3d^3 - 2ac^2d^3 * (- (4ac - b^2)^3)^{1/2} - a^2 \\
&* b^3e^3 * (- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^3e^3 + b^2cd^3 * (- (4ac - b^ \\
&2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e + 6a^2cd^2e^2 * (- (4ac - \\
&b^2)^3)^{1/2} + 24a^2b^2c^2d^2e - 3abc^2d^2e(- (4ac - b^2)^3)^{1/2} \\
&/ (54(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 - 48a^4b^2c^3)))^{1/3} \\
&+ \log(3c^2 * x * (2c^3d^4 + ab^2e^4 - 2a^2ce^4 - b^3d^2e^3 + 3b^2cd^2e^2 - \\
&4bc^2d^3e) + (2^{2/3} * (3^{1/2} * i - 1) * (- (b^5cd^3 + a^2b^4e^3 + \\
&16a^4c^2e^3 - 8a^* b^3c^2d^3 + 16a^2b^3c^3d^3 + 2ac^2d^3 * (- (4ac - \\
&b^2)^3)^{1/2} + a^2b^3e^3 * (- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^3e^3 - b^2 \\
&cd^3 * (- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2 \\
&cd^2e^2 * (- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3abc^2d^2e * \\
&(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3)^{1/3} * (36a^5d^3 - \\
&9b^2c^4d^3 - 9ab^3c^2e^3 + 36a^2b^3c^3e^3 - 108a^2c^4d^2e^2 \\
&+ (2^{1/3} * (3^{1/2} * i + 1) * (81c^3 * x * (4ac - b^2)^2 * (ae - bd) - (81 * 2^{ \\
&2/3} * ab^3c^3 * (3^{1/2} * i - 1) * (4ac - b^2)^2 * (- (b^5cd^3 + a^2b^4e^3 + \\
&16a^4c^2e^3 - 8a^* b^3c^2d^3 + 16a^2b^3c^3d^3 + 2ac^2d^3 * (- (4ac - \\
&b^2)^3)^{1/2} + a^2b^3e^3 * (- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^3e^3 - b^2 \\
&cd^3 * (- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e - 6a^2 \\
&cd^2e^2 * (- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3abc^2d^2e * \\
&(- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3)^{1/3} / 4 * (- (b^5cd^3 + \\
&a^2b^4e^3 + 16a^4c^2e^3 - 8ab^3c^2d^3 + 16a^2b^3c^3d^3 + 2ac^2 \\
&d^3 * (- (4ac - b^2)^3)^{1/2} + a^2b^3e^3 * (- (4ac - b^2)^3)^{1/2} - 8a^3 \\
&* b^2c^3e^3 - b^2cd^3 * (- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4 \\
&cd^2e - 6a^2cd^2e^2 * (- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + \\
&3abc^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b^2)^3)^{2/3} / 36 \\
&+ 27ab^2c^3d^2e^2) / 12 * ((3^{1/2} * i) / 2 - 1/2) * (- (b^5cd^3 + a^2b^4e^ \\
&^3 + 16a^4c^2e^3 - 8a^* b^3c^2d^3 + 16a^2b^3c^3d^3 + 2ac^2d^3 * (- (4 \\
&a^* c - b^2)^3)^{1/2} + a^2b^3e^3 * (- (4ac - b^2)^3)^{1/2} - 8a^3b^2c^3e^3 \\
&- b^2cd^3 * (- (4ac - b^2)^3)^{1/2} - 48a^3c^3d^2e - 3ab^4cd^2e \\
&- 6a^2cd^2e^2 * (- (4ac - b^2)^3)^{1/2} + 24a^2b^2c^2d^2e + 3abc^2d^2e \\
&* (- (4ac - b^2)^3)^{1/2}) / (54(64a^5c^4 - a^2b^6c + 12a^3b^4c^2 \\
&- 48a^4b^2c^3)))^{1/3} - \log(3c^2 * x * (2c^3d^4 + ab^2e^4 - 2a^2ce^4 - \\
&b^3d^2e^3 + 3b^2cd^2e^2 - 4bc^2d^3e) + (2^{2/3} * (3^{1/2} * i + \\
&1) * (- (b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8a^* b^3c^2d^3 + 16a^2b^3 \\
&c^3d^3 - 2ac^2d^3 * (- (4ac - b^2)^3)^{1/2} - a^2b^3e^3 * (- (4ac - b^2) \\
&^3)^{1/2} - 8a^3b^2c^3e^3 + b^2cd^3 * (- (4ac - b^2)^3)^{1/2} - 48a^3c^ \\
&^3d^2e - 3ab^4cd^2e + 6a^2cd^2e^2 * (- (4ac - b^2)^3)^{1/2} + 24a^ \\
&^2b^2c^2d^2e - 3abc^2d^2e * (- (4ac - b^2)^3)^{1/2}) / (a^2c(4ac - b \\
&^2)^3)^{1/3} * (9b^2c^4d^3 - 36a^5d^3 + 9ab^3c^2e^3 - 36a^2b^3c^ \\
&^3e^3 + 108a^2c^4d^2e^2 + (2^{1/3} * (3^{1/2} * i - 1) * (81c^3 * x * (4ac - b^ \\
&2)^2 * (ae - bd) + (81 * 2^{2/3} * ab^3c^3 * (3^{1/2} * i + 1) * (4ac - b^2)^2 * (- (\\
&b^5cd^3 + a^2b^4e^3 + 16a^4c^2e^3 - 8a^* b^3c^2d^3 + 16a^2b^3c^3d^
\end{aligned}$$

$$\begin{aligned}
&^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} \\
&/2 - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d^2 \\
&*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2* \\
&c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(a^2*c*(4*a*c - b^2)^3) \\
&)^{(1/3)}/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + \\
&16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*b*e^3*(-(4*a \\
&c - b^2)^3)^{(1/2)} - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - \\
&48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(a^2*c*(\\
&4*a*c - b^2)^3))^{(2/3)}/36 - 27*a*b^2*c^3*d*e^2)/12)*((3^{(1/2)}*1i)/2 + 1/2 \\
&)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b* \\
&c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - a^2*b*e^3*(-(4*a*c - b^2)^ \\
&3)^{(1/2)} - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^ \\
&3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2 \\
&*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)})/(54*(64*a^5*c^4 - \\
&a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^{(1/3)} - \log(3*c^2*x*(2*c^3*d \\
&^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) \\
&+ (2^{(2/3)}*(3^{(1/2)}*1i + 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - \\
&8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
&a^2*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c \\
&- b^2)^3)^{(1/2)} - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a \\
&c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3 \\
&)^{(1/2)})/(a^2*c*(4*a*c - b^2)^3))^{(1/3)}*(9*b^2*c^4*d^3 - 36*a*c^5*d^3 + 9*a \\
&*b^3*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 + (2^{(1/3)}*(3^{(1/2)}*1i \\
&- 1)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) + (81*2^{(2/3)}*a*b*c^3*(3^{(1/2)}*1 \\
&i + 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^ \\
&3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + a^2*b \\
&*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2) \\
&^3)^{(1/2)} - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b \\
&^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2} \\
&)))/(a^2*c*(4*a*c - b^2)^3))^{(1/3)}/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c \\
&^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + a^2*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*b^2*c*e^3 - b^2*c*d^3* \\
&(- (4*a*c - b^2)^3)^{(1/2)} - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e \\
&^2*(-(4*a*c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c \\
&- b^2)^3)^{(1/2)})/(a^2*c*(4*a*c - b^2)^3))^{(2/3)}/36 - 27*a*b^2*c^3*d*e^2) \\
&/12)*((3^{(1/2)}*1i)/2 + 1/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8 \\
&*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
&a^2*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - \\
&b^2)^3)^{(1/2)} - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a* \\
&c - b^2)^3)^{(1/2)} + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3) \\
&)^{(1/2)})/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^{(1 \\
&/3)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.18 \quad \int \frac{d+ex^3}{x^2(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=653

$$\frac{\sqrt[3]{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) \sqrt[3]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \log \left(\dots \right)}{6 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}}$$

[Out] $-d/a/x + 1/6 * c^{(1/3)} * \ln(2^{(1/3)} * c^{(1/3)} * x + (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * (d + (-2*a*e + b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a / (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)} - 1/12 * c^{(1/3)} * \ln(2^{(2/3)} * c^{(2/3)} * x^2 - 2^{(1/3)} * c^{(1/3)} * x * (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)} + (b - (-4*a*c + b^2)^{(1/2)})^{(2/3)}) * (d + (-2*a*e + b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a / (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)} + 1/6 * c^{(1/3)} * \arctan(1/3 * (1 - 2 * 2^{(1/3)} * c^{(1/3)} * x) / (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * 3^{(1/2)} * (d + (-2*a*e + b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a * 3^{(1/2)} / (b - (-4*a*c + b^2)^{(1/2)})^{(1/3)} + 1/6 * c^{(1/3)} * \ln(2^{(1/3)} * c^{(1/3)} * x + (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * (d + (2*a*e - b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a / (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)} - 1/12 * c^{(1/3)} * \ln(2^{(2/3)} * c^{(2/3)} * x^2 - 2^{(1/3)} * c^{(1/3)} * x * (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)} + (b + (-4*a*c + b^2)^{(1/2)})^{(2/3)}) * (d + (2*a*e - b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a / (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)} + 1/6 * c^{(1/3)} * \arctan(1/3 * (1 - 2 * 2^{(1/3)} * c^{(1/3)} * x) / (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)}) * 3^{(1/2)} * (d + (2*a*e - b*d) / (-4*a*c + b^2)^{(1/2)}) * 2^{(1/3)} / a * 3^{(1/2)} / (b + (-4*a*c + b^2)^{(1/2)})^{(1/3)}$

Rubi [A] time = 1.18, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1504, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right) \sqrt[3]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \log \left(\dots \right)}{6 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)), x]

[Out] $-(d/(a*x)) + (c^{(1/3)} * (d + (b*d - 2*a*e) / \text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTan}[(1 - (2 * 2^{(1/3)} * c^{(1/3)} * x) / (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) / \text{Sqrt}[3]]) / (2^{(2/3)} * \text{Sqrt}[3] * a * (b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)}) + (c^{(1/3)} * (d - (b*d - 2*a*e) / \text{Sqrt}[b^2 -$

$$4ac) \cdot \text{ArcTan}\left[\frac{1 - (2 \cdot 2^{1/3}) \cdot c^{1/3} \cdot x}{(b + \sqrt{b^2 - 4ac})^{1/3}}\right] / \sqrt[3]{\dots}$$

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^-1, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^3}{x^2(a + bx^3 + cx^6)} dx &= -\frac{d}{ax} - \frac{\int \frac{x(bd - ae + cd^3)}{a + bx^3 + cx^6} dx}{a} \\
&= -\frac{d}{ax} - \frac{\left(c\left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{x}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx}{2a} - \frac{\left(c\left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{x}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^3} dx}{2a} \\
&= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} a \sqrt[3]{b + \sqrt{b^2 - 4ac}}} - \frac{\left(c^{2/3}\left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}{\sqrt[3]{2}} - \sqrt[3]{c}x} dx}{3 \cdot 2^{2/3} a \sqrt[3]{b + \sqrt{b^2 - 4ac}}} \\
&= -\frac{d}{ax} + \frac{\sqrt[3]{c} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt[3]{c} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} - \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}} \\
&= -\frac{d}{ax} + \frac{\sqrt[3]{c} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} + \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt[3]{c} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2 - 4ac}} - \sqrt[3]{2} \sqrt[3]{c} x\right)}{3 \cdot 2^{2/3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}} \\
&= -\frac{d}{ax} + \frac{\sqrt[3]{c} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} a \sqrt[3]{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt[3]{c} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2 - 4ac}}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3} a \sqrt[3]{b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.13

$$-\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3cd \log(x - \#1) - ae \log(x - \#1) + bd \log(x - \#1)}{2\#1^4c + \#1b}\right]}{3a} - \frac{d}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)), x]

[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 &, (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) &]/(3*a)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{(cx^6 + bx^3 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^2), x)

maple [C] time = 0.01, size = 70, normalized size = 0.11

$$\frac{\left(\text{RootOf}(-Z^6c + Z^3b + a)^4 cd + (-ae + bd) \text{RootOf}(-Z^6c + Z^3b + a)\right) \ln(-\text{RootOf}(-Z^6c + Z^3b + a)) + 3a \left(2 \text{RootOf}(-Z^6c + Z^3b + a)^5 c + \text{RootOf}(-Z^6c + Z^3b + a)^2 b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)

[Out] -1/3/a*sum((c*d*_R^4+(-a*e+b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(-Z^6*c+_Z^3*b+a))-1/a*d/x

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 38.02, size = 11174, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)$

[Out] $\log\left(\left(2^{1/3}\right)\left(-\left(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 3*a*b^3*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 3*a^2*b^2*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\left(a^4*\left(4*a*c - b^2\right)^3\right)^{2/3}\left(2^{2/3}\right)\left(27*a^7*c^3*x*\left(4*a*c - b^2\right)*\left(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e\right) - \left(27*2^{1/3}\right)*a^{10}*b*c^3*\left(4*a*c - b^2\right)^2*\left(-\left(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 3*a*b^3*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 3*a^2*b^2*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\left(a^4*\left(4*a*c - b^2\right)^3\right)^{2/3}\right)/2*\left(-\left(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 3*a*b^3*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 3*a^2*b^2*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\left(a^4*\left(4*a*c - b^2\right)^3\right)^{1/3}\right)/6 + 36*a^9*c^6*d^3 - 108*a^{10}*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2)/18 + a^7*c^4*e*x*\left(a*e^2 + c*d^2 - b*d*e\right)^2*\left(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 3*a*b^3*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 3*a^2*b^2*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\left(54*\left(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2\right)\right)^{1/3} + \log\left(\left(2^{1/3}\right)\left(-\left(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 3*a*b^3*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} + 3*a^2*b^2*d*e^2*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*\left(-\left(4*a*c - b^2\right)^3\right)^{1/2}\right)\left(a^4*\left(4*a*c - b^2\right)^3\right)^{1/3}\right)$

$$\begin{aligned}
& 3c^2d^3 - 2a^2c^2d^3(-4ac - b^2)^{3/2} - 10ab^5cd^3 - 3ab^6d^2e + 4ab^2c^2d^3(-4ac - b^2)^{3/2} + 3ab^3d^2e(-4ac - b^2)^{3/2} + 27a^2b^4c^2d^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^2d^2e^2 + 6a^3c^2d^2e^2(-4ac - b^2)^{3/2} - 3a^2b^2d^2e^2(-4ac - b^2)^{3/2} - 72a^3b^2c^2d^2e - 9a^2b^2cd^2e(-4ac - b^2)^{3/2} \\
&)/(a^4(4ac - b^2)^{2/3})^{2/3} * ((2^{2/3}) * (27a^7c^3 * x * (4ac - b^2) * (b^4d^2 - 2a^3ce^2 + a^2b^2e^2 + 2a^2c^2d^2 - 2ab^3d^2e - 4ab^2cd^2 + 6a^2b^2cd^2e) - (27 * 2^{1/3}) * a^{10} * b^3 * c^3 * (4ac - b^2)^2 * (-b^7d^3 - a^3b^4e^3 - b^4d^3(-4ac - b^2)^{3/2} - 16a^5c^2e^3 - 32a^3b^3c^3d^3 + a^3b^2e^3(-4ac - b^2)^{3/2} + 8a^4b^2c^2e^3 + 3a^2b^5d^2e^2 + 48a^4c^3d^2e + 32a^2b^3c^2d^3 - 2a^2c^2d^3(-4ac - b^2)^{3/2} - 10ab^5cd^3 - 3ab^6d^2e + 4ab^2c^2d^3(-4ac - b^2)^{3/2} + 3ab^3d^2e(-4ac - b^2)^{3/2} + 27a^2b^4c^2d^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^2d^2e^2 + 6a^3c^2d^2e^2(-4ac - b^2)^{3/2} - 3a^2b^2d^2e^2(-4ac - b^2)^{3/2} - 72a^3b^2c^2d^2e - 9a^2b^2cd^2e(-4ac - b^2)^{3/2}) / (a^4(4ac - b^2)^{2/3}) / 2 * \\
& (-b^7d^3 - a^3b^4e^3 - b^4d^3(-4ac - b^2)^{3/2} - 16a^5c^2e^3 - 32a^3b^3c^3d^3 + a^3b^2e^3(-4ac - b^2)^{3/2} + 8a^4b^2c^2e^3 + 3a^2b^5d^2e^2 + 48a^4c^3d^2e + 32a^2b^3c^2d^3 - 2a^2c^2d^3(-4ac - b^2)^{3/2} - 10ab^5cd^3 - 3ab^6d^2e + 4ab^2c^2d^3(-4ac - b^2)^{3/2} + 3ab^3d^2e(-4ac - b^2)^{3/2} + 27a^2b^4c^2d^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^2d^2e^2 + 6a^3c^2d^2e^2(-4ac - b^2)^{3/2} - 3a^2b^2d^2e^2(-4ac - b^2)^{3/2} - 72a^3b^2c^2d^2e - 9a^2b^2cd^2e(-4ac - b^2)^{3/2}) / (a^4(4ac - b^2)^{2/3}) / 6 + \\
& 36a^9c^6d^3 - 108a^{10}c^5d^2e^2 + 9a^7b^4c^4d^3 - 45a^8b^2c^5d^3 + 108a^9b^2c^5d^2e - 27a^8b^3c^4d^2e + 27a^9b^2c^4d^2e^2) / 18 + a^7c^4e * x * (a^2 + cd^2 - bde)^2 * ((b^7d^3 - a^3b^4e^3 - b^4d^3(-4ac - b^2)^{3/2} - 16a^5c^2e^3 - 32a^3b^3c^3d^3 + a^3b^2e^3(-4ac - b^2)^{3/2} + 8a^4b^2c^2e^3 + 3a^2b^5d^2e^2 + 48a^4c^3d^2e + 32a^2b^3c^2d^3 - 2a^2c^2d^3(-4ac - b^2)^{3/2} - 10ab^5cd^3 - 3ab^6d^2e + 4ab^2c^2d^3(-4ac - b^2)^{3/2} + 3ab^3d^2e(-4ac - b^2)^{3/2} + 27a^2b^4c^2d^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^2d^2e^2 + 6a^3c^2d^2e^2(-4ac - b^2)^{3/2} - 3a^2b^2d^2e^2(-4ac - b^2)^{3/2} - 72a^3b^2c^2d^2e - 9a^2b^2cd^2e(-4ac - b^2)^{3/2}) / (54 * (a^4b^6 - 64a^7c^3 - 12a^5b^4c + 48a^6b^2c^2))^{1/3} - \log((2^{1/3}) * (3^{1/2}) * 1i - 1) * (-b^7d^3 - a^3b^4e^3 + b^4d^3(-4ac - b^2)^{3/2} - 16a^5c^2e^3 - 32a^3b^3c^3d^3 - a^3b^2e^3(-4ac - b^2)^{3/2} + 8a^4b^2c^2e^3 + 3a^2b^5d^2e^2 + 48a^4c^3d^2e + 32a^2b^3c^2d^3 + 2a^2c^2d^3(-4ac - b^2)^{3/2} - 10ab^5cd^3 - 3ab^6d^2e - 4ab^2c^2d^3(-4ac - b^2)^{3/2} - 3ab^3d^2e(-4ac - b^2)^{3/2} + 27a^2b^4c^2d^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^2d^2e^2 - 6a^3c^2d^2e^2(-4ac - b^2)^{3/2} - 3a^2b^2d^2e^2(-4ac - b^2)^{3/2} - 72a^3b^2c^2d^2e + 9a^2b^2cd^2e(-4ac - b^2)^{3/2}) / (a^4(4ac - b^2)^{2/3}) * (36a^9c^6d^3 - 108a^{10}c^5d^2e^2 + 9a^7b^4c^4d^3 - 45a^8b^2c^5d^3 - (2^{2/3}
\end{aligned}$$

$$\begin{aligned}
& 3) * (3^{(1/2)} * i + 1) * (27 * a^7 * c^3 * x * (4 * a * c - b^2) * (b^4 * d^2 - 2 * a^3 * c * e^2 + a^2 * b^2 * e^2 + 2 * a^2 * c^2 * d^2 - 2 * a * b^3 * d * e - 4 * a * b^2 * c * d^2 + 6 * a^2 * b * c * d * e) - \\
& (27 * 2^{(1/3)} * a^{10} * b * c^3 * (3^{(1/2)} * i - 1) * (4 * a * c - b^2)^2 * (- (b^7 * d^3 - a^3 * b^4 * e^3 + b^4 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 16 * a^5 * c^2 * e^3 - 32 * a^3 * b * c^3 * d^3 - a^3 * b * e^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 8 * a^4 * b^2 * c * e^3 + 3 * a^2 * b^5 * d * e^2 + 48 * a^4 * c^3 * d^2 * e + 32 * a^2 * b^3 * c^2 * d^3 + 2 * a^2 * c^2 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a * b^5 * c * d^3 - 3 * a * b^6 * d^2 * e - 4 * a * b^2 * c * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 3 * a * b^3 * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)} + 27 * a^2 * b^4 * c * d^2 * e - 24 * a^3 * b^3 * c * d * e^2 + 48 * a^4 * b * c^2 * d * e^2 - 6 * a^3 * c * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^2 * b^2 * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 72 * a^3 * b^2 * c^2 * d^2 * e + 9 * a^2 * b * c * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)}) / (a^4 * (4 * a * c - b^2)^3))^{(2/3)} / 4) * (- (b^7 * d^3 - a^3 * b^4 * e^3 + b^4 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 16 * a^5 * c^2 * e^3 - 32 * a^3 * b * c^3 * d^3 - a^3 * b * e^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 8 * a^4 * b^2 * c * e^3 + 3 * a^2 * b^5 * d * e^2 + 48 * a^4 * c^3 * d^2 * e + 32 * a^2 * b^3 * c^2 * d^3 + 2 * a^2 * c^2 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a * b^5 * c * d^3 - 3 * a * b^6 * d^2 * e - 4 * a * b^2 * c * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 3 * a * b^3 * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)} + 27 * a^2 * b^4 * c * d^2 * e - 24 * a^3 * b^3 * c * d * e^2 + 48 * a^4 * b * c^2 * d * e^2 - 6 * a^3 * c * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^2 * b^2 * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 72 * a^3 * b^2 * c^2 * d^2 * e + 9 * a^2 * b * c * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)}) / (a^4 * (4 * a * c - b^2)^3))^{(1/3)} / 12 + 108 * a^9 * b * c^5 * d^2 * e - 27 * a^8 * b^3 * c^4 * d^2 * e + 27 * a^9 * b^2 * c^4 * d * e^2) / 36 + a^7 * c^4 * e * x * (a * e^2 + c * d^2 - b * d * e)^2 * ((3^{(1/2)} * i) / 2 + 1/2) * ((b^7 * d^3 - a^3 * b^4 * e^3 + b^4 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 16 * a^5 * c^2 * e^3 - 32 * a^3 * b * c^3 * d^3 - a^3 * b * e^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 8 * a^4 * b^2 * c * e^3 + 3 * a^2 * b^5 * d * e^2 + 48 * a^4 * c^3 * d^2 * e + 32 * a^2 * b^3 * c^2 * d^3 + 2 * a^2 * c^2 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a * b^5 * c * d^3 - 3 * a * b^6 * d^2 * e - 4 * a * b^2 * c * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 3 * a * b^3 * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)} + 27 * a^2 * b^4 * c * d^2 * e - 24 * a^3 * b^3 * c * d * e^2 + 48 * a^4 * b * c^2 * d * e^2 - 6 * a^3 * c * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a^2 * b^2 * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 72 * a^3 * b^2 * c^2 * d^2 * e + 9 * a^2 * b * c * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)}) / (54 * (a^4 * b^6 - 64 * a^7 * c^3 - 12 * a^5 * b^4 * c + 48 * a^6 * b^2 * c^2)))^{(1/3)} - \log((2^{(1/3)} * (3^{(1/2)} * i - 1) * (- (b^7 * d^3 - a^3 * b^4 * e^3 - b^4 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 16 * a^5 * c^2 * e^3 - 32 * a^3 * b * c^3 * d^3 + a^3 * b * e^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 8 * a^4 * b^2 * c * e^3 + 3 * a^2 * b^5 * d * e^2 + 48 * a^4 * c^3 * d^2 * e + 32 * a^2 * b^3 * c^2 * d^3 - 2 * a^2 * c^2 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 10 * a * b^5 * c * d^3 - 3 * a * b^6 * d^2 * e + 4 * a * b^2 * c * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 3 * a * b^3 * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)} + 27 * a^2 * b^4 * c * d^2 * e - 24 * a^3 * b^3 * c * d * e^2 + 48 * a^4 * b * c^2 * d * e^2 + 6 * a^3 * c * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 3 * a^2 * b^2 * d * e^2 * (- (4 * a * c - b^2)^3)^{(1/2)} - 72 * a^3 * b^2 * c^2 * d^2 * e - 9 * a^2 * b * c * d^2 * e * (- (4 * a * c - b^2)^3)^{(1/2)}) / (a^4 * (4 * a * c - b^2)^3))^{(2/3)}) * (36 * a^9 * c^6 * d^3 - 108 * a^{10} * c^5 * d * e^2 + 9 * a^7 * b^4 * c^4 * d^3 - 45 * a^8 * b^2 * c^5 * d^3 - (2^{(2/3)} * (3^{(1/2)} * i + 1) * (27 * a^7 * c^3 * x * (4 * a * c - b^2) * (b^4 * d^2 - 2 * a^3 * c * e^2 + a^2 * b^2 * e^2 + 2 * a^2 * c^2 * d^2 - 2 * a * b^3 * d * e - 4 * a * b^2 * c * d^2 + 6 * a^2 * b * c * d * e) - (27 * 2^{(1/3)} * a^{10} * b * c^3 * (3^{(1/2)} * i - 1) * (4 * a * c - b^2)^2 * (- (b^7 * d^3 - a^3 * b^4 * e^3 - b^4 * d^3 * (- (4 * a * c - b^2)^3)^{(1/2)} - 16 * a^5 * c^2 * e^3 - 32 * a^3 * b * c^3 * d^3 + a^3 * b * e^3 * (- (4 * a * c - b^2)^3)^{(1/2)} + 8 * a^4 * b^2 * c * e^3 + 3 * a^2 * b^5 * d * e^2 + 48 * a^4 * c^3 * d^2 * e + 32 * a^2 * b^3 * c^2 * d^3 - 2 * a^2 * c^2 * d^3 * (- (4 * a
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2))}/(a^4*(4*a*c - b^2)^3))^{(2/3)})/4)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)))/(a^4*(4*a*c - b^2)^3))^{(1/3)})/12 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2)/36 + a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2*((3^(1/2)*1i)/2 + 1/2)*((b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} + \log(a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2 - (2^(1/3)*(3^(1/2)*1i + 1))*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)))/(a^4*(4*a*c - b^2)^3))^{(2/3)}*(36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2^(2/3)*(3^(1/2)*1i - 1))*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)))/(a^4*(4*a*c - b^2)^3))^{(2/3)})/4)*(-(b^7*d^3 - a^3
\end{aligned}$$

$$\begin{aligned}
& *b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3 \\
& *d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e \\
& ^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(a^4*(4*a*c - b^2)^3)^{(1/3)}/12 + 10*8*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2)/36)*((3^(1/2)*1i)/2 - 1/2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^{(1/3)} + \log(a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2 - (2^{(1/3)}*(3^{(1/2)*1i} + 1)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(a^4*(4*a*c - b^2)^3))^{(2/3)}*(36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^3 + (2^{(2/3)}*(3^{(1/2)*1i} - 1)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*b*c*d*e) + (27*2^{(1/3)}*a^10*b*c^3*(3^{(1/2)*1i} + 1)*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)}/(a^4*(4*a*c - b^2)^3))^{(2/3)})/4)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^3)^{1/2} - 3a^2b^2d^2e^2(-4ac - b^2)^3)^{1/2} - 72a^3b^2c^2 \\
& *d^2e - 9a^2b^2cd^2e(-4ac - b^2)^3)^{1/2})/(a^4(4ac - b^2)^3))^{1/3} \\
&)/12 + 108a^9b^2c^5d^2e - 27a^8b^3c^4d^2e + 27a^9b^2c^4d^2e^2 \\
&)/36)*((3^{1/2}*i)/2 - 1/2)*((b^7d^3 - a^3b^4e^3 - b^4d^3(-4ac - \\
& b^2)^3)^{1/2} - 16a^5c^2e^3 - 32a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b \\
& ^2)^3)^{1/2} + 8a^4b^2c^2e^3 + 3a^2b^5d^2e^2 + 48a^4c^3d^2e + 32a^ \\
& 2b^3c^2d^3 - 2a^2c^2d^3(-4ac - b^2)^3)^{1/2} - 10ab^5cd^3 - 3 \\
& *ab^6d^2e + 4ab^2cd^3(-4ac - b^2)^3)^{1/2} + 3ab^3d^2e(-4ac \\
& - b^2)^3)^{1/2} + 27a^2b^4cd^2e - 24a^3b^3cd^2e^2 + 48a^4b^2c^ \\
& 2d^2e^2 + 6a^3cd^2e^2(-4ac - b^2)^3)^{1/2} - 3a^2b^2d^2e^2(-4ac \\
& - b^2)^3)^{1/2} - 72a^3b^2c^2d^2e - 9a^2b^2cd^2e(-4ac - b^2)^3 \\
&)^{1/2})/(54(a^4b^6 - 64a^7c^3 - 12a^5b^4c + 48a^6b^2c^2))^{1/3} \\
& - d/(ax)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.19 \quad \int \frac{d+ex^3}{x^3(a+bx^3+cx^6)} dx$$

Optimal. Leaf size=655

$$\frac{c^{2/3} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{c^{2/3} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}}$$

[Out] $-1/2*d/a/x^2-1/6*c^{(2/3)*\ln(2^{(1/3)*c^{(1/3)*x+(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)})}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/a/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}+1/12*c^{(2/3)*\ln(2^{(2/3)*c^{(2/3)*x^2-2^{(1/3)*c^{(1/3)*x*(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)})}+(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)})}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/a/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}+1/6*c^{(2/3)*\arctan(1/3*(1-2*2^{(1/3)*c^{(1/3)*x/(b-(-4*a*c+b^2)^{(1/2)})^{(1/3)})})*3^{(1/2)})}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/a*3^{(1/2)/(b-(-4*a*c+b^2)^{(1/2)})^{(2/3)}-1/6*c^{(2/3)*\ln(2^{(1/3)*c^{(1/3)*x+(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)})}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})})*2^{(2/3)}/a/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}+1/12*c^{(2/3)*\ln(2^{(2/3)*c^{(2/3)*x^2-2^{(1/3)*c^{(1/3)*x*(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)})}+(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)})}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/a/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}+1/6*c^{(2/3)*\arctan(1/3*(1-2*2^{(1/3)*c^{(1/3)*x/(b+(-4*a*c+b^2)^{(1/2)})^{(1/3)})})*3^{(1/2)})}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(2/3)}/a*3^{(1/2)/(b+(-4*a*c+b^2)^{(1/2)})^{(2/3)}}$

Rubi [A] time = 1.11, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1504, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}} + \frac{c^{2/3} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \log \left(-\sqrt[3]{2} \sqrt[3]{c} x \sqrt[3]{b - \sqrt{b^2 - 4ac}} + \left(b - \sqrt{b^2 - 4ac} \right)^{2/3} + 2^{2/3} c^{2/3} x^2 \right)}{6\sqrt[3]{2} a \left(b - \sqrt{b^2 - 4ac} \right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]

[Out] $-d/(2*a*x^2) + (c^{(2/3)*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c] * \text{ArcTan}[(1 - (2*2^{(1/3)*c^{(1/3)*x}/(b - \text{Sqrt}[b^2 - 4*a*c])^{(1/3)})/\text{Sqrt}[3]])/(2^{(1/3)*\text{Sqrt}[3]*a*(b - \text{Sqrt}[b^2 - 4*a*c])^{(2/3)})} + (c^{(2/3)*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])})$

$$\begin{aligned}
& - 4*a*c]) * \text{ArcTan}[(1 - (2*2^{1/3}) * c^{1/3} * x) / (b + \text{Sqrt}[b^2 - 4*a*c])^{1/3}) \\
& / \text{Sqrt}[3]] / (2^{1/3} * \text{Sqrt}[3] * a * (b + \text{Sqrt}[b^2 - 4*a*c])^{2/3}) - (c^{2/3} * (d \\
& + (b*d - 2*a*e) / \text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{1/3} + 2^{1/3} \\
& * c^{1/3} * x]) / (3*2^{1/3} * a * (b - \text{Sqrt}[b^2 - 4*a*c])^{2/3}) - (c^{2/3} * (d - \\
& (b*d - 2*a*e) / \text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{1/3} + 2^{1/3} \\
& * c^{1/3} * x]) / (3*2^{1/3} * a * (b + \text{Sqrt}[b^2 - 4*a*c])^{2/3}) + (c^{2/3} * (d + \\
& (b*d - 2*a*e) / \text{Sqrt}[b^2 - 4*a*c]) * \text{Log}[(b - \text{Sqrt}[b^2 - 4*a*c])^{2/3} - 2^{1/3} \\
&) * c^{1/3} * (b - \text{Sqrt}[b^2 - 4*a*c])^{1/3} * x + 2^{2/3} * c^{2/3} * x^2]) / (6*2^{1/3} \\
&) * a * (b - \text{Sqrt}[b^2 - 4*a*c])^{2/3}) + (c^{2/3} * (d - (b*d - 2*a*e) / \text{Sqrt}[b^2 - \\
& 4*a*c]) * \text{Log}[(b + \text{Sqrt}[b^2 - 4*a*c])^{2/3} - 2^{1/3} * c^{1/3} * (b + \text{Sqrt}[b^2 \\
& - 4*a*c])^{1/3} * x + 2^{2/3} * c^{2/3} * x^2]) / (6*2^{1/3} * a * (b + \text{Sqrt}[b^2 - 4*a* \\
& c])^{2/3})
\end{aligned}$$

Rule 31

$\text{Int}[(a + (b \cdot x)^{-1}), x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x, x]] / b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 200

$\text{Int}[(a + (b \cdot x)^3)^{-1}, x_Symbol] \rightarrow \text{Dist}[1 / (3 \cdot \text{Rt}[a, 3]^2), \text{Int}[1 / (\text{Rt}[a, 3] + \text{Rt}[b, 3] \cdot x), x], x] + \text{Dist}[1 / (3 \cdot \text{Rt}[a, 3]^2), \text{Int}[(2 \cdot \text{Rt}[a, 3] - \text{Rt}[b, 3] \cdot x) / (\text{Rt}[a, 3]^2 - \text{Rt}[a, 3] \cdot \text{Rt}[b, 3] \cdot x + \text{Rt}[b, 3]^2 \cdot x^2), x], x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] \text{ ; FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 617

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4 \cdot \text{Simplify}[(a \cdot c) / b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1 / (q - x^2), x], x, 1 + (2 \cdot c \cdot x) / b], x] \text{ ; RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4 \cdot a \cdot c])] \text{ ; FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4 \cdot a \cdot c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x)) / (a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] \text{ ; FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])
```

Rule 1504

```
Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^(p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^3}{x^3(a + bx^3 + cx^6)} dx &= -\frac{d}{2ax^2} - \frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6} dx}{2a} \\
&= -\frac{d}{2ax^2} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} - \frac{\left(c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^3} dx}{2a} \\
&= -\frac{d}{2ax^2} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}} + \sqrt[3]{c}x} dx}{3\sqrt[3]{2}a\left(b + \sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}\right)^{2/3}} dx}{3\sqrt[3]{2}a\left(b + \sqrt{b^2-4ac}\right)^{2/3}} \\
&= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \log\left(\sqrt[3]{b - \sqrt{b^2-4ac}} + \sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}} \\
&= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b - \sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b - \sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b + \sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b + \sqrt{b^2-4ac}\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.14

$$-\frac{\text{RootSum}\left[\#1^6c + \#1^3b + a\&, \frac{\#1^3cd \log(x-\#1) - ae \log(x-\#1) + bd \log(x-\#1)}{2\#1^5c + \#1^2b}\&\right]}{3a} - \frac{d}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)), x]

[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) &]/(3*a)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^3 + d}{(cx^6 + bx^3 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)/((c*x^6 + b*x^3 + a)*x^3), x)

maple [C] time = 0.01, size = 68, normalized size = 0.10

$$\frac{\left(-\text{RootOf}(_Z^6c + _Z^3b + a)^3 cd + ae - bd\right) \ln\left(-\text{RootOf}(_Z^6c + _Z^3b + a) + x\right)}{3a\left(2\text{RootOf}(_Z^6c + _Z^3b + a)^5 c + \text{RootOf}(_Z^6c + _Z^3b + a)^2 b\right)} - \frac{d}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)

[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(-_R+x),_R=RootOf(_Z^6*c+_Z^3*b+a))-1/2/a*d/x^2

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 37.90, size = 13466, normalized size = 20.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)), x)$

[Out] $\log(- (2^{(2/3)}*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a^5*(4*a*c - b^2)^3))^{(1/3)}*((2^{(1/3)}*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (81*2^{(2/3)}*a^{10}*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a^5*(4*a*c - b^2)^3))^{(1/3)})/2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)))/(a^5*(4*a*c - b^2)^3))^{(2/3)})/18 + 36*a^{10}*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2)/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e -$

$$\begin{aligned}
& 27a^3b^4c^2d^2e^2 + 96a^4b^3c^3d^2e + 5a^2b^3c^2d^3(-4ac - b^2)^3 \\
& \left(\frac{1}{2} + 3a^2b^3d^2e^2(-4ac - b^2)^3 \right)^{\frac{1}{2}} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 \\
& - 6a^3c^2d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 12a^2b^2c^2d^2e(-4ac - b^2)^3 \right)^{\frac{1}{2}} - 9a^3b^3c^2d^2e^2 \\
& \left(\frac{1}{2} + 12a^2b^2c^2d^2e(-4ac - b^2)^3 \right)^{\frac{1}{2}} \Big/ (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{\frac{1}{3}} + \log(-2^{\frac{2}{3}}) \\
& \left((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3(-4ac - b^2)^3)^{\frac{1}{2}} + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 - 2a^4c^3e^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b^2)^3 \left(\frac{1}{2} - 11ab^6cd^3 - 3ab^7d^2e + 5ab^3cd^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3ab^4d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 30a^2b^5cd^2e - 27a^3b^4c^2d^2e + 96a^4b^3c^3d^2e - 5a^2b^3c^2d^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} - 3a^2b^3d^2e^2(-4ac - b^2)^3 \left(\frac{1}{2} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& - 12a^2b^2c^2d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 9a^3b^3cd^2e^2(-4ac - b^2)^3 \right)^{\frac{1}{2}} \Big/ (a^5(4ac - b^2)^3)^{\frac{1}{3}} \left(2^{\frac{1}{3}}(81a^8c^3x(4ac - b^2)^2(ab^2e - b^2d + acd) + (81x^{\frac{2}{3}})^{\frac{1}{3}}) \right. \\
& \left. + a^{10}b^3c^3(4ac - b^2)^2(b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3(-4ac - b^2)^3)^{\frac{1}{2}} + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 - 2a^4c^3e^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b^2)^3 \left(\frac{1}{2} - 11ab^6cd^3 - 3ab^7d^2e + 5ab^3cd^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3ab^4d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 30a^2b^5cd^2e - 27a^3b^4c^2d^2e + 96a^4b^3c^3d^2e - 5a^2b^3c^2d^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} - 3a^2b^3d^2e^2(-4ac - b^2)^3 \left(\frac{1}{2} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 9a^3b^3cd^2e^2(-4ac - b^2)^3 \left(\frac{1}{2} + 9a^3b^3cd^2e^2(-4ac - b^2)^3 \right)^{\frac{1}{2}} \Big/ (a^5(4ac - b^2)^3)^{\frac{1}{3}} \Big/ 2 \left((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3(-4ac - b^2)^3)^{\frac{1}{2}} + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 - 2a^4c^3e^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b^2)^3 \left(\frac{1}{2} - 11ab^6cd^3 - 3ab^7d^2e + 5ab^3cd^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& + 3ab^4d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 30a^2b^5cd^2e - 27a^3b^4c^2d^2e + 96a^4b^3c^3d^2e - 5a^2b^3c^2d^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} - 3a^2b^3d^2e^2(-4ac - b^2)^3 \left(\frac{1}{2} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e(-4ac - b^2)^3 \right)^{\frac{1}{2}} \\
& - 12a^2b^2c^2d^2e(-4ac - b^2)^3 \left(\frac{1}{2} + 9a^3b^3cd^2e^2(-4ac - b^2)^3 \right)^{\frac{1}{2}} \Big/ (a^5(4ac - b^2)^3)^{\frac{2}{3}} \Big/ 18 + 36a^{10}c^5e^3 + 72a^8b^3c^6d^3 - 108a^9c^6d^2e + 9a^6b^5c^4d^3 - 54a^7b^3c^5d^3 - 9a^9b^2c^4e^3 - 108a^9b^3c^5d^2e^2 - 27a^7b^4c^4d^2e + 135a^8b^2c^5d^2e + 27a^8b^3c^4d^2e^2) \Big/ 6 - 3a^6c^5x(2a^3e^4 - 2ac^2d^4 + b^2cd^4 - b^3d^3e + 3ab^2d^2e^2 - 4a^2b^3d^2e^3) \left((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3(-4ac - b^2)^3)^{\frac{1}{2}} + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 - 2a^4c^3e^3(-4ac - b^2)^3 \right)^{\frac{1}{2}} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b^2)^3 \left(\frac{1}{2} - 11ab^6cd^3 - 3ab^7d^2e + 5ab^3cd^3(-4ac - b^2)^3 \right)^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
 & 2)^3)^{(1/2)} + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - \\
 & 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - \\
 & 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e \\
 & + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 12*a^2* \\
 & b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))} \\
 & / (54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2))^((1/3) - \\
 & d/(2*a*x^2) + \log((2^{(2/3)}*(3^{(1/2)}*1i - 1)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + \\
 & b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
 & 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - \\
 & b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - \\
 & 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + \\
 & 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
 & 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} \\
 & + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))} / (a^5*(4*a*c - b^2)^3))^((1/3)) / \\
 & 4)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + \\
 & 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - \\
 & 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^{(1/2)} - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + \\
 & 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2*d^2*e + \\
 & 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - \\
 & 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^{(1/2))} / (a^5*(4*a*c - b^2)^3))^((2/3)) / 36 - \\
 & 9*a^6*b^5*c^4*d^3 + 54*a^7*b^3*c^5*d^3 + 9*a^9*b^2*c^4*e^3 + 108*a^9*b*c^5*d*e^2 + 27*a^7*b^4*c^4*d^2*e - 135*a^8*b^2*c^5*d^2*e - 27*a^8*b^3*c^4*d*e^2) / 12 - \\
 & 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3)) * ((3^{(1/2)}*1i) / 2 - 1/2) * (-b^8*d^3 - a^3*b^5*e^3 + \\
 & 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2
 \end{aligned}$$

$$\begin{aligned}
& - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 - a^3b^2e^3 * \\
& (-4ac - b^2)^3)^{(1/2)} - 11ab^6cd^3 - 3ab^7d^2e - 5ab^3cd^3 * \\
& (-4ac - b^2)^3)^{(1/2)} - 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + 30a^2b^5cd^2e - \\
& 27a^3b^4cd^2e^2 + 96a^4b^3cd^2e + 5a^2b^3cd^3 * (-4ac - b^2)^3)^{(1/2)} + \\
& 3a^2b^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 - \\
& 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} + 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} - \\
& 9a^3b^3cd^2e * (-4ac - b^2)^3)^{(1/2)} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2 \\
&))^{(1/3)} + \log((2^{(2/3)} * (3^{(1/2)} * i - 1) * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - \\
& b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + 8a^4b^3ce^3 - 16a^5b^3c^2e^3 - 2a^4ce^3 * \\
& (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - \\
& 56a^3b^2c^3d^3 + a^3b^2e^3 * (-4ac - b^2)^3)^{(1/2)} - 11ab^6cd^3 - 3ab^7d^2e + \\
& 5ab^3cd^3 * (-4ac - b^2)^3)^{(1/2)} + 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + \\
& 30a^2b^5cd^2e - 27a^3b^4cd^2e^2 + 96a^4b^3cd^2e - 5a^2b^3cd^3 * (-4ac - b^2)^3)^{(1/2)} - \\
& 3a^2b^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 + \\
& 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} - 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} + \\
& 9a^3b^3cd^2e * (-4ac - b^2)^3)^{(1/2)} / (a^5(4ac - b^2)^3)^{(1/3)} * (108a^9c^6d^2e - \\
& 72a^8b^3c^6d^3 - 36a^10c^5e^3 + (2^{(1/3)} * (3^{(1/2)} * i + 1) * (81a^8c^3 * x * (4ac - b^2)^2 * \\
& (a * b * e - b^2 * d + a * c * d) + (81 * 2^{(2/3)} * a^10 * b^3 * c^3 * (3^{(1/2)} * i - 1) * \\
& (4ac - b^2)^2 * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + \\
& 8a^4b^3ce^3 - 16a^5b^3c^2e^3 - 2a^4ce^3 * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - \\
& 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3 * (-4ac - b^2)^3)^{(1/2)} - \\
& 11ab^6cd^3 - 3ab^7d^2e + 5ab^3cd^3 * (-4ac - b^2)^3)^{(1/2)} + 3ab^4d^2e * (-4ac - \\
& b^2)^3)^{(1/2)} + 30a^2b^5cd^2e - 27a^3b^4cd^2e^2 + 96a^4b^3cd^2e - 5a^2b^3cd^3 * \\
& (-4ac - b^2)^3)^{(1/2)} - 3a^2b^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + \\
& 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} - 12a^2b^2cd^2e * (-4ac - \\
& b^2)^3)^{(1/2)} + 9a^3b^3cd^2e * (-4ac - b^2)^3)^{(1/2)} / (a^5(4ac - b^2)^3)^{(1/3)} / 4) \\
& * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + 8a^4b^3ce^3 - \\
& 16a^5b^3c^2e^3 - 2a^4ce^3 * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + \\
& 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3 * (-4ac - b^2)^3)^{(1/2)} - 11ab^6cd^3 - \\
& 3ab^7d^2e + 5ab^3cd^3 * (-4ac - b^2)^3)^{(1/2)} + 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + \\
& 30a^2b^5cd^2e - 27a^3b^4cd^2e^2 + 96a^4b^3cd^2e - 5a^2b^3cd^3 * (-4ac - b^2)^3)^{(1/2)} - \\
& 3a^2b^3d^2e * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e * \\
& (-4ac - b^2)^3)^{(1/2)} - 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} + 9a^3b^3cd^2e * \\
& (-4ac - b^2)^3)^{(1/2)} / (a^5(4ac - b^2)^3)^{(2/3)} / 36 - 9a^6b^5c^4d^3 + 54a^7b^3c^5d^3 + \\
& 9a^9b^2c^4e^3 + 108a^9b^3c^5d^2e^2 + 27a^7b^4c^4d^2e - 135a^8b^2c^5d^2e - 27a^8b^3c^4d^2e^2) \\
& / 12 - 3a^6c^5 * x * (2a^3e^4 - 2ac^2d^4 + b^2cd^4 - b^3d^3e + 3ab^2d^2e^2 - 4a^2b^2d^2e^3) * \\
& ((3^{(1/2)} * i) / 2 - 1/2) * (-b^8d^3 - a^3b^5e^3
\end{aligned}$$

$$\begin{aligned}
& 3 + 16a^4c^4d^3 - b^5d^3(-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5b^2c^2e^3 - 2a^4c^3e^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3(-4ac - b^2)^3)^{(1/2)} - 11a^2b^6c^3d^3 - 3a^2b^7d^2e^2 + 5a^2b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^4d^2e^2(-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e^2 - 27a^3b^4c^3d^2e^2 + 96a^4b^3c^3d^2e^2 - 5a^2b^2c^2d^3(-4ac - b^2)^3)^{(1/2)} - 3a^2b^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e^2 + 72a^4b^2c^2d^2e^2 + 6a^3c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} - 12a^2b^2c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} + 9a^3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2))} / (5a^4b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{(1/3)} - \log((2^{(2/3)}(3^{(1/2)}i + 1)((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 + b^5d^3(-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5b^2c^2e^3 + 2a^4c^3e^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 - a^3b^2e^3(-4ac - b^2)^3)^{(1/2)} - 11a^2b^6c^3d^3 - 3a^2b^7d^2e^2 - 5a^2b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} - 3a^2b^4d^2e^2(-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e^2 - 27a^3b^4c^3d^2e^2 + 96a^4b^3c^3d^2e^2 + 5a^2b^2c^2d^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e^2 + 72a^4b^2c^2d^2e^2 - 6a^3c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 9a^3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2))} / (a^5(4ac - b^2)^3)^{(1/3)} * (36a^10c^5e^3 + 72a^8b^3c^6d^3 - 108a^9c^6d^2e + (2^{(1/3)}(3^{(1/2)}i - 1)(81a^8c^3x(4ac - b^2)^2(ab^2e - b^2d + acd) - (81*2^{(2/3)}a^10b^3c^3(3^{(1/2)}i + 1)(4ac - b^2)^2((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 + b^5d^3(-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5b^2c^2e^3 + 2a^4c^3e^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 - a^3b^2e^3(-4ac - b^2)^3)^{(1/2)} - 11a^2b^6c^3d^3 - 3a^2b^7d^2e^2 - 5a^2b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} - 3a^2b^4d^2e^2(-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e^2 - 27a^3b^4c^3d^2e^2 + 96a^4b^3c^3d^2e^2 + 5a^2b^2c^2d^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e^2 + 72a^4b^2c^2d^2e^2 - 6a^3c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 9a^3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2))} / (a^5(4ac - b^2)^3)^{(1/3))} / 4 * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 + b^5d^3(-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5b^2c^2e^3 + 2a^4c^3e^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^2e^2 - 48a^5c^3d^2e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 - a^3b^2e^3(-4ac - b^2)^3)^{(1/2)} - 11a^2b^6c^3d^3 - 3a^2b^7d^2e^2 - 5a^2b^3c^3d^3(-4ac - b^2)^3)^{(1/2)} - 3a^2b^4d^2e^2(-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e^2 - 27a^3b^4c^3d^2e^2 + 96a^4b^3c^3d^2e^2 + 5a^2b^2c^2d^3(-4ac - b^2)^3)^{(1/2)} + 3a^2b^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e^2 + 72a^4b^2c^2d^2e^2 - 6a^3c^2d^2e^2(-4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^3d^2e^2(-4ac - b^2)^3)^{(1/2)} - 9a^3b^3c^3d^2e^2(-4ac - b^2)^3)^{(1/2))} / (a^5(4ac - b^2)^3)^{(2/3))} / 36 + 9a^6b^5c^4d^3 - 54a^7b^3c^5d^3 - 9a^9b^2c^4e^3 - 108a^9b^3c^5d^2e - 27a^7b^4c^4d^2e + 135a^8b^2c^5d^2e + 27a^8b^3c^4d^2e)) /
\end{aligned}$$

$$\begin{aligned}
& 12 - 3a^6c^5x(2a^3e^4 - 2ac^2d^4 + b^2cd^4 - b^3d^3e + 3ab^2 \\
& *d^2e^2 - 4a^2bd^3e^3) * ((3^{(1/2)} * i) / 2 + 1/2) * (-b^8d^3 - a^3b^5e^3 \\
& + 16a^4c^4d^3 + b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5 \\
& b^3c^2e^3 + 2a^4c^3e^3 * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^3e^2 - 48 \\
& a^5c^3d^3e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 - a^3b^2e^3 * (-4 \\
& ac - b^2)^3)^{(1/2)} - 11ab^6cd^3 - 3ab^7d^2e - 5ab^3c^3d^3 * (-4 \\
& ac - b^2)^3)^{(1/2)} - 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c \\
& *d^2e - 27a^3b^4c^3d^3e^2 + 96a^4b^3c^3d^2e + 5a^2b^3c^2d^3 * (-4ac \\
& - b^2)^3)^{(1/2)} + 3a^2b^3d^3e^2 * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2 \\
& *d^2e + 72a^4b^2c^2d^3e^2 - 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} + \\
& 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} - 9a^3b^3cd^2e * (-4ac - b \\
& ^2)^3)^{(1/2)} / (54(a^5b^6 - 64a^8c^3 - 12a^6b^4c + 48a^7b^2c^2))^{(1/3)} - \\
& \log((2^{(2/3)} * (3^{(1/2)} * i + 1) * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - \\
& b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 \\
& - 2a^4c^3e^3 * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^3e^2 - 48a^5c^3d^3e^2 \\
& + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3 * (-4ac - b^2)^3)^{(1/2)} - \\
& 11ab^6cd^3 - 3ab^7d^2e + 5ab^3c^3d^3 * (-4ac - b^2)^3)^{(1/2)} + \\
& 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e - 27a^3b^4c^3d^3e^2 \\
& + 96a^4b^3c^3d^2e - 5a^2b^3c^2d^3 * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^3d^3e^2 \\
& * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^3e^2 + 6a^3c^2d^2e \\
& * (-4ac - b^2)^3)^{(1/2)} - 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} + 9a^3b^3cd^2e \\
& * (-4ac - b^2)^3)^{(1/2)} / (a^5(4ac - b^2)^3)^{(1/3)} * (36a^{10}c^5e^3 + 72a^8b^3c^6d^3 - \\
& 108a^9c^6d^2e + (2^{(1/3)} * (3^{(1/2)} * i - 1) * (81a^8c^3x(4ac - b^2)^2 * (ab^2e \\
& - b^2d + acd) - (81 * 2^{(2/3)} * a^{10}b^3c^3 * (3^{(1/2)} * i + 1) * (4ac - b^2)^2 \\
& * ((b^8d^3 - a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3 * (-4ac - b^2)^3)^{(1/2)} \\
& + 8a^4b^3c^3e^3 - 16a^5b^3c^2e^3 - 2a^4c^3e^3 * (-4ac - b^2)^3)^{(1/2)} \\
& + 3a^2b^6d^3e^2 - 48a^5c^3d^3e^2 + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3 * \\
& (-4ac - b^2)^3)^{(1/2)} - 11ab^6cd^3 - 3ab^7d^2e + 5ab^3c^3d^3 * (-4ac - \\
& b^2)^3)^{(1/2)} + 3ab^4d^2e * (-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e - \\
& 27a^3b^4c^3d^3e^2 + 96a^4b^3c^3d^2e - 5a^2b^3c^2d^3 * (-4ac - b^2)^3)^{(1/2)} \\
& - 3a^2b^3d^3e^2 * (-4ac - b^2)^3)^{(1/2)} - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^3e^2 \\
& + 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} - 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} \\
& + 9a^3b^3cd^2e * (-4ac - b^2)^3)^{(1/2)} / (a^5(4ac - b^2)^3)^{(1/3)} / 4 * ((b^8d^3 - \\
& a^3b^5e^3 + 16a^4c^4d^3 - b^5d^3 * (-4ac - b^2)^3)^{(1/2)} + 8a^4b^3c^3e^3 - \\
& 16a^5b^3c^2e^3 - 2a^4c^3e^3 * (-4ac - b^2)^3)^{(1/2)} + 3a^2b^6d^3e^2 - 48a^5c^3d^3e^2 \\
& + 41a^2b^4c^2d^3 - 56a^3b^2c^3d^3 + a^3b^2e^3 * (-4ac - b^2)^3)^{(1/2)} - \\
& 11ab^6cd^3 - 3ab^7d^2e + 5ab^3c^3d^3 * (-4ac - b^2)^3)^{(1/2)} + 3ab^4d^2e * \\
& (-4ac - b^2)^3)^{(1/2)} + 30a^2b^5c^3d^2e - 27a^3b^4c^3d^3e^2 + 96a^4b^3c^3d^2e - \\
& 5a^2b^3c^2d^3 * (-4ac - b^2)^3)^{(1/2)} - 3a^2b^3d^3e^2 * (-4ac - b^2)^3)^{(1/2)} \\
& - 96a^3b^3c^2d^2e + 72a^4b^2c^2d^3e^2 + 6a^3c^2d^2e * (-4ac - b^2)^3)^{(1/2)} - \\
& 12a^2b^2cd^2e * (-4ac - b^2)^3)^{(1/2)} + 9a^3b^3cd^2e * (-4ac - b^2)^3)^{(1/2)} / \\
& (a^5(4ac - b^2)^3)^{(2/3)} / 36 + 9a^6
\end{aligned}$$

$$\begin{aligned} & *b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 108*a^9*b*c^5*d*e^2 \\ & - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*b^3*c^4*d*e^2)/12 \\ & - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*d^3*e + 3*a*b^2*d \\ & ^2*e^2 - 4*a^2*b*d*e^3))*((3^{(1/2)}*i)/2 + 1/2)*(-(b^8*d^3 - a^3*b^5*e^3 + \\ & 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c - b^2)^3)^{(1/2)} + 8*a^4*b^3*c*e^3 - 16*a^ \\ & 5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^{(1/2)} + 3*a^2*b^6*d*e^2 - 48*a \\ & ^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a \\ & *c - b^2)^3)^{(1/2)} - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a* \\ & c - b^2)^3)^{(1/2)} + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 30*a^2*b^5*c*d \\ & ^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - \\ & b^2)^3)^{(1/2)} - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - 96*a^3*b^3*c^2* \\ & d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} - 1 \\ & 2*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^3)^{(1/2)} + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2 \\ &)^3)^{(1/2)})/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2))^{(1 \\ & /3)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)

[Out] Timed out

$$3.20 \quad \int \frac{x^8(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=46

$$-\frac{x^6}{6} - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{6} \log(x^6 - x^3 + 1)$$

[Out] $-1/6*x^6+1/6*\ln(x^6-x^3+1)-1/9*\arctan(1/3*(-2*x^3+1)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{x^6}{6} + \frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] $-x^6/6 - \text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[1 - x^3 + x^6]/6$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 800

`Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]`

Rule 1474

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]`

Rubi steps

$$\begin{aligned}
 \int \frac{x^8(1-x^3)}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(1-x)x^2}{1-x+x^2} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(-x + \frac{x}{1-x+x^2} \right) dx, x, x^3 \right) \\
 &= -\frac{x^6}{6} + \frac{1}{3} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^3 \right) \\
 &= -\frac{x^6}{6} + \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) + \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\
 &= -\frac{x^6}{6} + \frac{1}{6} \log(1-x^3+x^6) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
 &= -\frac{x^6}{6} - \frac{\tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{6} \log(1-x^3+x^6)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$-\frac{x^6}{6} + \frac{\tan^{-1} \left(\frac{2x^3-1}{\sqrt{3}} \right)}{3\sqrt{3}} + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] $-1/6*x^6 + \text{ArcTan}[(-1 + 2*x^3)/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[1 - x^3 + x^6]/6$

fricas [A] time = 0.72, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) + \frac{1}{6}\log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")

[Out] $-1/6*x^6 + 1/9*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^3 - 1)) + 1/6*\text{log}(x^6 - x^3 + 1)$

giac [A] time = 0.42, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) + \frac{1}{6}\log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out] $-1/6*x^6 + 1/9*\text{sqrt}(3)*\text{arctan}(1/3*\text{sqrt}(3)*(2*x^3 - 1)) + 1/6*\text{log}(x^6 - x^3 + 1)$

maple [A] time = 0.00, size = 38, normalized size = 0.83

$$-\frac{x^6}{6} + \frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} + \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x)

[Out] $-1/6*x^6+1/6*\ln(x^6-x^3+1)+1/9*3^{(1/2)}*\text{arctan}(1/3*(2*x^3-1)*3^{(1/2)})$

maxima [A] time = 0.99, size = 37, normalized size = 0.80

$$-\frac{1}{6}x^6 + \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) + \frac{1}{6}\log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] $-1/6*x^6 + 1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) + 1/6*\log(x^6 - x^3 + 1)$

mupad [B] time = 0.06, size = 39, normalized size = 0.85

$$\frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9} - \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1), x)`

[Out] $\log(x^6 - x^3 + 1)/6 - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^3)/3))/9 - x^6/6$

sympy [A] time = 0.14, size = 42, normalized size = 0.91

$$-\frac{x^6}{6} + \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(-x**3+1)/(x**6-x**3+1), x)`

[Out] $-x**6/6 + \log(x**6 - x**3 + 1)/6 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**3/3 - \sqrt{3}/3)/9$

$$3.21 \quad \int \frac{x^5(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=31

$$-\frac{x^3}{3} - \frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] $-1/3*x^3-2/9*\arctan(1/3*(-2*x^3+1)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1474, 773, 618, 204}

$$-\frac{x^3}{3} - \frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] `Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]`

[Out] $-x^3/3 - (2*\text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3])$

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 773

`Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 1474

`Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)`

$/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] \&\& EqQ[n2, 2*n] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^5(1-x^3)}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{(1-x)x}{1-x+x^2} dx, x, x^3 \right) \\ &= -\frac{x^3}{3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) \\ &= -\frac{x^3}{3} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\ &= -\frac{x^3}{3} - \frac{2 \tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2 \tan^{-1} \left(\frac{2x^3-1}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] -1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])

fricas [A] time = 0.50, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3} \arctan \left(\frac{1}{3}\sqrt{3}(2x^3 - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1), x, algorithm="fricas")

[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

giac [A] time = 0.58, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3} \arctan \left(\frac{1}{3}\sqrt{3}(2x^3 - 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

maple [A] time = 0.00, size = 25, normalized size = 0.81

$$-\frac{x^3}{3} + \frac{2\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x)

[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$-\frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9} - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)

[Out] - (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3

sympy [A] time = 0.12, size = 32, normalized size = 1.03

$$-\frac{x^3}{3} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)

[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9

$$3.22 \quad \int \frac{x^2(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1)$$

[Out] $-1/6*\ln(x^6-x^3+1)-1/9*\arctan(1/3*(-2*x^3+1)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] $-\text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) - \text{Log}[1 - x^3 + x^6]/6$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (
e_)*(x_)^(n_)]^(q_), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2(1-x^3)}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1-x}{1-x+x^2} dx, x, x^3 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\ &= -\frac{1}{6} \log(1-x^3+x^6) - \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{1}{6} \log(1-x^3+x^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^3-1}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6), x]
```

```
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

fricas [A] time = 0.61, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^3 - 1) \right) - \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1), x, algorithm="fricas")
```

[Out] $1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) - 1/6*\log(x^6 - x^3 + 1)$

giac [A] time = 0.57, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")`

[Out] $1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) - 1/6*\log(x^6 - x^3 + 1)$

maple [A] time = 0.00, size = 33, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9} - \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-x^3+1)/(x^6-x^3+1),x)`

[Out] $-1/6*\ln(x^6-x^3+1)+1/9*3^{(1/2)}*\arctan(1/3*(2*x^3-1)*3^{(1/2)})$

maxima [A] time = 0.95, size = 32, normalized size = 0.82

$$\frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")`

[Out] $1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) - 1/6*\log(x^6 - x^3 + 1)$

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)`

[Out] $-\log(x^6 - x^3 + 1)/6 - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^3)/3))/9$

sympy [A] time = 0.14, size = 37, normalized size = 0.95

$$-\frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)

[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9

$$3.23 \quad \int \frac{1-x^3}{x(1-x^3+x^6)} dx$$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

[Out] $\ln(x) - 1/6 * \ln(x^6 - x^3 + 1) + 1/9 * \arctan(1/3 * (-2 * x^3 + 1) * 3^{(1/2)}) * 3^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^6 - x^3 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x^3)/(x*(1 - x^3 + x^6)), x]$

[Out] $\text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]]/(3*\text{Sqrt}[3]) + \text{Log}[x] - \text{Log}[1 - x^3 + x^6]/6$

Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_)]/[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x(1-x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1-x}{x(1-x+x^2)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2} \right) dx, x, x^3 \right) \\
&= \log(x) - \frac{1}{3} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^3 \right) \\
&= \log(x) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) \\
&= \log(x) - \frac{1}{6} \log(1-x^3+x^6) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
&= \frac{\tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}} + \log(x) - \frac{1}{6} \log(1-x^3+x^6)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x) - \frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1)}{2\#1^3 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)), x]

[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) &]/3

fricas [A] time = 0.86, size = 34, normalized size = 0.83

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log(x^6-x^3+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x/(x^6-x^3+1), x, algorithm="fricas")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(x)

giac [A] time = 0.59, size = 35, normalized size = 0.85

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log(x^6-x^3+1)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x/(x^6-x^3+1), x, algorithm="giac")

[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(abs(x))

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln(x^6-x^3+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)/x/(x^6-x^3+1), x)

[Out] -1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{6}\log(x^6-x^3+1)+\frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")

[Out] $-1/9*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) - 1/6*\log(x^6 - x^3 + 1) + 1/3*\log(x^3)$

mupad [B] time = 1.86, size = 36, normalized size = 0.88

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)

[Out] $\log(x) - \log(x^6 - x^3 + 1)/6 + (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^3)/3))/9$

sympy [A] time = 0.15, size = 41, normalized size = 1.00

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)

[Out] $\log(x) - \log(x**6 - x**3 + 1)/6 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**3/3 - \sqrt{3}/3)/9$

$$3.24 \quad \int \frac{1-x^3}{x^4(1-x^3+x^6)} dx$$

Optimal. Leaf size=31

$$\frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3x^3}$$

[Out] $-1/3/x^3+2/9*\arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1474, 800, 618, 204}

$$\frac{2 \tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]

[Out] $-1/(3*x^3) + (2*\text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]])/(3*\text{Sqrt}[3])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 800

Int[(((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1474

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*((d_) + (
e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x^4(1-x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1-x}{x^2(1-x+x^2)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x^2} + \frac{1}{-1+x-x^2} \right) dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{-1+x-x^2} dx, x, x^3 \right) \\
&= -\frac{1}{3x^3} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1-2x^3 \right) \\
&= -\frac{1}{3x^3} + \frac{2 \tan^{-1} \left(\frac{1-2x^3}{\sqrt{3}} \right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 45, normalized size = 1.45

$$-\frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\log(x - \#1)}{2\#1^3 - 1} \& \right] - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)), x]

[Out] -1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) &]/3

fricas [A] time = 0.80, size = 28, normalized size = 0.90

$$\frac{2\sqrt{3}x^3 \arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) + 3}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^4/(x^6-x^3+1), x, algorithm="fricas")

[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3

giac [A] time = 0.45, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")

[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3

maple [A] time = 0.01, size = 25, normalized size = 0.81

$$-\frac{2\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)/x^4/(x^6-x^3+1),x)

[Out] -2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3

maxima [A] time = 0.96, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")

[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3

mupad [B] time = 0.04, size = 26, normalized size = 0.84

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}x^3}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)

[Out] (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)

sympy [A] time = 0.14, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)

[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)

$$3.25 \quad \int \frac{x^6(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=418

$$\frac{x^4}{4} \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

[Out] $-1/4*x^4+1/6*\arctan(1/3*(1+2*2^{(1/3)}*x/(1+I*3^{(1/2)})^{(1/3)})*3^{(1/2)}*(I-3^{(1/2)})*2^{(2/3)/(1+I*3^{(1/2)})^{(2/3)}+1/18*\ln(-2^{(1/3)}*x+(1+I*3^{(1/2)})^{(1/3)})*(3-I*3^{(1/2)})*2^{(2/3)/(1+I*3^{(1/2)})^{(2/3)}-1/36*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1+I*3^{(1/2)})^{(1/3)}+(1+I*3^{(1/2)})^{(2/3)})*(3-I*3^{(1/2)})*2^{(2/3)/(1+I*3^{(1/2)})^{(2/3)}+1/18*\ln(-2^{(1/3)}*x+(1-I*3^{(1/2)})^{(1/3)})*(3+I*3^{(1/2)})*2^{(2/3)/(1-I*3^{(1/2)})^{(2/3)}-1/36*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1-I*3^{(1/2)})^{(1/3)}+(1-I*3^{(1/2)})^{(2/3)})*(3+I*3^{(1/2)})*2^{(2/3)/(1-I*3^{(1/2)})^{(2/3)}-1/6*\arctan(1/3*(1+2*2^{(1/3)}*x/(1-I*3^{(1/2)})^{(1/3)})*3^{(1/2)}*(3^{(1/2)}+I)*2^{(2/3)/(1-I*3^{(1/2)})^{(2/3)})$

Rubi [A] time = 0.54, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1502, 12, 1374, 200, 31, 634, 617, 204, 628}

$$\frac{x^4}{4} \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] $-x^4/4 - ((I + \text{Sqrt}[3])*\text{ArcTan}[(1 + (2*x)/((1 - I*\text{Sqrt}[3])/2)^{(1/3)})]/\text{Sqrt}[3]))/(3*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) + ((I - \text{Sqrt}[3])*\text{ArcTan}[(1 + (2*x)/((1 + I*\text{Sqrt}[3])/2)^{(1/3)})]/\text{Sqrt}[3]))/(3*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)}) + ((3 + I*\text{Sqrt}[3])*Log[(1 - I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) + ((3 - I*\text{Sqrt}[3])*Log[(1 + I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)}) - ((3 + I*\text{Sqrt}[3])*Log[(1 - I*\text{Sqrt}[3])^{(2/3)} + (2*(1 - I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) - ((3 - I*\text{Sqrt}[3])*Log[(1 + I*\text{Sqrt}[3])^{(2/3)} + (2*(1 + I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)⁽⁻¹⁾, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1374

Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -

$n)/(b/2 + q/2 + c*x^n), x], x] - \text{Dist}[(d^n*(b/q - 1))/2, \text{Int}[(d*x)^(m - n) / (b/2 - q/2 + c*x^n), x], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GeQ}[m, n]$

Rule 1502

$\text{Int}[(f_*)*(x_)^(m_)*((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2n_))^(p_), x_Symbol] \rightarrow \text{Simp}[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - \text{Dist}[f^n/(c*(m + n*(2*p + 1) + 1)), \text{Int}[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*\text{Simp}[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \&\& \text{EqQ}[n^2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*(2*p + 1) + 1, 0] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
\int \frac{x^6(1-x^3)}{1-x^3+x^6} dx &= -\frac{x^4}{4} - \frac{1}{4} \int -\frac{4x^3}{1-x^3+x^6} dx \\
&= -\frac{x^4}{4} + \int \frac{x^3}{1-x^3+x^6} dx \\
&= -\frac{x^4}{4} - \frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx + \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
&= -\frac{x^4}{4} + \frac{(3-i\sqrt{3}) \int \frac{1}{-\frac{3\sqrt[3]{1+i\sqrt{3}}}{2}+x} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}x+x^2} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \int \frac{1}{-\frac{3\sqrt[3]{1-i\sqrt{3}}}{2}+x} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= -\frac{x^4}{4} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= -\frac{x^4}{4} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} \\
&= -\frac{x^4}{4} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6 - \#1^3 + 1\&, \frac{\#1 \log(x - \#1)}{2\#1^3 - 1}\&\right] - \frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] -1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) &]/3

fricas [B] time = 1.02, size = 1036, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")

[Out]
$$-1/4*x^4 + 1/54*18^{(2/3)}*12^{(1/6)}*\cos(2/3*\arctan(\sqrt{3} + 2))*\log(2*18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2) + 2/27*18^{(2/3)}*12^{(1/6)}*\arctan(1/216*(18^{(1/3)}*12^{(5/6)}*\sqrt{3}*\sqrt{2}*\sqrt{2*18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2) - 6*18^{(1/3)}*12^{(5/6)}*\sqrt{3}*x - 216*\sin(2/3*\arctan(\sqrt{3} + 2)))/\cos(2/3*\arctan(\sqrt{3} + 2)))*\sin(2/3*\arctan(\sqrt{3} + 2)) - 1/27*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2)) - 18^{(2/3)}*12^{(1/6)}*\sin(2/3*\arctan(\sqrt{3} + 2)))*\arctan(-1/108*(6*18^{(1/3)}*12^{(5/6)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 108*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 108*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 - 18*(18^{(1/3)}*12^{(5/6)}*x + 24*\cos(2/3*\arctan(\sqrt{3} + 2)))*\sin(2/3*\arctan(\sqrt{3} + 2)) - \sqrt{-18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2)*(18^{(1/3)}*12^{(5/6)}*\sqrt{3}*\sqrt{2}*\cos(2/3*\arctan(\sqrt{3} + 2)) - 3*18^{(1/3)}*12^{(5/6)}*\sqrt{2}*\sin(2/3*\arctan(\sqrt{3} + 2))))/(\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*\sin(2/3*\arctan(\sqrt{3} + 2))^2) - 1/27*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2)) + 18^{(2/3)}*12^{(1/6)}*\sin(2/3*\arctan(\sqrt{3} + 2)))*\arctan(1/108*(6*18^{(1/3)}*12^{(5/6)}*\sqrt{3}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) - 108*\sqrt{3}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 108*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*(18^{(1/3)}*12^{(5/6)}*x - 24*\cos(2/3*\arctan(\sqrt{3} + 2)))*\sin(2/3*\arctan(\sqrt{3} + 2)) - \sqrt{-18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) - 3*18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2)*(18^{(1/3)}*12^{(5/6)}*\sqrt{3}*\sqrt{2}*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(5/6)}*\sqrt{2}*\sin(2/3*\arctan(\sqrt{3} + 2))))/(\cos(2/3*\arctan(\sqrt{3} + 2))^2 - 3*\sin(2/3*\arctan(\sqrt{3} + 2))^2) - 1/108*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2)) + 18^{(2/3)}*12^{(1/6)}*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(-18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2) + 1/108*(18^{(2/3)}*12^{(1/6)}*\sqrt{3}*\sin(2/3*\arctan(\sqrt{3} + 2)) - 18^{(2/3)}*12^{(1/6)}*\cos(2/3*\arctan(\sqrt{3} + 2)))*\log(-18^{(2/3)}*12^{(1/6)}*\sqrt{3}*x*\sin(2/3*\arctan(\sqrt{3} + 2)) - 3*18^{(2/3)}*12^{(1/6)}*x*\cos(2/3*\arctan(\sqrt{3} + 2)) + 3*18^{(1/3)}*12^{(1/3)}*\cos(2/3*\arctan(\sqrt{3} + 2))^2 + 3*18^{(1/3)}*12^{(1/3)}*\sin(2/3*\arctan(\sqrt{3} + 2))^2 + 18*x^2)$$

giac [B] time = 0.58, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$-1/4*x^4 - 1/9*(2*\sqrt{3}*\cos(4/9*\pi)^4 - 12*\sqrt{3}*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 + 2*\sqrt{3}*\sin(4/9*\pi)^4 + 8*\cos(4/9*\pi)^3*\sin(4/9*\pi) - 8*\cos(4/9*\pi)*\sin(4/9*\pi)^3 + \sqrt{3}*\cos(4/9*\pi) + \sin(4/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(4/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(4/9*\pi))) - 1/9*(2*\sqrt{3}*\cos(2/9*\pi)^4 - 12*\sqrt{3}*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 + 2*\sqrt{3}*\sin(2/9*\pi)^4 + 8*\cos(2/9*\pi)^3*\sin(2/9*\pi) - 8*\cos(2/9*\pi)*\sin(2/9*\pi)^3 + \sqrt{3}*\cos(2/9*\pi) + \sin(2/9*\pi))*\arctan(-((\sqrt{3}*i + 1)*\cos(2/9*\pi) - 2*x)/((\sqrt{3}*i + 1)*\sin(2/9*\pi))) - 1/9*(2*\sqrt{3}*\cos(1/9*\pi)^4 - 12*\sqrt{3}*\cos(1/9*\pi)^2*\sin(1/9*\pi)^2 + 2*\sqrt{3}*\sin(1/9*\pi)^4 - 8*\cos(1/9*\pi)^3*\sin(1/9*\pi) + 8*\cos(1/9*\pi)*\sin(1/9*\pi)^3 - \sqrt{3}*\cos(1/9*\pi) + \sin(1/9*\pi))*\arctan(((\sqrt{3}*i + 1)*\cos(1/9*\pi) + 2*x)/((\sqrt{3}*i + 1)*\sin(1/9*\pi))) - 1/18*(8*\sqrt{3}*\cos(4/9*\pi)^3*\sin(4/9*\pi) - 8*\sqrt{3}*\cos(4/9*\pi)*\sin(4/9*\pi)^3 - 2*\cos(4/9*\pi)^4 + 12*\cos(4/9*\pi)^2*\sin(4/9*\pi)^2 - 2*\sin(4/9*\pi)^4 + \sqrt{3}*\sin(4/9*\pi) - \cos(4/9*\pi))*\log(-(\sqrt{3}*i*\cos(4/9*\pi) + \cos(4/9*\pi))*x + x^2 + 1) - 1/18*(8*\sqrt{3}*\cos(2/9*\pi)^3*\sin(2/9*\pi) - 8*\sqrt{3}*\cos(2/9*\pi)*\sin(2/9*\pi)^3 - 2*\cos(2/9*\pi)^4 + 12*\cos(2/9*\pi)^2*\sin(2/9*\pi)^2 - 2*\sin(2/9*\pi)^4 + \sqrt{3}*\sin(2/9*\pi) - \cos(2/9*\pi))*\log(-(\sqrt{3}*i*\cos(2/9*\pi) + \cos(2/9*\pi))*x + x^2 + 1) + 1/18*(8*\sqrt{3}*\cos(1/9*\pi)^3*\sin(1/9*\pi) - 8*\sqrt{3}*\cos(1/9*\pi)*\sin(1/9*\pi)^3 + 2*\cos(1/9*\pi)^4 - 12*\cos(1/9*\pi)^2*\sin(1/9*\pi)^2 + 2*\sin(1/9*\pi)^4 - \sqrt{3}*\sin(1/9*\pi) - \cos(1/9*\pi))*\log((\sqrt{3}*i*\cos(1/9*\pi) + \cos(1/9*\pi))*x + x^2 + 1)$$

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$-\frac{x^4}{4} + \frac{\text{RootOf}(-Z^6 - Z^3 + 1)^3 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x)

[Out]
$$-1/4*x^4 + 1/3*\sum(1/(2*_R^5 - _R^2)*_R^3*\ln(-_R+x), _R=\text{RootOf}(-Z^6 - Z^3 + 1))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}x^4 + \int \frac{x^3}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)

mupad [B] time = 0.65, size = 332, normalized size = 0.79

$$\frac{\ln\left(x + \frac{2^{2/3} 3^{5/6} (-3 - \sqrt{3} 1i)^{1/3} 1i}{6}\right) (-36 - \sqrt{3} 12i)^{1/3}}{18} + \frac{\ln\left(x - \frac{2^{2/3} 3^{5/6} (-3 + \sqrt{3} 1i)^{1/3} 1i}{6}\right) (-36 + \sqrt{3} 12i)^{1/3}}{18} - \frac{x^4}{4} - \frac{2^{2/3} \ln\left(\dots\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)

[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36

sympy [A] time = 0.18, size = 31, normalized size = 0.07

$$-\frac{x^4}{4} - \text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log\left(-1458t^4 + 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)

[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 + 9*_t + x)))

$$3.26 \quad \int \frac{x^4(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=382

$$\frac{x^2}{2} \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1-i\sqrt{3})} x + (1-i\sqrt{3})^{2/3} \right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1-i\sqrt{3}}} + \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1+i\sqrt{3})} x + (1+i\sqrt{3})^{2/3} \right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1+i\sqrt{3}}} + \frac{i \log \left(-\sqrt[3]{3} \right)}{3\sqrt{3}}$$

[Out] $-1/2*x^2+1/3*I*2^{(1/3)}*\arctan(1/3*(1+2*2^{(1/3)}*x/(1-I*3^{(1/2)})^{(1/3)})*3^{(1/2)})/(1-I*3^{(1/2)})^{(1/3)}-1/3*I*2^{(1/3)}*\arctan(1/3*(1+2*2^{(1/3)}*x/(1+I*3^{(1/2)})^{(1/3)})*3^{(1/2)})/(1+I*3^{(1/2)})^{(1/3)}+1/9*I*2^{(1/3)}*\ln(-2^{(1/3)}*x+(1-I*3^{(1/2)})^{(1/3)})/(1-I*3^{(1/2)})^{(1/3)}*3^{(1/2)}-1/18*I*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1-I*3^{(1/2)})^{(1/3)}+(1-I*3^{(1/2)})^{(2/3)})*2^{(1/3)}/(1-I*3^{(1/2)})^{(1/3)}*3^{(1/2)}-1/9*I*2^{(1/3)}*\ln(-2^{(1/3)}*x+(1+I*3^{(1/2)})^{(1/3)})/(1+I*3^{(1/2)})^{(1/3)}*3^{(1/2)}+1/18*I*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1+I*3^{(1/2)})^{(1/3)}+(1+I*3^{(1/2)})^{(2/3)})*2^{(1/3)}/(1+I*3^{(1/2)})^{(1/3)}*3^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1502, 12, 1375, 292, 31, 634, 617, 204, 628}

$$\frac{x^2}{2} \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1-i\sqrt{3})} x + (1-i\sqrt{3})^{2/3} \right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1-i\sqrt{3}}} + \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1+i\sqrt{3})} x + (1+i\sqrt{3})^{2/3} \right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt[3]{1+i\sqrt{3}}} + \frac{i \log \left(-\sqrt[3]{3} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] $-x^2/2 + ((I/3)*\text{ArcTan}[(1 + (2*x))/((1 - I*\text{Sqrt}[3])/2)^{(1/3)})/\text{Sqrt}[3])/((1 - I*\text{Sqrt}[3])/2)^{(1/3)} - ((I/3)*\text{ArcTan}[(1 + (2*x))/((1 + I*\text{Sqrt}[3])/2)^{(1/3)})/\text{Sqrt}[3])/((1 + I*\text{Sqrt}[3])/2)^{(1/3)} + ((I/3)*\text{Log}[(1 - I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(\text{Sqrt}[3]*((1 - I*\text{Sqrt}[3])/2)^{(1/3)}) - ((I/3)*\text{Log}[(1 + I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(\text{Sqrt}[3]*((1 + I*\text{Sqrt}[3])/2)^{(1/3)}) - ((I/3)*\text{Log}[(1 - I*\text{Sqrt}[3])^{(2/3)} + (2*(1 - I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/((2^{(2/3)}*\text{Sqrt}[3]*(1 - I*\text{Sqrt}[3])^{(1/3)}) + ((I/3)*\text{Log}[(1 + I*\text{Sqrt}[3])^{(2/3)} + (2*(1 + I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/((2^{(2/3)}*\text{Sqrt}[3]*(1 + I*\text{Sqrt}[3])^{(1/3)})$

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :=> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :=> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(n_
- 1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] :=> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1375

```
Int[((d_.)*(x_)^(m_))/((a_) + (c_.)*(x_)^(n2_) + (b_.)*(x_)^(n_)), x_Symb
ol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rule 1502

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{x^4(1-x^3)}{1-x^3+x^6} dx &= -\frac{x^2}{2} - \frac{1}{2} \int -\frac{2x}{1-x^3+x^6} dx \\
&= -\frac{x^2}{2} + \int \frac{x}{1-x^3+x^6} dx \\
&= -\frac{x^2}{2} - \frac{i \int \frac{x}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx}{\sqrt{3}} + \frac{i \int \frac{x}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx}{\sqrt{3}} \\
&= -\frac{x^2}{2} + \frac{i \int \frac{1}{-\sqrt{\frac{1}{2}(1-i\sqrt{3})} + x} dx}{3\sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \int \frac{-\sqrt{\frac{1}{2}(1-i\sqrt{3})} + x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{1}{2}(1-i\sqrt{3})}x + x^2} dx}{3\sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \int \frac{1}{-\sqrt{\frac{1}{2}(1+i\sqrt{3})} + x} dx}{3\sqrt{3} \sqrt{\frac{1}{2}(1+i\sqrt{3})}} + \dots \\
&= -\frac{x^2}{2} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \int \frac{1}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{1}{2}(1-i\sqrt{3})}x}}{2\sqrt{3}} \\
&= -\frac{x^2}{2} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1+i\sqrt{3})}} - \frac{i \log\left(\left(1-i\sqrt{3}\right)^{2/3} + \sqrt[3]{2}\left(1-i\sqrt{3}\right)\right)}{3 \cdot 2^{2/3} \sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} \\
&= -\frac{x^2}{2} + \frac{i \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \tan^{-1}\left(\frac{1 + \frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3 \sqrt{\frac{1}{2}(1+i\sqrt{3})}} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1-i\sqrt{3})}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \sqrt{\frac{1}{2}(1+i\sqrt{3})}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.13

$$\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\log(x - \#1)}{2\#1^4 - \#1} \&\right] - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] -1/2*x^2 + RootSum[1 - #1^3 + #1^6 &, Log[x - #1]/(-#1 + 2*#1^4) &]/3

fricas [B] time = 1.14, size = 1588, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3)
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/2*x^2 - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2
/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*ar
ctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*s
qrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3)
) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2
/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^
(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqr
t(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*
sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
- 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)
*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sq
rt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin
(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*
arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sq
rt(3) - 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) -
2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(
```

$$\begin{aligned} & \frac{2}{3} \cdot 12^{2/3} \cdot x - 216 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4} \\ & + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \\ & + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \\ & + 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 \\ & + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) \\ & + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \\ & + 36 \cdot x^2) - 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^4 \\ & + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 \\ & + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 36 \cdot x^2) \end{aligned}$$

giac [B] time = 0.69, size = 817, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2 \cdot x^2 - 1/9 \cdot (\sqrt{3} \cdot \cos(4/9 \cdot \pi))^5 - 10 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi)^2 + 5 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^4 - 5 \cdot \cos(4/9 \cdot \pi)^4 \cdot \sin(4/9 \cdot \pi) + 1 \\ & 0 \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^3 - \sin(4/9 \cdot \pi)^5 - \sqrt{3} \cdot \cos(4/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(4/9 \cdot \pi)^2 + 2 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \cdot \pi))) \\ & - 1/9 \cdot (\sqrt{3} \cdot \cos(2/9 \cdot \pi))^5 - 10 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^3 \cdot \sin(2/9 \cdot \pi)^2 + 5 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi)^4 - 5 \cdot \cos(2/9 \cdot \pi)^4 \cdot \sin(2/9 \cdot \pi) + 10 \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^3 - \sin(2/9 \cdot \pi)^5 \\ & - \sqrt{3} \cdot \cos(2/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(2/9 \cdot \pi)^2 + 2 \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(2/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(2/9 \cdot \pi))) \\ & + 1/9 \cdot (\sqrt{3} \cdot \cos(1/9 \cdot \pi))^5 - 10 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^3 \cdot \sin(1/9 \cdot \pi)^2 + 5 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi)^4 + 5 \cdot \cos(1/9 \cdot \pi)^4 \cdot \sin(1/9 \cdot \pi) - 10 \cdot \cos(1/9 \cdot \pi)^2 \cdot \sin(1/9 \cdot \pi)^3 + \sin(1/9 \cdot \pi)^5 \\ & + \sqrt{3} \cdot \cos(1/9 \cdot \pi)^2 - \sqrt{3} \cdot \sin(1/9 \cdot \pi)^2 + 2 \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi) \cdot \arctan(((\sqrt{3} \cdot i + 1) \cdot \cos(1/9 \cdot \pi) + 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(1/9 \cdot \pi))) - 1/18 \cdot (5 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi))^4 \cdot \sin(4/9 \cdot \pi) - 10 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^3 + \sqrt{3} \cdot \sin(4/9 \cdot \pi)^5 + \cos(4/9 \cdot \pi)^5 - 10 \cdot \cos(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi)^2 + 5 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^4 - 2 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi) - \cos(4/9 \cdot \pi)^2 + \sin(4/9 \cdot \pi)^2) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(4/9 \cdot \pi) + \cos(4/9 \cdot \pi)) \cdot x + x^2 + 1) - 1/18 \cdot (5 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi))^4 \cdot \sin(2/9 \cdot \pi) - 10 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^3 + \end{aligned}$$

$\sqrt{3}\sin(2/9\pi)^5 + \cos(2/9\pi)^5 - 10\cos(2/9\pi)^3\sin(2/9\pi)^2 + 5\cos(2/9\pi)\sin(2/9\pi)^4 - 2\sqrt{3}\cos(2/9\pi)\sin(2/9\pi) - \cos(2/9\pi)^2 + \sin(2/9\pi)^2 \cdot \log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) - 1/18(5\sqrt{3}\cos(1/9\pi)^4\sin(1/9\pi) - 10\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^3 + \sqrt{3}\sin(1/9\pi)^5 - \cos(1/9\pi)^5 + 10\cos(1/9\pi)^3\sin(1/9\pi)^2 - 5\cos(1/9\pi)\sin(1/9\pi)^4 + 2\sqrt{3}\cos(1/9\pi)\sin(1/9\pi) - \cos(1/9\pi)^2 + \sin(1/9\pi)^2) \cdot \log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1)$

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$-\frac{x^2}{2} + \frac{\text{RootOf}(-Z^6 - Z^3 + 1) \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{6 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - 3 \text{RootOf}(-Z^6 - Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(-x^3+1)/(x^6-x^3+1), x)

[Out] -1/2*x^2+1/3*sum(1/(2*_R^5-_R^2)*_R*ln(-_R+x), _R=RootOf(-Z^6-Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}x^2 + \int \frac{x}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1), x, algorithm="maxima")

[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)

mupad [B] time = 2.28, size = 309, normalized size = 0.81

$$\frac{\ln\left(x + \left(81x - \frac{27(36 - \sqrt{3}12i)^{2/3}}{4}\right)\left(-\frac{1}{162} + \frac{\sqrt{3}1i}{486}\right)\right)(36 - \sqrt{3}12i)^{1/3}}{18} + \frac{\ln\left(x - \left(81x - \frac{27(36 + \sqrt{3}12i)^{2/3}}{4}\right)\left(\frac{1}{162} + \frac{\sqrt{3}1i}{486}\right)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1), x)

[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/2 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/3))

$$\begin{aligned}
& 6) * 1i)) / 36 - (2^{(2/3)} * \log(x + (2^{(1/3)} * 3^{(2/3)} * (3^{(1/2)} * 1i + 3)^{(2/3)})) / 12 - \\
& (2^{(1/3)} * 3^{(1/6)} * (3^{(1/2)} * 1i + 3)^{(2/3)} * 1i) / 4 * (3^{(1/2)} * 1i + 3)^{(1/3)} * (3^{(1/3)} - 3^{(5/6)} * 1i)) / 36 - (2^{(2/3)} * \log(x - (2^{(1/3)} * 3^{(2/3)} * (3 - 3^{(1/2)} * 1i)^{(2/3)})) / 6 * (3 - 3^{(1/2)} * 1i)^{(1/3)} * (3^{(1/3)} - 3^{(5/6)} * 1i)) / 36 - (2^{(2/3)} * \log(x - (2^{(1/3)} * 3^{(2/3)} * (3^{(1/2)} * 1i + 3)^{(2/3)})) / 6 * (3^{(1/2)} * 1i + 3)^{(1/3)} * (3^{(1/3)} + 3^{(5/6)} * 1i)) / 36
\end{aligned}$$

sympy [A] time = 0.19, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(-6561t^5 - 27t^2 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1), x)

[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 - 27*_t**2 + x)))

$$3.27 \quad \int \frac{x^3(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=378

$$\frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1-i\sqrt{3})} x + (1-i\sqrt{3})^{2/3} \right)}{3\sqrt[3]{2}\sqrt{3}(1-i\sqrt{3})^{2/3}} + \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1+i\sqrt{3})} x + (1+i\sqrt{3})^{2/3} \right)}{3\sqrt[3]{2}\sqrt{3}(1+i\sqrt{3})^{2/3}} - x + \frac{i \log \left(-\sqrt[3]{2} \right)}{3\sqrt{3} \left(\frac{1}{2} \right)}$$

[Out] $-x - \frac{1}{3} I 2^{2/3} \arctan \left(\frac{1}{3} (1 + 2 \cdot 2^{1/3}) x / ((1 - I 3^{1/2})^{1/3}) \cdot 3^{1/2} \right) / ((1 - I 3^{1/2})^{2/3}) + \frac{1}{3} I 2^{2/3} \arctan \left(\frac{1}{3} (1 + 2 \cdot 2^{1/3}) x / ((1 + I 3^{1/2})^{1/3}) \cdot 3^{1/2} \right) / ((1 + I 3^{1/2})^{2/3}) + \frac{1}{9} I 2^{2/3} \ln \left(-2^{1/3} x + (1 - I 3^{1/2})^{1/3} \right) / ((1 - I 3^{1/2})^{2/3}) \cdot 3^{1/2} - \frac{1}{18} I \ln \left(2^{2/3} x^2 + 2^{1/3} x (1 - I 3^{1/2})^{1/3} + (1 - I 3^{1/2})^{2/3} \right) / ((1 - I 3^{1/2})^{2/3}) \cdot 3^{1/2} - \frac{1}{9} I 2^{2/3} \ln \left(-2^{1/3} x + (1 + I 3^{1/2})^{1/3} \right) / ((1 + I 3^{1/2})^{2/3}) \cdot 3^{1/2} + \frac{1}{18} I \ln \left(2^{2/3} x^2 + 2^{1/3} x (1 + I 3^{1/2})^{1/3} + (1 + I 3^{1/2})^{2/3} \right) / ((1 + I 3^{1/2})^{2/3}) \cdot 3^{1/2}$

Rubi [A] time = 0.26, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1502, 1347, 200, 31, 634, 617, 204, 628}

$$\frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1-i\sqrt{3})} x + (1-i\sqrt{3})^{2/3} \right)}{3\sqrt[3]{2}\sqrt{3}(1-i\sqrt{3})^{2/3}} + \frac{i \log \left(2^{2/3} x^2 + \sqrt[3]{2(1+i\sqrt{3})} x + (1+i\sqrt{3})^{2/3} \right)}{3\sqrt[3]{2}\sqrt{3}(1+i\sqrt{3})^{2/3}} - x + \frac{i \log \left(-\sqrt[3]{2} \right)}{3\sqrt{3} \left(\frac{1}{2} \right)}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] $-x - \left(\frac{I}{3} \operatorname{ArcTan} \left[\frac{1 + (2x)}{\left(\frac{1 - I \operatorname{Sqrt}[3]}{2} \right)^{1/3}} \right] / \operatorname{Sqrt}[3] \right) / \left(\left(\frac{1 - I \operatorname{Sqrt}[3]}{2} \right)^{2/3} \right) + \left(\frac{I}{3} \operatorname{ArcTan} \left[\frac{1 + (2x)}{\left(\frac{1 + I \operatorname{Sqrt}[3]}{2} \right)^{1/3}} \right] / \operatorname{Sqrt}[3] \right) / \left(\left(\frac{1 + I \operatorname{Sqrt}[3]}{2} \right)^{2/3} \right) + \left(\frac{I}{3} \operatorname{Log} \left[\frac{1 - I \operatorname{Sqrt}[3]}{2} \right]^{1/3} - 2^{1/3} x \right) / \left(\operatorname{Sqrt}[3] \cdot \left(\frac{1 - I \operatorname{Sqrt}[3]}{2} \right)^{2/3} \right) - \left(\frac{I}{3} \operatorname{Log} \left[\frac{1 + I \operatorname{Sqrt}[3]}{2} \right]^{1/3} - 2^{1/3} x \right) / \left(\operatorname{Sqrt}[3] \cdot \left(\frac{1 + I \operatorname{Sqrt}[3]}{2} \right)^{2/3} \right) - \left(\frac{I}{3} \operatorname{Log} \left[\frac{1 - I \operatorname{Sqrt}[3]}{2} \right]^{2/3} + (2 \cdot (1 - I \operatorname{Sqrt}[3]))^{1/3} x + 2^{2/3} x^2 \right) / \left(2^{1/3} \operatorname{Sqrt}[3] \cdot (1 - I \operatorname{Sqrt}[3])^{2/3} \right) + \left(\frac{I}{3} \operatorname{Log} \left[\frac{1 + I \operatorname{Sqrt}[3]}{2} \right]^{2/3} + (2 \cdot (1 + I \operatorname{Sqrt}[3]))^{1/3} x + 2^{2/3} x^2 \right) / \left(2^{1/3} \operatorname{Sqrt}[3] \cdot (1 + I \operatorname{Sqrt}[3])^{2/3} \right)$

Rule 31

$\text{Int}[\frac{(a_+ + (b_+)(x_+))^{-1}}{b, x}] /; \text{FreeQ}\{a, b\}, x] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 200

$\text{Int}[\frac{(a_+ + (b_+)(x_+)^3)^{-1}}{Rt[a, 3] + Rt[b, 3]*x}, x] \rightarrow \text{Dist}[1/(3*Rt[a, 3]^2), \text{Int}[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + \text{Dist}[1/(3*Rt[a, 3]^2), \text{Int}[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 204

$\text{Int}[\frac{(a_+ + (b_+)(x_+)^2)^{-1}}{Rt[-a, 2]*Rt[-b, 2]}, x] \rightarrow -\text{Simp}[\text{ArcTan}[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])]$

Rule 617

$\text{Int}[\frac{(a_+ + (b_+)(x_+) + (c_+)(x_+)^2)^{-1}}{RationalQ[q]}, x] \rightarrow \text{With}\{q = 1 - 4*\text{Simplify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_+ + (e_+)(x_+))}{(a_+ + (b_+)(x_+) + (c_+)(x_+)^2)}, x] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_+ + (e_+)(x_+))}{(a_+ + (b_+)(x_+) + (c_+)(x_+)^2)}, x] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1347

$\text{Int}[\frac{(a_+ + (b_+)(x_+)^{n_+} + (c_+)(x_+)^{n2_+})^{-1}}{Rt[b^2 - 4*a*c, 2]}, x] \rightarrow \text{With}\{q = Rt[b^2 - 4*a*c, 2]\}, \text{Dist}[c/q, \text{Int}[1/(b/2 - q/2 + c*x^n), x], x] - \text{Dist}[c/q, \text{Int}[1/(b/2 + q/2 + c*x^n), x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1502

```

Int[((f_)*(x_))^(m_)*((d_)+(e_)*(x_)^n)*((a_)+(b_)*(x_)^n)+(
c_)*(x_)^n2)^(p_), x_Symbol] := Simp[(e*f^(n-1)*(f*x)^(m-n+1)*(a
+b*x^n+c*x^(2*n))^(p+1))/(c*(m+n*(2*p+1)+1)), x] - Dist[f^n/(c*(
m+n*(2*p+1)+1)), Int[(f*x)^(m-n)*(a+b*x^n+c*x^(2*n))^p*Simp[a*e
*(m-n+1)+(b*e*(m+n*p+1)-c*d*(m+n*(2*p+1)+1))*x^n, x], x],
x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2-4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*(2*p+1)+1, 0] && Integer
Q[p]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1-x^3)}{1-x^3+x^6} dx &= -x + \int \frac{1}{1-x^3+x^6} dx \\
&= -x - \frac{i \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx}{\sqrt{3}} + \frac{i \int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx}{\sqrt{3}} \\
&= -x + \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x} dx}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i \int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}+x} dx}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} \\
&= -x + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i \int \frac{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+2x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{3\sqrt{2}\sqrt{3}(1-i\sqrt{3})^{2/3}} \\
&= -x + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} - \frac{i \log\left((1-i\sqrt{3})^{2/3} + \sqrt[3]{2}(1-i\sqrt{3})\right)}{3\sqrt{2}\sqrt{3}(1-i\sqrt{3})^{2/3}} \\
&= -x - \frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} + \frac{i \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}} + \frac{i \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}} - \frac{i \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{3\sqrt{3} \left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.12

$$\frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\log(x - \#1)}{2\#1^5 - \#1^2} \& \right] - x$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]

[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) &]/3

fricas [B] time = 0.99, size = 1030, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")

[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2))))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2))))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2))))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2))))*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2))

$$\begin{aligned}
& t(3) - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \\
& \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x \\
& + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) / \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \\
& + 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot \\
& 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(\\
& 2/3 \cdot \arctan(\sqrt{3} - 2)) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2 \\
&)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \\
& \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2 - 1/108 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \\
& \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} \\
& - 2))) \cdot \log(-2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \\
& \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin \\
& (2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - x
\end{aligned}$$

giac [B] time = 0.63, size = 632, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/9 \cdot (\sqrt{3} \cdot \cos(4/9 \cdot \pi))^4 - 6 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^2 \cdot \sin(4/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(4/9 \cdot \pi)^4 \\
& + 4 \cdot \cos(4/9 \cdot \pi)^3 \cdot \sin(4/9 \cdot \pi) - 4 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^3 - \sqrt{3} \cdot \cos(4/9 \cdot \pi) - \sin(4/9 \cdot \pi) \\
& \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \cdot \pi))) - 1/9 \cdot (\sqrt{3} \cdot \cos(2/9 \cdot \pi))^4 - 6 \cdot \sqrt{3} \\
& \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(2/9 \cdot \pi)^4 + 4 \cdot \cos(2/9 \cdot \pi)^3 \cdot \sin(2/9 \cdot \pi) - 4 \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi)^3 \\
& - \sqrt{3} \cdot \cos(2/9 \cdot \pi) - \sin(2/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(2/9 \cdot \pi) - 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(2/9 \cdot \pi) \\
&))) - 1/9 \cdot (\sqrt{3} \cdot \cos(1/9 \cdot \pi))^4 - 6 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^2 \cdot \sin(1/9 \cdot \pi)^2 + \sqrt{3} \cdot \sin(1/9 \cdot \pi)^4 - 4 \cdot \cos(1/9 \cdot \pi)^3 \cdot \sin(1/9 \cdot \pi) \\
& + 4 \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi)^3 + \sqrt{3} \cdot \cos(1/9 \cdot \pi) - \sin(1/9 \cdot \pi) \cdot \arctan(((\sqrt{3} \cdot i + 1) \cdot \cos(1/9 \cdot \pi) + 2 \cdot x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(1/9 \cdot \pi) \\
&))) - 1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi))^3 \cdot \sin(4/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^3 - \cos(4/9 \cdot \pi)^4 + 6 \cdot \cos(4/9 \cdot \pi)^2 \\
& \cdot \sin(4/9 \cdot \pi)^2 - \sin(4/9 \cdot \pi)^4 - \sqrt{3} \cdot \sin(4/9 \cdot \pi) + \cos(4/9 \cdot \pi) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(4/9 \cdot \pi) + \cos(4/9 \cdot \pi)) \cdot x + x^2 + 1) \\
& - 1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi))^3 \cdot \sin(2/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi)^3 - \cos(2/9 \cdot \pi)^4 + 6 \cdot \cos(2/9 \cdot \pi)^2 \cdot \sin(2/9 \cdot \pi)^2 \\
& - \sin(2/9 \cdot \pi)^4 - \sqrt{3} \cdot \sin(2/9 \cdot \pi) + \cos(2/9 \cdot \pi) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(2/9 \cdot \pi) + \cos(2/9 \cdot \pi)) \cdot x + x^2 + 1) + 1/18 \cdot (4 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi))^3 \\
& \cdot \sin(1/9 \cdot \pi) - 4 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi)^3 + \cos(1/9 \cdot \pi)^4 - 6 \cdot \cos(1/9 \cdot \pi)^2 \cdot \sin(1/9 \cdot \pi)^2 + \sin(1/9 \cdot \pi)^4 + \sqrt{3} \cdot \sin(1/9 \cdot \pi) \\
& + \cos(1/9 \cdot \pi) \cdot \log((\sqrt{3} \cdot i \cdot \cos(1/9 \cdot \pi) + \cos(1/9 \cdot \pi)) \cdot x + x^2 + 1) - x
\end{aligned}$$

maple [C] time = 0.01, size = 41, normalized size = 0.11

$$-x + \frac{\ln\left(-\text{RootOf}\left(_Z^6 - _Z^3 + 1\right) + x\right)}{6 \text{RootOf}\left(_Z^6 - _Z^3 + 1\right)^5 - 3 \text{RootOf}\left(_Z^6 - _Z^3 + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x)

[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(_Z^6-_Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x + \int \frac{1}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] -x + integrate(1/(x^6 - x^3 + 1), x)

mupad [B] time = 2.38, size = 330, normalized size = 0.87

$$-x + \frac{\ln\left(x + \frac{2^{2/3} 3^{1/3} (3 - \sqrt{3} 1i)^{1/3}}{4} - \frac{2^{2/3} 3^{5/6} (3 - \sqrt{3} 1i)^{1/3} 1i}{12}\right) (36 - \sqrt{3} 12i)^{1/3}}{18} + \frac{\ln\left(x + \frac{2^{2/3} 3^{1/3} (3 + \sqrt{3} 1i)^{1/3}}{4} + \frac{2^{2/3} 3^{5/6} (3 + \sqrt{3} 1i)^{1/3} 1i}{12}\right) (36 + \sqrt{3} 12i)^{1/3}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)

[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36

sympy [A] time = 0.18, size = 24, normalized size = 0.06

$$-x - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(729t^4 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

$$3.28 \quad \int \frac{x(1-x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=411

$$\frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

[Out] $\frac{1}{6} \arctan\left(\frac{1}{3} \frac{(1+2 \cdot 2^{1/3})x}{(1-I \cdot 3^{1/2})^{1/3}} \cdot 3^{1/2}\right) \cdot (I-3^{1/2}) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3} - \frac{1}{18} \ln\left(-2^{1/3} \cdot x + (1-I \cdot 3^{1/2})^{1/3}\right) \cdot (3-I \cdot 3^{1/2}) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3} + \frac{1}{36} \ln\left(2^{2/3} \cdot x^2 + 2^{1/3} \cdot x \cdot (1-I \cdot 3^{1/2})^{1/3} + (1-I \cdot 3^{1/2})^{2/3}\right) \cdot (3-I \cdot 3^{1/2}) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3} - \frac{1}{18} \ln\left(-2^{1/3} \cdot x + (1+I \cdot 3^{1/2})^{1/3}\right) \cdot (3+I \cdot 3^{1/2}) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3} + \frac{1}{36} \ln\left(2^{2/3} \cdot x^2 + 2^{1/3} \cdot x \cdot (1+I \cdot 3^{1/2})^{1/3} + (1+I \cdot 3^{1/2})^{2/3}\right) \cdot (3+I \cdot 3^{1/2}) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3} - \frac{1}{6} \arctan\left(\frac{1}{3} \frac{(1+2 \cdot 2^{1/3})x}{(1+I \cdot 3^{1/2})^{1/3}} \cdot 3^{1/2}\right) \cdot (3^{1/2}+I) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3}$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] $\frac{(I - \sqrt{3}) \cdot \text{ArcTan}\left[\frac{1 + (2x)}{\left(\frac{1 - I \cdot \sqrt{3}}{2}\right)^{1/3}}\right] / \sqrt{3}}{(3 \cdot 2^{2/3}) \cdot (1 - I \cdot \sqrt{3})^{1/3}} - \frac{(I + \sqrt{3}) \cdot \text{ArcTan}\left[\frac{1 + (2x)}{\left(\frac{1 + I \cdot \sqrt{3}}{2}\right)^{1/3}}\right] / \sqrt{3}}{(3 \cdot 2^{2/3}) \cdot (1 + I \cdot \sqrt{3})^{1/3}} - \frac{(3 - I \cdot \sqrt{3}) \cdot \text{Log}\left[\frac{1 - I \cdot \sqrt{3}}{2} - 2^{1/3} \cdot x\right]}{(9 \cdot 2^{2/3}) \cdot (1 - I \cdot \sqrt{3})^{1/3}} - \frac{(3 + I \cdot \sqrt{3}) \cdot \text{Log}\left[\frac{1 + I \cdot \sqrt{3}}{2} - 2^{1/3} \cdot x\right]}{(9 \cdot 2^{2/3}) \cdot (1 + I \cdot \sqrt{3})^{1/3}} + \frac{(3 - I \cdot \sqrt{3}) \cdot \text{Log}\left[\frac{1 - I \cdot \sqrt{3}}{2} + (2 \cdot (1 - I \cdot \sqrt{3}))^{1/3} \cdot x + 2^{2/3} \cdot x^2\right]}{(18 \cdot 2^{2/3}) \cdot (1 - I \cdot \sqrt{3})^{1/3}} + \frac{(3 + I \cdot \sqrt{3}) \cdot \text{Log}\left[\frac{1 + I \cdot \sqrt{3}}{2} + (2 \cdot (1 + I \cdot \sqrt{3}))^{1/3} \cdot x + 2^{2/3} \cdot x^2\right]}{(18 \cdot 2^{2/3}) \cdot (1 + I \cdot \sqrt{3})^{1/3}}$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1510

Int[((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^{(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*xⁿ), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*xⁿ), x], x]] /; FreeQ[{a, b}

, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x(1-x^3)}{1-x^3+x^6} dx &= \frac{1}{6}(-3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2} + \frac{i\sqrt{3}}{2} + x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^3} dx \\
 &= -\frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{3}{2}}(1-i\sqrt{3})+x} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \int \frac{-\sqrt{\frac{3}{2}}(1-i\sqrt{3})+x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{3}{2}}(1-i\sqrt{3})x+x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{3}{2}}(1+i\sqrt{3})+x} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} \\
 &= -\frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} \\
 &= -\frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} \\
 &= \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{3}{2}}(1-i\sqrt{3})}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{3}{2}}(1+i\sqrt{3})}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.13

$$-\frac{1}{3} \text{RootSum}\left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - \log(x - \#1)}{2\#1^4 - \#1} \&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6), x]

[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2*#1^4) &]

fricas [B] time = 1.22, size = 1583, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \log(18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 36 \cdot x^2) + 2/27 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan(-1/432 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot x - 216 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 36 \cdot x^2)) / (\cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 12 \cdot (18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^3) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 36 \cdot x^2) \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 2 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) / (3 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 - 10 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4) + 1/27 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 - 864 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \sqrt{3} \cdot x - 36 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 12 \cdot (18^{2/3} \cdot 12^{2/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 72 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^3) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 18^{2/3} \cdot 12^{2/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 6 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))$

$$\begin{aligned}
& 2))^{2} + 36x^{2}) \cdot (18^{(2/3)} \cdot 12^{(2/3)} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} - \\
& 18^{(2/3)} \cdot 12^{(2/3)} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 2 \cdot 18^{(2/3)} \cdot 12^{(2/3)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) / (3 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{4} - 10 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{4}) + 1/108 \cdot (18^{(2/3)} \cdot 12^{(1/6)} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{(2/3)} \cdot 12^{(1/6)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \log(18^{(2/3)} \cdot 12^{(2/3)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{4} + 18^{(2/3)} \cdot 12^{(2/3)} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{4} + 12 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 6 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 2 \cdot (18^{(2/3)} \cdot 12^{(2/3)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} - 3 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 36x^{2}) - 1/108 \cdot (18^{(2/3)} \cdot 12^{(1/6)} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 18^{(2/3)} \cdot 12^{(1/6)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \log(18^{(2/3)} \cdot 12^{(2/3)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{4} + 18^{(2/3)} \cdot 12^{(2/3)} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{4} - 12 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 6 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 2 \cdot (18^{(2/3)} \cdot 12^{(2/3)} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} - 3 \cdot 18^{(1/3)} \cdot 12^{(1/3)} \cdot x) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^{2} + 36x^{2})
\end{aligned}$$

giac [B] time = 0.58, size = 821, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out] $\begin{aligned}
& 1/9 \cdot (\sqrt{3} \cdot \cos(4/9 \cdot \pi))^{5} - 10 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^{3} \cdot \sin(4/9 \cdot \pi)^{2} + 5 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^{4} - 5 \cdot \cos(4/9 \cdot \pi)^{4} \cdot \sin(4/9 \cdot \pi) + 10 \cdot \cos(4/9 \cdot \pi)^{2} \cdot \sin(4/9 \cdot \pi)^{3} - \sin(4/9 \cdot \pi)^{5} + 2 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^{2} - 2 \cdot \sqrt{3} \cdot \sin(4/9 \cdot \pi)^{2} - 4 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(4/9 \cdot \pi) - 2x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(4/9 \cdot \pi))) + 1/9 \cdot (\sqrt{3} \cdot \cos(2/9 \cdot \pi))^{5} - 10 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^{3} \cdot \sin(2/9 \cdot \pi)^{2} + 5 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi)^{4} - 5 \cdot \cos(2/9 \cdot \pi)^{4} \cdot \sin(2/9 \cdot \pi) + 10 \cdot \cos(2/9 \cdot \pi)^{2} \cdot \sin(2/9 \cdot \pi)^{3} - \sin(2/9 \cdot \pi)^{5} + 2 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^{2} - 2 \cdot \sqrt{3} \cdot \sin(2/9 \cdot \pi)^{2} - 4 \cdot \cos(2/9 \cdot \pi) \cdot \sin(2/9 \cdot \pi) \cdot \arctan(-((\sqrt{3} \cdot i + 1) \cdot \cos(2/9 \cdot \pi) - 2x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(2/9 \cdot \pi))) - 1/9 \cdot (\sqrt{3} \cdot \cos(1/9 \cdot \pi))^{5} - 10 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^{3} \cdot \sin(1/9 \cdot \pi)^{2} + 5 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi)^{4} + 5 \cdot \cos(1/9 \cdot \pi)^{4} \cdot \sin(1/9 \cdot \pi) - 10 \cdot \cos(1/9 \cdot \pi)^{2} \cdot \sin(1/9 \cdot \pi)^{3} + \sin(1/9 \cdot \pi)^{5} - 2 \cdot \sqrt{3} \cdot \cos(1/9 \cdot \pi)^{2} + 2 \cdot \sqrt{3} \cdot \sin(1/9 \cdot \pi)^{2} - 4 \cdot \cos(1/9 \cdot \pi) \cdot \sin(1/9 \cdot \pi) \cdot \arctan(((\sqrt{3} \cdot i + 1) \cdot \cos(1/9 \cdot \pi) + 2x) / ((\sqrt{3} \cdot i + 1) \cdot \sin(1/9 \cdot \pi))) + 1/18 \cdot (5 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^{4} \cdot \sin(4/9 \cdot \pi) - 10 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi)^{2} \cdot \sin(4/9 \cdot \pi)^{3} + \sqrt{3} \cdot \sin(4/9 \cdot \pi)^{5} + \cos(4/9 \cdot \pi)^{5} - 10 \cdot \cos(4/9 \cdot \pi)^{3} \cdot \sin(4/9 \cdot \pi)^{2} + 5 \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi)^{4} + 4 \cdot \sqrt{3} \cdot \cos(4/9 \cdot \pi) \cdot \sin(4/9 \cdot \pi) + 2 \cdot \cos(4/9 \cdot \pi)^{2} - 2 \cdot \sin(4/9 \cdot \pi)^{2}) \cdot \log(-(\sqrt{3} \cdot i \cdot \cos(4/9 \cdot \pi) + \cos(4/9 \cdot \pi)) \cdot x + x^{2} + 1) + 1/18 \cdot (5 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^{4} \cdot \sin(2/9 \cdot \pi) - 10 \cdot \sqrt{3} \cdot \cos(2/9 \cdot \pi)^{2} \cdot \sin(2/9 \cdot \pi)
\end{aligned}$

$$\begin{aligned} &^3 + \sqrt{3} \sin(2/9\pi)^5 + \cos(2/9\pi)^5 - 10 \cos(2/9\pi)^3 \sin(2/9\pi)^2 \\ &+ 5 \cos(2/9\pi) \sin(2/9\pi)^4 + 4 \sqrt{3} \cos(2/9\pi) \sin(2/9\pi) + 2 \cos(2/9\pi)^2 \\ &- 2 \sin(2/9\pi)^2 \log(-(\sqrt{3}i \cos(2/9\pi) + \cos(2/9\pi)) * x + x^2 + 1) \\ &+ 1/18 * (5 \sqrt{3} \cos(1/9\pi)^4 \sin(1/9\pi) - 10 \sqrt{3} \cos(1/9\pi)^2 \sin(1/9\pi)^3 \\ &+ \sqrt{3} \sin(1/9\pi)^5 - \cos(1/9\pi)^5 + 10 \cos(1/9\pi)^3 \sin(1/9\pi)^2 \\ &- 5 \cos(1/9\pi) \sin(1/9\pi)^4 - 4 \sqrt{3} \cos(1/9\pi) \sin(1/9\pi) \\ &+ 2 \cos(1/9\pi)^2 - 2 \sin(1/9\pi)^2) \log((\sqrt{3}i \cos(1/9\pi) + \cos(1/9\pi)) * x + x^2 + 1) \end{aligned}$$

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$\frac{\left(\text{RootOf}(-Z^6 - Z^3 + 1)^4 - \text{RootOf}(-Z^6 - Z^3 + 1)\right) \ln\left(-\text{RootOf}(-Z^6 - Z^3 + 1) + x\right)}{3 \left(2 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - \text{RootOf}(-Z^6 - Z^3 + 1)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^3+1)/(x^6-x^3+1),x)

[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(-_R+x),_R=RootOf(-Z^6-Z^3+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(x^3 - 1)x}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)

mupad [B] time = 2.26, size = 281, normalized size = 0.68

$$\frac{\ln\left(x - \frac{2^{1/3} 3^{2/3} (-3 + \sqrt{3} 1i)^{2/3}}{6}\right) (-36 + \sqrt{3} 12i)^{1/3}}{18} + \frac{\ln\left(x - \frac{(-36 - \sqrt{3} 12i)^{2/3}}{12}\right) (-36 - \sqrt{3} 12i)^{1/3}}{18} - \frac{2^{2/3} \ln\left(x - \frac{2^{1/3} (-3 - \sqrt{3} 1i)^{2/3}}{6}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)

[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (-3^(1/2)*12i - 36)^(2/3)/12)*(-3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(-3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(-3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)

```
*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(-
  3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/
  3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1
  /3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)
  ^2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
  /6)*1i))/36
```

sympy [A] time = 0.18, size = 22, normalized size = 0.05

$$-\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log(-27t^2 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

$$3.29 \quad \int \frac{1-x^3}{1-x^3+x^6} dx$$

Optimal. Leaf size=411

$$\frac{(3 - i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 - i\sqrt{3})}x + (1 - i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1 - i\sqrt{3})^{2/3}} + \frac{(3 + i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 + i\sqrt{3})}x + (1 + i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1 + i\sqrt{3})^{2/3}}$$

[Out] $-1/6*\arctan(1/3*(1+2*2^{(1/3)}*x/(1-I*3^{(1/2)})^{(1/3)})*3^{(1/2)}*(I-3^{(1/2)})*2^{(2/3)}/(1-I*3^{(1/2)})^{(2/3)}-1/18*\ln(-2^{(1/3)}*x+(1-I*3^{(1/2)})^{(1/3)})*(3-I*3^{(1/2)})*2^{(2/3)}/(1-I*3^{(1/2)})^{(2/3)}+1/36*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1-I*3^{(1/2)})^{(1/3)}+(1-I*3^{(1/2)})^{(2/3)})*(3-I*3^{(1/2)})*2^{(2/3)}/(1-I*3^{(1/2)})^{(2/3)}-1/18*\ln(-2^{(1/3)}*x+(1+I*3^{(1/2)})^{(1/3)})*(3+I*3^{(1/2)})*2^{(2/3)}/(1+I*3^{(1/2)})^{(2/3)}+1/36*\ln(2^{(2/3)}*x^2+2^{(1/3)}*x*(1+I*3^{(1/2)})^{(1/3)}+(1+I*3^{(1/2)})^{(2/3)})*(3+I*3^{(1/2)})*2^{(2/3)}/(1+I*3^{(1/2)})^{(2/3)}+1/6*\arctan(1/3*(1+2*2^{(1/3)}*x/(1+I*3^{(1/2)})^{(1/3)})*3^{(1/2)}*(3^{(1/2)}+I)*2^{(2/3)}/(1+I*3^{(1/2)})^{(2/3)})$

Rubi [A] time = 0.28, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$\frac{(3 - i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 - i\sqrt{3})}x + (1 - i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1 - i\sqrt{3})^{2/3}} + \frac{(3 + i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1 + i\sqrt{3})}x + (1 + i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1 + i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)/(1 - x^3 + x^6), x]

[Out] $-((I - \text{Sqrt}[3])*ArcTan[(1 + (2*x))/((1 - I*\text{Sqrt}[3])/2)^{(1/3)}]/\text{Sqrt}[3]))/(3*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) + ((I + \text{Sqrt}[3])*ArcTan[(1 + (2*x))/((1 + I*\text{Sqrt}[3])/2)^{(1/3)}]/\text{Sqrt}[3]))/(3*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)}) - ((3 - I*\text{Sqrt}[3])*Log[(1 - I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) - ((3 + I*\text{Sqrt}[3])*Log[(1 + I*\text{Sqrt}[3])^{(1/3)} - 2^{(1/3)}*x])/(9*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)}) + ((3 - I*\text{Sqrt}[3])*Log[(1 - I*\text{Sqrt}[3])^{(2/3)} + (2*(1 - I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(1/3)}*(1 - I*\text{Sqrt}[3])^{(2/3)}) + ((3 + I*\text{Sqrt}[3])*Log[(1 + I*\text{Sqrt}[3])^{(2/3)} + (2*(1 + I*\text{Sqrt}[3]))^{(1/3)}*x + 2^{(2/3)}*x^2])/(18*2^{(1/3)}*(1 + I*\text{Sqrt}[3])^{(2/3)})$

Rule 31

`Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rule 200

`Int[((a_) + (b_)*(x_)^3)^-1, x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]`

Rule 204

`Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 617

`Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 634

`Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1422

`Int[((d_) + (e_)*(x_)^n)/((a_) + (b_)*(x_)^n + (c_)*(x_)^(n2)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])`

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{1-x^3+x^6} dx &= \frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx \\
&= -\frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+x} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} \\
&= -\frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \int \frac{1}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{18} \\
&= -\frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}+\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}x+x^2\right)}{18} \\
&= -\frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&, \frac{\#1^3 \log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)/(1 - x^3 + x^6), x]

[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 + 2*#1^5) &]

fricas [B] time = 1.12, size = 1031, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")

[Out] $\frac{1}{54} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) \cdot \log(-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2 - \frac{2}{27} \cdot 18^{2/3} \cdot 12^{1/6} \cdot \arctan(-1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6}) \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) + 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x + 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))} + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + \frac{1}{27} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6}) \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) - 108 \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x - 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - \sqrt{-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))} - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)))) / (\cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2) - \frac{1}{27} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \arctan(1/216 \cdot (18^{1/3} \cdot 12^{5/6}) \cdot \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))} + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2 - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x - 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) / \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + \frac{1}{108} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) - 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \log(2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2 - \frac{1}{108} \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2)) + 18^{2/3} \cdot 12^{1/6} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))) \cdot \log(-18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} + 2))^2 + 18 \cdot x^2$

giac [B] time = 0.72, size = 637, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{9}(\sqrt{3}\cos(4/9\pi)^4 - 6\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^2 + \sqrt{3}\sin(4/9\pi)^4 + 4\cos(4/9\pi)^3\sin(4/9\pi) - 4\cos(4/9\pi)\sin(4/9\pi)^3 + 2\sqrt{3}\cos(4/9\pi) + 2\sin(4/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(4/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(4/9\pi))) + \frac{1}{9}(\sqrt{3}\cos(2/9\pi)^4 - 6\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^2 + \sqrt{3}\sin(2/9\pi)^4 + 4\cos(2/9\pi)^3\sin(2/9\pi) - 4\cos(2/9\pi)\sin(2/9\pi)^3 + 2\sqrt{3}\cos(2/9\pi) + 2\sin(2/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(2/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(2/9\pi))) + \frac{1}{9}(\sqrt{3}\cos(1/9\pi)^4 - 6\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^2 + \sqrt{3}\sin(1/9\pi)^4 - 4\cos(1/9\pi)^3\sin(1/9\pi) + 4\cos(1/9\pi)\sin(1/9\pi)^3 - 2\sqrt{3}\cos(1/9\pi) + 2\sin(1/9\pi))\arctan(((\sqrt{3}i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}i + 1)\sin(1/9\pi))) + \frac{1}{18}(4\sqrt{3}\cos(4/9\pi)^3\sin(4/9\pi) - 4\sqrt{3}\cos(4/9\pi)\sin(4/9\pi)^3 - \cos(4/9\pi)^4 + 6\cos(4/9\pi)^2\sin(4/9\pi)^2 - \sin(4/9\pi)^4 + 2\sqrt{3}\sin(4/9\pi) - 2\cos(4/9\pi))\log(-(\sqrt{3}i\cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) + \frac{1}{18}(4\sqrt{3}\cos(2/9\pi)^3\sin(2/9\pi) - 4\sqrt{3}\cos(2/9\pi)\sin(2/9\pi)^3 - \cos(2/9\pi)^4 + 6\cos(2/9\pi)^2\sin(2/9\pi)^2 - \sin(2/9\pi)^4 + 2\sqrt{3}\sin(2/9\pi) - 2\cos(2/9\pi))\log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) - \frac{1}{18}(4\sqrt{3}\cos(1/9\pi)^3\sin(1/9\pi) - 4\sqrt{3}\cos(1/9\pi)\sin(1/9\pi)^3 + \cos(1/9\pi)^4 - 6\cos(1/9\pi)^2\sin(1/9\pi)^2 + \sin(1/9\pi)^4 - 2\sqrt{3}\sin(1/9\pi) - 2\cos(1/9\pi))\log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1)$

maple [C] time = 0.00, size = 44, normalized size = 0.11

$$\frac{(-\text{RootOf}(_Z^6 - _Z^3 + 1)^3 + 1)\ln(-\text{RootOf}(_Z^6 - _Z^3 + 1) + x)}{6\text{RootOf}(_Z^6 - _Z^3 + 1)^5 - 3\text{RootOf}(_Z^6 - _Z^3 + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)/(x^6-x^3+1),x)

[Out] $\frac{1}{3}\sum((-R^3+1)/(2R^5-R^2)*\ln(-R+x), R=\text{RootOf}(_Z^6-_Z^3+1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3 - 1}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")

[Out] `-integrate((x^3 - 1)/(x^6 - x^3 + 1), x)`

mupad [B] time = 2.30, size = 319, normalized size = 0.78

$$\frac{\ln\left(x - \frac{\left(-\frac{27}{2} + \frac{\sqrt{3}9i}{2}\right)\left(-36 - \sqrt{3}12i\right)^{1/3}}{54}\right)\left(-36 - \sqrt{3}12i\right)^{1/3}}{18} + \frac{\ln\left(x + \frac{\left(\frac{27}{2} + \frac{\sqrt{3}9i}{2}\right)\left(-36 + \sqrt{3}12i\right)^{1/3}}{54}\right)\left(-36 + \sqrt{3}12i\right)^{1/3}}{18} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3 - 1)/(x^6 - x^3 + 1), x)`

[Out] $(\log(x - (((3^{1/2})9i)/2 - 27/2)*(-3^{1/2}12i - 36)^{1/3})/54)*(-3^{1/2}12i - 36)^{1/3})/18 + (\log(x + (((3^{1/2})9i)/2 + 27/2)*(3^{1/2}12i - 36)^{1/3})/54)*(3^{1/2}12i - 36)^{1/3})/18 - (2^{2/3}*\log(x - (2^{2/3}*(-3^{1/2}1i - 3)^{1/3}*(3^{1/3} + 3^{5/6}1i))*((3*(3^{1/2}1i + 3)*(3^{1/3} + 3^{5/6}1i))^3)/16 + 27))/108)*(-3^{1/2}1i - 3)^{1/3}*(3^{1/3} + 3^{5/6}1i))/36 - (2^{2/3}*\log(x + (2^{2/3}*(3^{1/2}1i - 3)^{1/3}*(3^{1/3} - 3^{5/6}1i))*((3*(3^{1/2}1i - 3)*(3^{1/3} - 3^{5/6}1i))^3)/16 - 27))/108)*(3^{1/2}1i - 3)^{1/3}*(3^{1/3} - 3^{5/6}1i))/36 - (2^{2/3}*\log(x + (2^{2/3})*3^{5/6}*(-3^{1/2}1i - 3)^{1/3}1i)/6)*(-3^{1/2}1i - 3)^{1/3}*(3^{1/3} - 3^{5/6}1i))/36 - (2^{2/3}*\log(x - (2^{2/3})*3^{5/6}*(3^{1/2}1i - 3)^{1/3}1i)/6)*(3^{1/2}1i - 3)^{1/3}*(3^{1/3} + 3^{5/6}1i))/36$

sympy [A] time = 0.18, size = 26, normalized size = 0.06

$$-\text{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t \log(729t^4 - 9t + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)/(x**6-x**3+1), x)`

[Out] `-RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t + x)))`

$$3.30 \quad \int \frac{1-x^3}{x^2(1-x^3+x^6)} dx$$

Optimal. Leaf size=416

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

[Out] $-1/x + 1/6 \cdot \arctan\left(\frac{1/3(1+2 \cdot 2^{1/3})x}{(1+I \cdot 3^{1/2})^{1/3}}\right) \cdot 3^{1/2} \cdot (I-3^{1/2}) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3} - 1/18 \cdot \ln(-2^{1/3}x + (1+I \cdot 3^{1/2})^{1/3}) \cdot (3-I \cdot 3^{1/2}) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3} + 1/36 \cdot \ln(2^{2/3}x^2 + 2^{1/3}x \cdot (1+I \cdot 3^{1/2})^{1/3} + (1+I \cdot 3^{1/2})^{2/3}) \cdot (3-I \cdot 3^{1/2}) \cdot 2^{1/3} / (1+I \cdot 3^{1/2})^{1/3} - 1/18 \cdot \ln(-2^{1/3}x + (1-I \cdot 3^{1/2})^{1/3}) \cdot (3+I \cdot 3^{1/2}) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3} + 1/36 \cdot \ln(2^{2/3}x^2 + 2^{1/3}x \cdot (1-I \cdot 3^{1/2})^{1/3} + (1-I \cdot 3^{1/2})^{2/3}) \cdot (3+I \cdot 3^{1/2}) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3} - 1/6 \cdot \arctan\left(\frac{1/3(1+2 \cdot 2^{1/3})x}{(1-I \cdot 3^{1/2})^{1/3}}\right) \cdot 3^{1/2} \cdot (3^{1/2}+I) \cdot 2^{1/3} / (1-I \cdot 3^{1/2})^{1/3}$

Rubi [A] time = 0.28, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1504, 1374, 292, 31, 634, 617, 204, 628}

$$\frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]

[Out] $-x^{-1} - ((I + \text{Sqrt}[3]) \cdot \text{ArcTan}[(1 + (2x)/((1 - I \cdot \text{Sqrt}[3])/2)^{1/3})]/\text{Sqrt}[3])) / (3 \cdot 2^{2/3} \cdot (1 - I \cdot \text{Sqrt}[3])^{1/3}) + ((I - \text{Sqrt}[3]) \cdot \text{ArcTan}[(1 + (2x)/((1 + I \cdot \text{Sqrt}[3])/2)^{1/3})]/\text{Sqrt}[3])) / (3 \cdot 2^{2/3} \cdot (1 + I \cdot \text{Sqrt}[3])^{1/3}) - ((3 + I \cdot \text{Sqrt}[3]) \cdot \text{Log}[(1 - I \cdot \text{Sqrt}[3])^{1/3} - 2^{1/3}x]) / (9 \cdot 2^{2/3} \cdot (1 - I \cdot \text{Sqrt}[3])^{1/3}) - ((3 - I \cdot \text{Sqrt}[3]) \cdot \text{Log}[(1 + I \cdot \text{Sqrt}[3])^{1/3} - 2^{1/3}x]) / (9 \cdot 2^{2/3} \cdot (1 + I \cdot \text{Sqrt}[3])^{1/3}) + ((3 + I \cdot \text{Sqrt}[3]) \cdot \text{Log}[(1 - I \cdot \text{Sqrt}[3])^{2/3} + (2 \cdot (1 - I \cdot \text{Sqrt}[3]))^{1/3}x + 2^{2/3}x^2]) / (18 \cdot 2^{2/3} \cdot (1 - I \cdot \text{Sqrt}[3])^{1/3}) + ((3 - I \cdot \text{Sqrt}[3]) \cdot \text{Log}[(1 + I \cdot \text{Sqrt}[3])^{2/3} + (2 \cdot (1 + I \cdot \text{Sqrt}[3]))^{1/3}x + 2^{2/3}x^2]) / (18 \cdot 2^{2/3} \cdot (1 + I \cdot \text{Sqrt}[3])^{1/3})$

Rule 31

$\text{Int}[\frac{(a_.) + (b_.)(x_.)^{-1}}{x}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] \text{ ; FreeQ}[\{a, b\}, x]$

Rule 204

$\text{Int}[\frac{(a_.) + (b_.)(x_.)^2}{(a_.) + (b_.)(x_.)^2}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[-b, 2], x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 292

$\text{Int}[\frac{x}{(a_.) + (b_.)(x_.)^3}, x_Symbol] \rightarrow -\text{Dist}[(3*\text{Rt}[a, 3]*\text{Rt}[b, 3])^{-1}], \text{Int}[1/(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x), x], x] + \text{Dist}[1/(3*\text{Rt}[a, 3]*\text{Rt}[b, 3]), \text{Int}[(\text{Rt}[a, 3] + \text{Rt}[b, 3]*x)/(\text{Rt}[a, 3]^2 - \text{Rt}[a, 3]*\text{Rt}[b, 3]*x + \text{Rt}[b, 3]^2*x^2), x], x] \text{ ; FreeQ}[\{a, b\}, x]$

Rule 617

$\text{Int}[\frac{(a_.) + (b_.)(x_.) + (c_.)(x_.)^2}{(a_.) + (b_.)(x_.) + (c_.)(x_.)^2}, x_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] \text{ ; RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c])] \text{ ; FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)(x_.)}{(a_.) + (b_.)(x_.) + (c_.)(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)(x_.)}{(a_.) + (b_.)(x_.) + (c_.)(x_.)^2}, x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] \text{ ; FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1374

$\text{Int}[\frac{(d_.)*(x_.)^{(m_.)}}{(a_.) + (c_.)(x_.)^{(n2_.)} + (b_.)(x_.)^{(n_.)}}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(d^n*(b/q + 1))/2, \text{Int}[(d*x)^{(m-n)}/(b/2 + q/2 + c*x^n), x], x] - \text{Dist}[(d^n*(b/q - 1))/2, \text{Int}[(d*x)^{(m-n)}/(b/2 - q/2 + c*x^n), x], x]] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\&$

NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

Rule 1504

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m+1)*(a + b*x^n + c*x^(2*n))^(p+1))/(a*f^(m+1)), x] + Dist[1/(a*f^n*(m+1)), Int[(f*x)^(m+n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m+1) - b*d*(m+n*(p+1)+1] - c*d*(m+2*n*(p+1)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{1-x^3}{x^2(1-x^3+x^6)} dx &= -\frac{1}{x} - \int \frac{x^4}{1-x^3+x^6} dx \\
 &= -\frac{1}{x} + \frac{1}{6}(-3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
 &= -\frac{1}{x} + \frac{(-3-i\sqrt{3}) \int \frac{-\sqrt{\frac{1}{2}(1-i\sqrt{3})+x}}{\left(\frac{1}{2}(1-i\sqrt{3})\right)^{2/3} + \sqrt{\frac{1}{2}(1-i\sqrt{3})}x+x^2} dx}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{1}{2}(1+i\sqrt{3})+x}} dx}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \\
 &= -\frac{1}{x} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \\
 &= -\frac{1}{x} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} + \\
 &= -\frac{1}{x} - \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1-i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}} + \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{1}{2}(1+i\sqrt{3})}}}{\sqrt{3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{1+i\sqrt{3}}} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9 \cdot 2^{2/3} \sqrt[3]{1-i\sqrt{3}}}
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6 - \#1^3 + 1\&, \frac{\#1^2 \log(x - \#1)}{2\#1^3 - 1}\&\right] - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)), x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

fricas [B] time = 1.32, size = 1598, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1), x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*s
in(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
- 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*
12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*
x*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
- 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3
) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3
*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(s
qrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(
2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)
*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*1
2^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2
+ 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(
2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*
cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arct
an(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(
3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)
) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*
sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 864*cos(2/3*arcta
```

$$\begin{aligned} & n(\sqrt{3} - 2) \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^3 - 6 \cdot (18^{2/3} \cdot 12^{2/3} \sqrt{3}) x - 36 \sqrt{3} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 12 \cdot (18^{2/3} \cdot 12^{2/3}) x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) + 72 \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^3 \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) - \sqrt{18^{2/3} \cdot 12^{2/3}} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \sqrt{3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) + 6 \cdot 18^{1/3} \cdot 12^{1/3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 36 x^2 \cdot (18^{2/3} \cdot 12^{2/3} \sqrt{3}) \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 - 18^{2/3} \cdot 12^{2/3} \sqrt{3} \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 2 \cdot 18^{2/3} \cdot 12^{2/3} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \Big/ (3 \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 - 10 \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 3 \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4) - 4 \cdot (18^{2/3} \cdot 12^{1/6} \sqrt{3}) x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) + 18^{2/3} \cdot 12^{1/6} x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \arctan\left(-\frac{1}{432} \cdot (6 \cdot 18^{2/3}) \cdot 12^{2/3} x - 216 \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 216 \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 - 18^{2/3} \cdot 12^{2/3} \sqrt{18^{2/3} \cdot 12^{2/3}} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 36 x^2) \Big/ (\cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)) - (18^{2/3} \cdot 12^{1/6} \sqrt{3}) x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) + 18^{2/3} \cdot 12^{1/6} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \log(18^{2/3} \cdot 12^{2/3} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 12 \cdot 18^{1/3} \cdot 12^{1/3} \sqrt{3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) + 6 \cdot 18^{1/3} \cdot 12^{1/3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 - 3 \cdot 18^{1/3} \cdot 12^{1/3} x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 36 x^2) + (18^{2/3} \cdot 12^{1/6} \sqrt{3}) x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) - 18^{2/3} \cdot 12^{1/6} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right) \log(18^{2/3} \cdot 12^{2/3} \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 + 18^{2/3} \cdot 12^{2/3} \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^4 - 12 \cdot 18^{1/3} \cdot 12^{1/3} x \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 2 \cdot (18^{2/3} \cdot 12^{2/3}) \cos\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 6 \cdot 18^{1/3} \cdot 12^{1/3} x \sin\left(\frac{2}{3} \arctan(\sqrt{3} - 2)\right)^2 + 36 x^2) - 108) / x \end{aligned}$$

giac [B] time = 0.71, size = 829, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{9} \cdot (2 \sqrt{3} \cos(4/9 \pi)^5 - 20 \sqrt{3} \cos(4/9 \pi)^3 \sin(4/9 \pi)^2 + 10 \sqrt{3} \cos(4/9 \pi) \sin(4/9 \pi)^4 - 10 \cos(4/9 \pi)^4 \sin(4/9 \pi) + 20 \cos(4/9 \pi)^2 \sin(4/9 \pi)^3 - 2 \sin(4/9 \pi)^5 + \sqrt{3} \cos(4/9 \pi)^2 - \sqrt{3} \sin(4/9 \pi)^2 - 2 \cos(4/9 \pi) \sin(4/9 \pi)) \arctan(-((\sqrt{3} i + 1) \cos(4/9 \pi))$

$\pi) - 2x)/((\sqrt{3}i + 1)\sin(4/9\pi))) + 1/9*(2\sqrt{3}\cos(2/9\pi)^5 - 20\sqrt{3}\cos(2/9\pi)^3\sin(2/9\pi)^2 + 10\sqrt{3}\cos(2/9\pi)\sin(2/9\pi)^4 - 10\cos(2/9\pi)^4\sin(2/9\pi) + 20\cos(2/9\pi)^2\sin(2/9\pi)^3 - 2\sin(2/9\pi)^5 + \sqrt{3}\cos(2/9\pi)^2 - \sqrt{3}\sin(2/9\pi)^2 - 2\cos(2/9\pi)\sin(2/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(2/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(2/9\pi))) - 1/9*(2\sqrt{3}\cos(1/9\pi)^5 - 20\sqrt{3}\cos(1/9\pi)^3\sin(1/9\pi)^2 + 10\sqrt{3}\cos(1/9\pi)\sin(1/9\pi)^4 + 10\cos(1/9\pi)^4\sin(1/9\pi) - 20\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sin(1/9\pi)^5 - \sqrt{3}\cos(1/9\pi)^2 + \sqrt{3}\sin(1/9\pi)^2 - 2\cos(1/9\pi)\sin(1/9\pi))\arctan(((\sqrt{3}i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}i + 1)\sin(1/9\pi))) + 1/18*(10\sqrt{3}\cos(4/9\pi)^4\sin(4/9\pi) - 20\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^3 + 2\sqrt{3}\sin(4/9\pi)^5 + 2\cos(4/9\pi)^5 - 20\cos(4/9\pi)^3\sin(4/9\pi)^2 + 10\cos(4/9\pi)\sin(4/9\pi)^4 + 2\sqrt{3}\cos(4/9\pi)\sin(4/9\pi) + \cos(4/9\pi)^2 - \sin(4/9\pi)^2)\log(-(\sqrt{3}i\cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) + 1/18*(10\sqrt{3}\cos(2/9\pi)^4\sin(2/9\pi) - 20\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^3 + 2\sqrt{3}\sin(2/9\pi)^5 + 2\cos(2/9\pi)^5 - 20\cos(2/9\pi)^3\sin(2/9\pi)^2 + 10\cos(2/9\pi)\sin(2/9\pi)^4 + 2\sqrt{3}\cos(2/9\pi)\sin(2/9\pi) + \cos(2/9\pi)^2 - \sin(2/9\pi)^2)\log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) + 1/18*(10\sqrt{3}\cos(1/9\pi)^4\sin(1/9\pi) - 20\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^3 + 2\sqrt{3}\sin(1/9\pi)^5 - 2\cos(1/9\pi)^5 + 20\cos(1/9\pi)^3\sin(1/9\pi)^2 - 10\cos(1/9\pi)\sin(1/9\pi)^4 - 2\sqrt{3}\cos(1/9\pi)\sin(1/9\pi) + \cos(1/9\pi)^2 - \sin(1/9\pi)^2)\log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1) - 1/x$

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$\frac{\text{RootOf}(-Z^6 - Z^3 + 1)^4 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{3 \left(2 \text{RootOf}(-Z^6 - Z^3 + 1)^5 - \text{RootOf}(-Z^6 - Z^3 + 1)^2 \right)} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)/x^2/(x^6-x^3+1), x)

[Out] -1/3*sum(1/(2*_R^5-_R^2)*_R^4*ln(-_R+x), _R=RootOf(-Z^6-_Z^3+1))-1/x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{x^4}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^2/(x^6-x^3+1), x, algorithm="maxima")

[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)

mupad [B] time = 0.40, size = 313, normalized size = 0.75

$$\frac{\ln\left(-x + \left(162x + \frac{27(36 + \sqrt{3}12i)^{2/3}}{4}\right)\left(\frac{1}{162} + \frac{\sqrt{3}1i}{486}\right)\right)(36 + \sqrt{3}12i)^{1/3}}{18} + \frac{\ln\left(-x - \left(162x + \frac{27(36 - \sqrt{3}12i)^{2/3}}{4}\right)\left(-\frac{1}{162} + \frac{\sqrt{3}1i}{486}\right)\right)(36 - \sqrt{3}12i)^{1/3}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)`

[Out] $(\log((162*x + (27*(3^{1/2}*12i + 36)^{2/3})/4)*((3^{1/2}*1i)/486 + 1/162) - x)*(3^{1/2}*12i + 36)^{1/3})/18 + (\log(-x - (162*x + (27*(36 - 3^{1/2}*12i)^{2/3})/4)*((3^{1/2}*1i)/486 - 1/162))*(36 - 3^{1/2}*12i)^{1/3})/18 - 1/x - (2^{2/3}*\log(x + (2^{1/3}*3^{2/3}*(3 - 3^{1/2}*1i)^{2/3}))/12 - (2^{1/3}*3^{1/6}*(3 - 3^{1/2}*1i)^{2/3}*1i)/4)*(3 - 3^{1/2}*1i)^{1/3}*(3^{1/3} - 3^{5/6}*1i))/36 - (2^{2/3}*\log(x + (2^{1/3}*3^{2/3}*(3^{1/2}*1i + 3)^{2/3}))/12 + (2^{1/3}*3^{1/6}*(3^{1/2}*1i + 3)^{2/3}*1i)/4)*(3^{1/2}*1i + 3)^{1/3}*(3^{1/3} + 3^{5/6}*1i))/36 - (2^{2/3}*\log(x - (2^{1/3}*3^{2/3}*(3 - 3^{1/2}*1i)^{2/3}))/6)*(3 - 3^{1/2}*1i)^{1/3}*(3^{1/3} + 3^{5/6}*1i))/36 - (2^{2/3}*\log(x - (2^{1/3}*3^{2/3}*(3^{1/2}*1i + 3)^{2/3}))/6)*(3^{1/2}*1i + 3)^{1/3}*(3^{1/3} - 3^{5/6}*1i))/36$

sympy [A] time = 0.19, size = 31, normalized size = 0.07

$$-\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(6561t^5 + 54t^2 + x)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**3+1)/x**2/(x**6-x**3+1),x)`

[Out] `-RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**2 + x))) - 1/x`

$$3.31 \quad \int \frac{1-x^3}{x^3(1-x^3+x^6)} dx$$

Optimal. Leaf size=418

$$-\frac{1}{2x^2} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

[Out] $-1/2/x^2 - 1/6 * \arctan(1/3 * (1 + 2 * 2^{1/3}) * x / ((1 + I * 3^{1/2})^{1/3}) * 3^{1/2}) * (I - 3^{1/2}) * 2^{2/3} / ((1 + I * 3^{1/2})^{2/3}) - 1/18 * \ln(-2^{1/3} * x + (1 + I * 3^{1/2})^{1/3}) * (3 - I * 3^{1/2}) * 2^{2/3} / ((1 + I * 3^{1/2})^{2/3}) + 1/36 * \ln(2^{2/3} * x^2 + 2^{1/3} * x * (1 + I * 3^{1/2})^{1/3} + (1 + I * 3^{1/2})^{2/3}) * (3 - I * 3^{1/2}) * 2^{2/3} / ((1 + I * 3^{1/2})^{2/3}) - 1/18 * \ln(-2^{1/3} * x + (1 - I * 3^{1/2})^{1/3}) * (3 + I * 3^{1/2}) * 2^{2/3} / ((1 - I * 3^{1/2})^{2/3}) + 1/36 * \ln(2^{2/3} * x^2 + 2^{1/3} * x * (1 - I * 3^{1/2})^{1/3} + (1 - I * 3^{1/2})^{2/3}) * (3 + I * 3^{1/2}) * 2^{2/3} / ((1 - I * 3^{1/2})^{2/3}) + 1/6 * \arctan(1/3 * (1 + 2 * 2^{1/3}) * x / ((1 - I * 3^{1/2})^{1/3}) * 3^{1/2}) * (3^{1/2} + I) * 2^{2/3} / ((1 - I * 3^{1/2})^{2/3})$

Rubi [A] time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1504, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2} + \frac{(3+i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1-i\sqrt{3})}x + (1-i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} + \frac{(3-i\sqrt{3}) \log\left(2^{2/3}x^2 + \sqrt[3]{2(1+i\sqrt{3})}x + (1+i\sqrt{3})^{2/3}\right)}{18\sqrt[3]{2}(1+i\sqrt{3})^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)), x]

[Out] $-1/(2*x^2) + ((I + \text{Sqrt}[3]) * \text{ArcTan}[(1 + (2*x)/((1 - I*\text{Sqrt}[3])/2)^{1/3})]/\text{Sqrt}[3])/((3*2^{1/3}) * (1 - I*\text{Sqrt}[3])^{2/3}) - ((I - \text{Sqrt}[3]) * \text{ArcTan}[(1 + (2*x)/((1 + I*\text{Sqrt}[3])/2)^{1/3})]/\text{Sqrt}[3])/((3*2^{1/3}) * (1 + I*\text{Sqrt}[3])^{2/3}) - ((3 + I*\text{Sqrt}[3]) * \text{Log}[(1 - I*\text{Sqrt}[3])^{1/3} - 2^{1/3}*x])/((9*2^{1/3}) * (1 - I*\text{Sqrt}[3])^{2/3}) - ((3 - I*\text{Sqrt}[3]) * \text{Log}[(1 + I*\text{Sqrt}[3])^{1/3} - 2^{1/3}*x])/((9*2^{1/3}) * (1 + I*\text{Sqrt}[3])^{2/3}) + ((3 + I*\text{Sqrt}[3]) * \text{Log}[(1 - I*\text{Sqrt}[3])^{2/3} + (2*(1 - I*\text{Sqrt}[3]))^{1/3}*x + 2^{2/3}*x^2])/((18*2^{1/3}) * (1 - I*\text{Sqrt}[3])^{2/3}) + ((3 - I*\text{Sqrt}[3]) * \text{Log}[(1 + I*\text{Sqrt}[3])^{2/3} + (2*(1 + I*\text{Sqrt}[3]))^{1/3}*x + 2^{2/3}*x^2])/((18*2^{1/3}) * (1 + I*\text{Sqrt}[3])^{2/3})$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)³)⁽⁻¹⁾, x_Symbol] := Dist[1/(3*Rt[a, 3]²), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]²), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]² - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]²*x²), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)²)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)²)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b²]}, Dist[-2/b, Subst[Int[1/(q - x²), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q², 1] || !RationalQ[b² - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b² - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)²), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x², x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)²), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x²), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x²), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b² - 4*a*c, 0] && !NiceSqrtQ[b² - 4*a*c]

Rule 1374

```

Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m - n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]

```

Rule 1504

```

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
:> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1)/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x^3(1-x^3+x^6)} dx &= -\frac{1}{2x^2} - \frac{1}{2} \int \frac{2x^3}{1-x^3+x^6} dx \\
&= -\frac{1}{2x^2} - \int \frac{x^3}{1-x^3+x^6} dx \\
&= -\frac{1}{2x^2} + \frac{1}{6}(-3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3} dx - \frac{1}{6}(3+i\sqrt{3}) \int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3} dx \\
&= -\frac{1}{2x^2} - \frac{(3-i\sqrt{3}) \int \frac{1}{-\sqrt{\frac{3}{2}}(1+i\sqrt{3})+x} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}(1+i\sqrt{3})\right)^{2/3} + \sqrt{\frac{3}{2}}(1+i\sqrt{3})x+x^2} dx}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} \\
&= -\frac{1}{2x^2} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \dots \\
&= -\frac{1}{2x^2} - \frac{(3+i\sqrt{3}) \log\left(\sqrt[3]{1-i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(3-i\sqrt{3}) \log\left(\sqrt[3]{1+i\sqrt{3}} - \sqrt[3]{2}x\right)}{9\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} + \dots \\
&= -\frac{1}{2x^2} + \frac{(i+\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{3}{2}}(1-i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1-i\sqrt{3})^{2/3}} - \frac{(i-\sqrt{3}) \tan^{-1}\left(\frac{1+\frac{2x}{\sqrt{\frac{3}{2}}(1+i\sqrt{3})}}{\sqrt{3}}\right)}{3\sqrt[3]{2}(1+i\sqrt{3})^{2/3}} - \frac{(3+i\sqrt{3}) \log}{9\sqrt[3]{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6 - \#1^3 + 1\&, \frac{\#1 \log(x - \#1)}{2\#1^3 - 1}\&\right] - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)), x]

[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) &] /3

fricas [B] time = 1.17, size = 1062, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")`

[Out]
$$\frac{1}{108} \cdot (2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) \cdot \log(-2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - 8 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \arctan(1/216 \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2}) \cdot \sqrt{-2 \cdot 18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - 6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x + 216 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 108 \cdot \sqrt{3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x + 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2}) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2) + 4 \cdot (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \arctan(-1/108 \cdot (6 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 108 \cdot \sqrt{3}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 108 \cdot \sqrt{3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 18 \cdot (18^{1/3} \cdot 12^{5/6} \cdot x - 24 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - \sqrt{18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))} - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) \cdot (18^{1/3} \cdot 12^{5/6} \cdot \sqrt{3} \cdot \sqrt{2}) \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) - 3 \cdot 18^{1/3} \cdot 12^{5/6} \cdot \sqrt{2}) \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))) / (\cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 - 3 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2) + (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - (18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x^2 \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) + 18^{2/3} \cdot 12^{1/6} \cdot x^2 \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))) \cdot \log(18^{2/3} \cdot 12^{1/6} \cdot \sqrt{3} \cdot x \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2)) - 3 \cdot 18^{2/3} \cdot 12^{1/6} \cdot x \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2)) + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \cos(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 3 \cdot 18^{1/3} \cdot 12^{1/3} \cdot \sin(2/3 \cdot \arctan(\sqrt{3} - 2))^2 + 18 \cdot x^2) - 54) / x^2$$

giac [B] time = 0.64, size = 642, normalized size = 1.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")

[Out] $\frac{1}{9}(2\sqrt{3}\cos(4/9\pi)^4 - 12\sqrt{3}\cos(4/9\pi)^2\sin(4/9\pi)^2 + 2\sqrt{3}\sin(4/9\pi)^4 + 8\cos(4/9\pi)^3\sin(4/9\pi) - 8\cos(4/9\pi)\sin(4/9\pi)^3 + \sqrt{3}\cos(4/9\pi) + \sin(4/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(4/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(4/9\pi))) + \frac{1}{9}(2\sqrt{3}\cos(2/9\pi)^4 - 12\sqrt{3}\cos(2/9\pi)^2\sin(2/9\pi)^2 + 2\sqrt{3}\sin(2/9\pi)^4 + 8\cos(2/9\pi)^3\sin(2/9\pi) - 8\cos(2/9\pi)\sin(2/9\pi)^3 + \sqrt{3}\cos(2/9\pi) + \sin(2/9\pi))\arctan(-((\sqrt{3}i + 1)\cos(2/9\pi) - 2x)/((\sqrt{3}i + 1)\sin(2/9\pi))) + \frac{1}{9}(2\sqrt{3}\cos(1/9\pi)^4 - 12\sqrt{3}\cos(1/9\pi)^2\sin(1/9\pi)^2 + 2\sqrt{3}\sin(1/9\pi)^4 - 8\cos(1/9\pi)^3\sin(1/9\pi) + 8\cos(1/9\pi)\sin(1/9\pi)^3 - \sqrt{3}\cos(1/9\pi) + \sin(1/9\pi))\arctan(((\sqrt{3}i + 1)\cos(1/9\pi) + 2x)/((\sqrt{3}i + 1)\sin(1/9\pi))) + \frac{1}{18}(8\sqrt{3}\cos(4/9\pi)^3\sin(4/9\pi) - 8\sqrt{3}\cos(4/9\pi)\sin(4/9\pi)^3 - 2\cos(4/9\pi)^4 + 12\cos(4/9\pi)^2\sin(4/9\pi)^2 - 2\sin(4/9\pi)^4 + \sqrt{3}\sin(4/9\pi) - \cos(4/9\pi))\log(-(\sqrt{3}i\cos(4/9\pi) + \cos(4/9\pi))x + x^2 + 1) + \frac{1}{18}(8\sqrt{3}\cos(2/9\pi)^3\sin(2/9\pi) - 8\sqrt{3}\cos(2/9\pi)\sin(2/9\pi)^3 - 2\cos(2/9\pi)^4 + 12\cos(2/9\pi)^2\sin(2/9\pi)^2 - 2\sin(2/9\pi)^4 + \sqrt{3}\sin(2/9\pi) - \cos(2/9\pi))\log(-(\sqrt{3}i\cos(2/9\pi) + \cos(2/9\pi))x + x^2 + 1) - \frac{1}{18}(8\sqrt{3}\cos(1/9\pi)^3\sin(1/9\pi) - 8\sqrt{3}\cos(1/9\pi)\sin(1/9\pi)^3 + 2\cos(1/9\pi)^4 - 12\cos(1/9\pi)^2\sin(1/9\pi)^2 + 2\sin(1/9\pi)^4 - \sqrt{3}\sin(1/9\pi) - \cos(1/9\pi))\log((\sqrt{3}i\cos(1/9\pi) + \cos(1/9\pi))x + x^2 + 1) - \frac{1}{2x^2}$

maple [C] time = 0.01, size = 46, normalized size = 0.11

$$\frac{\text{RootOf}(-Z^6 - Z^3 + 1)^3 \ln(-\text{RootOf}(-Z^6 - Z^3 + 1) + x)}{3\left(2\text{RootOf}(-Z^6 - Z^3 + 1)^5 - \text{RootOf}(-Z^6 - Z^3 + 1)^2\right)} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^3+1)/x^3/(x^6-x^3+1),x)

[Out] $-1/3\sum(1/(2*_R^5 - _R^2)*_R^3*\ln(-_R+x), _R=\text{RootOf}(-Z^6 - Z^3 + 1)) - 1/2/x^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2} - \int \frac{x^3}{x^6 - x^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")

[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)

mupad [B] time = 2.40, size = 332, normalized size = 0.79

$$\frac{\ln\left(x + \frac{2^{2/3} 3^{5/6} (3 - \sqrt{3} 1i)^{1/3} 1i}{6}\right) (36 - \sqrt{3} 12i)^{1/3}}{18} + \frac{\ln\left(x - \frac{2^{2/3} 3^{5/6} (3 + \sqrt{3} 1i)^{1/3} 1i}{6}\right) (36 + \sqrt{3} 12i)^{1/3}}{18} - \frac{1}{2x^2} - \frac{2^{2/3} \ln\left(x - \dots\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)

[Out] (log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36

sympy [A] time = 0.20, size = 32, normalized size = 0.08

$$-\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log(-1458t^4 - 9t + x)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)

[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t + x))) - 1/(2*x**2)

$$3.32 \quad \int \frac{x^2(-2+x^3)}{1-x^3+x^6} dx$$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{6} \log(x^6 - x^3 + 1)$$

[Out] 1/6*ln(x^6-x^3+1)+1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1468, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^6 - x^3 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6), x]

[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

`t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 1468

`Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_)]^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]`

Rubi steps

$$\begin{aligned} \int \frac{x^2(-2+x^3)}{1-x^3+x^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{-2+x}{1-x+x^2} dx, x, x^3 \right) \\ &= \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) \\ &= \frac{1}{6} \log(1-x^3+x^6) + \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\ &= -\frac{\tan^{-1} \left(\frac{-1+2x^3}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{1}{6} \log(1-x^3+x^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.03

$$\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1} \left(\frac{2x^3 - 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6), x]

[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6

fricas [A] time = 0.96, size = 32, normalized size = 0.89

$$-\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^3 - 1) \right) + \frac{1}{6} \log(x^6 - x^3 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(x^3-2)/(x^6-x^3+1), x, algorithm="fricas")

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) + 1/6*\log(x^6 - x^3 + 1)$

giac [A] time = 0.46, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) + \frac{1}{6}\log(x^6-x^3+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")`

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) + 1/6*\log(x^6 - x^3 + 1)$

maple [A] time = 0.00, size = 33, normalized size = 0.92

$$-\frac{\sqrt{3}\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3} + \frac{\ln(x^6-x^3+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(x^3-2)/(x^6-x^3+1),x)`

[Out] $1/6*\ln(x^6-x^3+1)-1/3*3^{(1/2)}*\arctan(1/3*(2*x^3-1)*3^{(1/2)})$

maxima [A] time = 0.98, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3-1)\right) + \frac{1}{6}\log(x^6-x^3+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")`

[Out] $-1/3*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^3 - 1)) + 1/6*\log(x^6 - x^3 + 1)$

mupad [B] time = 1.84, size = 34, normalized size = 0.94

$$\frac{\ln(x^6-x^3+1)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)`

[Out] $\log(x^6 - x^3 + 1)/6 + (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^3)/3))/3$

sympy [A] time = 0.13, size = 37, normalized size = 1.03

$$\frac{\log(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)

[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3

$$3.33 \quad \int \frac{1+x^3}{x(1-x^3+x^6)} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

[Out] $\ln(x) - 1/6 * \ln(x^6 - x^3 + 1) - 1/3 * \arctan(1/3 * (-2 * x^3 + 1) * 3^{(1/2)}) * 3^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 + x^3)/(x*(1 - x^3 + x^6)), x]$

[Out] $-(\text{ArcTan}[(1 - 2*x^3)/\text{Sqrt}[3]]/\text{Sqrt}[3]) + \text{Log}[x] - \text{Log}[1 - x^3 + x^6]/6$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x))/((a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1+x^3}{x(1-x^3+x^6)} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1+x}{x(1-x+x^2)} dx, x, x^3 \right) \\
 &= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{2-x}{1-x+x^2} \right) dx, x, x^3 \right) \\
 &= \log(x) + \frac{1}{3} \text{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, x^3 \right) \\
 &= \log(x) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) \\
 &= \log(x) - \frac{1}{6} \log(1-x^3+x^6) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
 &= \frac{\tan^{-1} \left(\frac{-1+2x^3}{\sqrt{3}} \right)}{\sqrt{3}} + \log(x) - \frac{1}{6} \log(1-x^3+x^6)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x) - \frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - 2 \log(x - \#1)}{2\#1^3 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]

[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-1 + 2*#1^3) &]/3

fricas [A] time = 1.05, size = 34, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(x)

giac [A] time = 0.54, size = 35, normalized size = 0.90

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(abs(x))

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3} + \ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/x/(x^6-x^3+1),x)

[Out] -1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)

maxima [A] time = 0.99, size = 38, normalized size = 0.97

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \frac{1}{3} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*log(x^3)

mupad [B] time = 1.85, size = 36, normalized size = 0.92

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)

[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3

sympy [A] time = 0.15, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/x/(x**6-x**3+1),x)

[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3

$$3.34 \quad \int \frac{1+x^3}{x-x^4+x^7} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

[Out] $\ln(x) - 1/6 * \ln(x^6 - x^3 + 1) - 1/3 * \arctan(1/3 * (-2 * x^3 + 1) * 3^{(1/2)}) * 3^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6} \log(x^6 - x^3 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] `Int[(1 + x^3)/(x - x^4 + x^7), x]`

[Out] `-(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628

`Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rule 634

`Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In`

$\text{Int}[(b + 2cx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2cd - be, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4ac]$

Rule 800

$\text{Int}[((d_.) + (e_.)x)^{m_}((f_.) + (g_.)x)/(a_.) + (b_.)x + (c_.)x^2), x_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(d + ex)^m(f + gx)/(a + bx + cx^2), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[cd^2 - bde + ae^2, 0] \&\& \text{IntegerQ}[m]$

Rule 1474

$\text{Int}[x^{m_}((a_.) + (c_.)x^{n2_}) + (b_.)x^{n_})^{p_}((d_.) + (e_.)x^{n_})^{q_}), x_Symbol] \text{:>} \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{\text{Simplify}[(m + 1)/n] - 1}(d + ex)^q(a + bx + cx^2)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{EqQ}[n2, 2n] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1594

$\text{Int}[(u_.)((a_.)x^{p_}) + (b_.)x^{q_}) + (c_.)x^{r_})^{n_}), x_Symbol] \text{:>} \text{Int}[u x^{n*p}(a + bx^{q-p} + cx^{r-p})^n, x] /; \text{FreeQ}\{a, b, c, p, q, r\}, x] \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p] \&\& \text{PosQ}[r - p]$

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x-x^4+x^7} dx &= \int \frac{1+x^3}{x(1-x^3+x^6)} dx \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1+x}{x(1-x+x^2)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{2-x}{1-x+x^2} \right) dx, x, x^3 \right) \\
&= \log(x) + \frac{1}{3} \text{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, x^3 \right) \\
&= \log(x) - \frac{1}{6} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^3 \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^3 \right) \\
&= \log(x) - \frac{1}{6} \log(1-x^3+x^6) - \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^3 \right) \\
&= \frac{\tan^{-1} \left(\frac{-1+2x^3}{\sqrt{3}} \right)}{\sqrt{3}} + \log(x) - \frac{1}{6} \log(1-x^3+x^6)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 1.41

$$\log(x) - \frac{1}{3} \text{RootSum} \left[\#1^6 - \#1^3 + 1 \&, \frac{\#1^3 \log(x - \#1) - 2 \log(x - \#1)}{2\#1^3 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^3)/(x - x^4 + x^7), x]

[Out] Log[x] - RootSum[1 - #1^3 + #1^6 &, (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-1 + 2*#1^3) &]/3

fricas [A] time = 1.05, size = 34, normalized size = 0.87

$$\frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^3 - 1) \right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^7-x^4+x), x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(x)

giac [A] time = 0.41, size = 35, normalized size = 0.90

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^3 - 1)\right) - \frac{1}{6} \log(x^6 - x^3 + 1) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(abs(x))

maple [A] time = 0.01, size = 35, normalized size = 0.90

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3} + \ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)/(x^7-x^4+x),x)

[Out] 1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))+ln(x)-1/6*ln(x^6-x^3+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^5 - 2x^2}{x^6 - x^3 + 1} dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")

[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)

mupad [B] time = 0.04, size = 36, normalized size = 0.92

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 + 1)/(x - x^4 + x^7),x)

[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3

sympy [A] time = 0.14, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)/(x**7-x**4+x),x)

[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3

3.35 $\int (d + ex^3)^{5/2} (a + bx^3 + cx^6) dx$

Optimal. Leaf size=396

$$\frac{2x(d + ex^3)^{5/2} (667ae^2 - 58bde + 16cd^2)}{11339e^2} + \frac{30dx(d + ex^3)^{3/2} (667ae^2 - 58bde + 16cd^2)}{124729e^2} + \frac{54d^2x\sqrt{d + ex^3} (667ae^2 - 58bde + 16cd^2)}{124729e^2}$$

[Out] $30/124729*d*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2+2/11339*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(5/2)/e^2-2/667*(-29*b*e+8*c*d)*x*(e*x^3+d)^(7/2)/e^2+2/29*c*x^4*(e*x^3+d)^(7/2)/e+54/124729*d^2*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+54/124729*3^(3/4)*d^3*(667*a*e^2-58*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))), I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^(1/2)$

Rubi [A] time = 0.42, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1411, 388, 195, 218}

$$\frac{2x(d + ex^3)^{5/2} (667ae^2 - 58bde + 16cd^2)}{11339e^2} + \frac{30dx(d + ex^3)^{3/2} (667ae^2 - 58bde + 16cd^2)}{124729e^2} + \frac{54d^2x\sqrt{d + ex^3} (667ae^2 - 58bde + 16cd^2)}{124729e^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x]

[Out] $(54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*sqrt[d + e*x^3])/(124729*e^2) + (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2) + (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7/2))/(29*e) + (54*3^(3/4)*sqrt[2 + sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*(d^(1/3) + e^(1/3)*x)*sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/((1 + sqrt[3])*d^(1/3) + e^(1/3)*x)^2*EllipticF[ArcSin[((1 - sqrt[3])*d^(1/3) + e^(1/3)*x)/(1 + sqrt[3])*d^(1/3) + e^(1/3)*x]), -7 - 4*sqrt[3]]/$

$$(124729e^{7/3}\sqrt{(d^{1/3})(d^{1/3} + e^{1/3}x)})/((1 + \sqrt{3})d^{1/3} + e^{1/3}x)^2\sqrt{d + ex^3}$$
Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
(c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (d + ex^3)^{5/2} (a + bx^3 + cx^6) dx &= \frac{2cx^4 (d + ex^3)^{7/2}}{29e} + \frac{2 \int (d + ex^3)^{5/2} \left(\frac{29ae}{2} - \left(4cd - \frac{29be}{2}\right) x^3 \right) dx}{29e} \\
&= -\frac{2(8cd - 29be)x (d + ex^3)^{7/2}}{667e^2} + \frac{2cx^4 (d + ex^3)^{7/2}}{29e} - \frac{1}{667} \left(-667a - \frac{2d(8cd - 29be)}{e^2} \right) \\
&= \frac{2 \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x (d + ex^3)^{5/2}}{11339} - \frac{2(8cd - 29be)x (d + ex^3)^{7/2}}{667e^2} + \frac{2cx^4 (d + ex^3)^{7/2}}{29e} \\
&= \frac{30d \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x (d + ex^3)^{3/2}}{124729} + \frac{2 \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x (d + ex^3)^{5/2}}{11339} \\
&= \frac{54d^2 \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x \sqrt{d + ex^3}}{124729} + \frac{30d \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x (d + ex^3)^{3/2}}{124729} \\
&= \frac{54d^2 \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x \sqrt{d + ex^3}}{124729} + \frac{30d \left(667a + \frac{2d(8cd - 29be)}{e^2} \right) x (d + ex^3)^{3/2}}{124729}
\end{aligned}$$

Mathematica [C] time = 0.18, size = 103, normalized size = 0.26

$$\frac{x \sqrt{d + ex^3} \left(\frac{{}_2F_1\left(-\frac{5}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{ex^3}{d}\right) (29d^2 e(23ae - 2bd) + 16cd^4)}{\sqrt{\frac{ex^3}{d} + 1}} - 2(d + ex^3)^3 (-29be + 8cd - 23cex^3) \right)}{667e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x]

[Out] (x*sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*c*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -(e*x^3)/d]))/sqrt[1 + (e*x^3)/d])/(667*e^2)

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((ce^2x^{12} + (2cde + be^2)x^9 + (cd^2 + 2bde + ae^2)x^6 + (bd^2 + 2ade)x^3 + ad^2\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] integral((c*e^2*x^12 + (2*c*d*e + b*e^2)*x^9 + (c*d^2 + 2*b*d*e + a*e^2)*x^6 + (b*d^2 + 2*a*d*e)*x^3 + a*d^2)*sqrt(e*x^3 + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)(ex^3 + d)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)

maple [B] time = 0.26, size = 1070, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x)

[Out] c*(2/29*e^2*x^13*(e*x^3+d)^(1/2)+122/667*d*e*x^10*(e*x^3+d)^(1/2)+1562/1133
9*d^2*x^7*(e*x^3+d)^(1/2)+810/124729*d^3/e*x^4*(e*x^3+d)^(1/2)-1296/124729*
d^4/e^2*x*(e*x^3+d)^(1/2)-864/124729*I*d^5/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x
+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(
1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*
e^2)^(1/3))^(1/2), (I*3^(1/2)/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))+b*(2/23*e^2*x^10*(e*x^3+d)^(1/2)+98/391*
d*e*x^7*(e*x^3+d)^(1/2)+974/4301*d^2*x^4*(e*x^3+d)^(1/2)+162/4301*d^3/e*x*(
e*x^3+d)^(1/2)+108/4301*I*d^4/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^
2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((
x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)
)^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e
*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2), (I*3^(1/2)/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d
e^2)^(1/3)))^(1/2))+a(2/17*e^2*x^7*(e*x^3+d)^(1/2)+74/187*d*e*x^4*(e*x^3
+d)^(1/2)+106/187*d^2*x*(e*x^3+d)^(1/2)-54/187*I*d^3*3^(1/2)/e*(-d*e^2)^(1/
3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d
e^2)^(1/3))^(1/2)((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-

$-d*e^2)^{(1/3)}*3^{(1/2)}*e/(-d*e^2)^{(1/3))^{(1/2)}/(e*x^3+d)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2/e*(-d*e^2)^{(1/3)}-1/2*I*3^{(1/2)}/e*(-d*e^2)^{(1/3)})*3^{(1/2)})*e/(-d*e^2)^{(1/3))^{(1/2)}, (I*3^{(1/2)}/e*(-d*e^2)^{(1/3)}/(-3/2/e*(-d*e^2)^{(1/3)}+1/2*I*3^{(1/2)}/e*(-d*e^2)^{(1/3))^{(1/2))}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)(ex^3 + d)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (ex^3 + d)^{5/2} (cx^6 + bx^3 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x)

[Out] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x)

sympy [A] time = 9.01, size = 400, normalized size = 1.01

$$\frac{ad^{\frac{5}{2}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2ad^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a\sqrt{d}e^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{bd^{\frac{5}{2}}x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)

[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))

```

) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar
(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hyper((-1/2, 1
0/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x
**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*
gamma(16/3))

```

$$3.36 \quad \int (d + ex^3)^{3/2} (a + bx^3 + cx^6) dx$$

Optimal. Leaf size=356

$$\frac{2x(d + ex^3)^{3/2} (391ae^2 - 46bde + 16cd^2)}{4301e^2} + \frac{18dx\sqrt{d + ex^3} (391ae^2 - 46bde + 16cd^2)}{21505e^2} + \frac{18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} d^2 (\sqrt[3]{d} + \dots)}{\dots}$$

[Out] $2/4301*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^{(3/2)}/e^2-2/391*(-23*b*e+8*c*d)*x*(e*x^3+d)^{(5/2)}/e^2+2/23*c*x^4*(e*x^3+d)^{(5/2)}/e+18/21505*d*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^{(1/2)}/e^2+18/21505*3^{(3/4)}*d^2*(391*a*e^2-46*b*d*e+16*c*d^2)*(d^{(1/3)}+e^{(1/3)}*x)*\text{EllipticF}((e^{(1/3)}*x+d^{(1/3)}*(1-3^{(1/2)}))/e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})),I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*((d^{(2/3)}-d^{(1/3)}*e^{(1/3)}*x+e^{(2/3)}*x^2)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^2)^{(1/2)}/e^{(7/3)}/(e*x^3+d)^{(1/2)}/(d^{(1/3)}*(d^{(1/3)}+e^{(1/3)}*x)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^2)^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1411, 388, 195, 218}

$$\frac{2x(d + ex^3)^{3/2} (391ae^2 - 46bde + 16cd^2)}{4301e^2} + \frac{18dx\sqrt{d + ex^3} (391ae^2 - 46bde + 16cd^2)}{21505e^2} + \frac{18 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} d^2 (\sqrt[3]{d} + \dots)}{\dots}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x]

[Out] $(18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*\text{Sqrt}[d + e*x^3])/(21505*e^2) + (2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^{(3/2)})/(4301*e^2) - (2*(8*c*d - 23*b*e)*x*(d + e*x^3)^{(5/2)})/(391*e^2) + (2*c*x^4*(d + e*x^3)^{(5/2)})/(23*e) + (18*3^{(3/4)}*\text{Sqrt}[2 + \text{Sqrt}[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*(d^{(1/3)} + e^{(1/3)}*x)*\text{Sqrt}[(d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}(((1 - \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)], -7 - 4*\text{Sqrt}[3])]/(21505*e^{(7/3)}*\text{Sqrt}[(d^{(1/3)}*(d^{(1/3)} + e^{(1/3)}*x))/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{Sqrt}[d + e*x^3])$

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
(c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (d + ex^3)^{3/2} (a + bx^3 + cx^6) dx &= \frac{2cx^4 (d + ex^3)^{5/2}}{23e} + \frac{2 \int (d + ex^3)^{3/2} \left(\frac{23ae}{2} - \left(4cd - \frac{23be}{2}\right) x^3 \right) dx}{23e} \\
&= -\frac{2(8cd - 23be)x (d + ex^3)^{5/2}}{391e^2} + \frac{2cx^4 (d + ex^3)^{5/2}}{23e} - \frac{1}{391} \left(-391a - \frac{2d(8cd - 23be)}{e^2} \right) \\
&= \frac{2 \left(391a + \frac{2d(8cd - 23be)}{e^2} \right) x (d + ex^3)^{3/2}}{4301} - \frac{2(8cd - 23be)x (d + ex^3)^{5/2}}{391e^2} + \frac{2cx^4}{e^2} \\
&= \frac{18d \left(391a + \frac{2d(8cd - 23be)}{e^2} \right) x \sqrt{d + ex^3}}{21505} + \frac{2 \left(391a + \frac{2d(8cd - 23be)}{e^2} \right) x (d + ex^3)^{3/2}}{4301} \\
&= \frac{18d \left(391a + \frac{2d(8cd - 23be)}{e^2} \right) x \sqrt{d + ex^3}}{21505} + \frac{2 \left(391a + \frac{2d(8cd - 23be)}{e^2} \right) x (d + ex^3)^{3/2}}{4301}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 101, normalized size = 0.28

$$\frac{x\sqrt{d+ex^3} \left(\frac{{}_2F_1\left(-\frac{3}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{ex^3}{d}\right) (23de(17ae-2bd)+16cd^3)}{\sqrt{\frac{ex^3}{d}+1}} - 2(d+ex^3)^2(-23be+8cd-17cex^3) \right)}{391e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x]

[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*c*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -(e*x^3)/d]))/Sqrt[1 + (e*x^3)/d])/(391*e^2)

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cex^9 + (cd + be)x^6 + (bd + ae)x^3 + ad\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] integral((c*e*x^9 + (c*d + b*e)*x^6 + (b*d + a*e)*x^3 + a*d)*sqrt(e*x^3 + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)(ex^3 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)

maple [B] time = 0.04, size = 1010, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x)

[Out] c*(2/23*e*x^10*(e*x^3+d)^(1/2)+52/391*d*x^7*(e*x^3+d)^(1/2)+54/4301*d^2/e*x^4*(e*x^3+d)^(1/2)-432/21505*d^3/e^2*x*(e*x^3+d)^(1/2)-288/21505*I*d^4/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + b*(2/17*e*x^7*(e*x^3+d)^(1/2)+40/187*d*x^4*(e*x^3+d)^(1/2)+54/935*d^2/e*x*(e*x^3+d)^(1/2)+36/935*I*d^3/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + a*(2/11*e*x^4*(e*x^3+d)^(1/2)+28/55*d*x*(e*x^3+d)^(1/2)-18/55*I*d^2*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)(ex^3 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (ex^3 + d)^{3/2} (cx^6 + bx^3 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x)

[Out] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x)

sympy [A] time = 5.92, size = 257, normalized size = 0.72

$$\frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a\sqrt{d}ex^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b\sqrt{d}ex^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)

[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))

3.37 $\int \sqrt{d + ex^3} (a + bx^3 + cx^6) dx$

Optimal. Leaf size=316

$$\frac{2x\sqrt{d + ex^3} (187ae^2 - 34bde + 16cd^2)}{935e^2} + \frac{2^{3/4} \sqrt{2 + \sqrt{3}} d (\sqrt[3]{d} + \sqrt[3]{ex}) \sqrt{\frac{d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex} + e^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{d} + \sqrt[3]{ex})^2}} (187ae^2 - 34bde + 16cd^2)}{935e^{7/3} \sqrt{\frac{\sqrt[3]{d} (\sqrt[3]{d} + \sqrt[3]{ex})}{((1 + \sqrt{3}) \sqrt[3]{d} + \sqrt[3]{ex})^2}} \sqrt{d + ex^3}}$$

[Out] $-2/187 * (-17 * b * e + 8 * c * d) * x * (e * x^3 + d)^{(3/2)} / e^2 + 2/17 * c * x^4 * (e * x^3 + d)^{(3/2)} / e^2 / 935 * (187 * a * e^2 - 34 * b * d * e + 16 * c * d^2) * x * (e * x^3 + d)^{(1/2)} / e^2 + 2/935 * 3^{(3/4)} * d * (187 * a * e^2 - 34 * b * d * e + 16 * c * d^2) * (d^{(1/3)} + e^{(1/3)} * x) * \text{EllipticF}((e^{(1/3)} * x + d^{(1/3)}) * (1 - 3^{(1/2)})) / (e^{(1/3)} * x + d^{(1/3)} * (1 + 3^{(1/2)})), I * 3^{(1/2)} + 2 * I) * (1/2 * 6^{(1/2)} + 1/2 * 2^{(1/2)}) * ((d^{(2/3)} - d^{(1/3)} * e^{(1/3)} * x + e^{(2/3)} * x^2) / (e^{(1/3)} * x + d^{(1/3)} * (1 + 3^{(1/2)})))^2)^{(1/2)} / e^{(7/3)} / (e * x^3 + d)^{(1/2)} / (d^{(1/3)} * (d^{(1/3)} + e^{(1/3)} * x) / (e^{(1/3)} * x + d^{(1/3)} * (1 + 3^{(1/2)})))^2)^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1411, 388, 195, 218}

$$\frac{2x\sqrt{d + ex^3} (187ae^2 - 34bde + 16cd^2)}{935e^2} + \frac{2^{3/4} \sqrt{2 + \sqrt{3}} d (\sqrt[3]{d} + \sqrt[3]{ex}) \sqrt{\frac{d^{2/3} - \sqrt[3]{d} \sqrt[3]{ex} + e^{2/3} x^2}{((1 + \sqrt{3}) \sqrt[3]{d} + \sqrt[3]{ex})^2}} (187ae^2 - 34bde + 16cd^2)}{935e^{7/3} \sqrt{\frac{\sqrt[3]{d} (\sqrt[3]{d} + \sqrt[3]{ex})}{((1 + \sqrt{3}) \sqrt[3]{d} + \sqrt[3]{ex})^2}} \sqrt{d + ex^3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]

[Out] $(2 * (16 * c * d^2 - 34 * b * d * e + 187 * a * e^2) * x * \text{Sqrt}[d + e * x^3]) / (935 * e^2) - (2 * (8 * c * d - 17 * b * e) * x * (d + e * x^3)^{(3/2)}) / (187 * e^2) + (2 * c * x^4 * (d + e * x^3)^{(3/2)}) / (17 * e) + (2 * 3^{(3/4)} * \text{Sqrt}[2 + \text{Sqrt}[3]] * d * (16 * c * d^2 - 34 * b * d * e + 187 * a * e^2) * (d^{(1/3)} + e^{(1/3)} * x) * \text{Sqrt}[(d^{(2/3)} - d^{(1/3)} * e^{(1/3)} * x + e^{(2/3)} * x^2)] / ((1 + \text{Sqrt}[3]) * d^{(1/3)} + e^{(1/3)} * x)^2 * \text{EllipticF}[\text{ArcSin}(((1 - \text{Sqrt}[3]) * d^{(1/3)} + e^{(1/3)} * x) / ((1 + \text{Sqrt}[3]) * d^{(1/3)} + e^{(1/3)} * x)), -7 - 4 * \text{Sqrt}[3]]) / (935 * e^{(7/3)} * \text{Sqrt}[(d^{(1/3)} * (d^{(1/3)} + e^{(1/3)} * x)) / ((1 + \text{Sqrt}[3]) * d^{(1/3)} + e^{(1/3)} * x)^2] * \text{Sqrt}[d + e * x^3])$

Rule 195


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] := Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
(c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d+ex^3} (a+bx^3+cx^6) dx &= \frac{2cx^4(d+ex^3)^{3/2}}{17e} + \frac{2 \int \sqrt{d+ex^3} \left(\frac{17ae}{2} - \left(4cd - \frac{17be}{2}\right)x^3 \right) dx}{17e} \\
&= -\frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e} - \frac{1}{187} \left(-187a - \frac{2d(8cd-17be)}{e^2} \right) \\
&= \frac{2}{935} \left(187a + \frac{2d(8cd-17be)}{e^2} \right) x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4}{187e} \\
&= \frac{2}{935} \left(187a + \frac{2d(8cd-17be)}{e^2} \right) x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4}{187e}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 98, normalized size = 0.31

$$\frac{x\sqrt{d+ex^3} \left(\frac{{}_2F_1\left(-\frac{1}{2}, \frac{1}{3}; \frac{4}{3}; -\frac{ex^3}{d}\right)(17e(11ae-2bd)+16cd^2)}{\sqrt{\frac{ex^3}{d}+1}} - 2(d+ex^3)(-17be+8cd-11cex^3) \right)}{187e^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6), x]

[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*d^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -(e*x^3)/d]))/Sqrt[1 + (e*x^3)/d])/(187*e^2)

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)\sqrt{ex^3 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)

maple [B] time = 0.04, size = 956, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x)

[Out] $c \cdot \left(\frac{2}{17} x^7 (e x^3 + d)^{1/2} + \frac{6}{187} d e x^4 (e x^3 + d)^{1/2} - \frac{48}{935} d^2 e^{-2} x^5 (e x^3 + d)^{1/2} - \frac{32}{935} I d^3 e^{-3} 3^{1/2} (-d e^2)^{1/3} (I (x + 1/2 (-d e^2)^{1/3}) / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2} \right) \cdot \left((x - (-d e^2)^{1/3} / e) / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) \right)^{1/2} \cdot \left(-I (x + 1/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2} \right) / (e x^3 + d)^{1/2} \cdot \text{EllipticF} \left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2}), (I 3^{1/2} (-d e^2)^{1/3} / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) / e \right)^{1/2} \right) + b \cdot \left(\frac{2}{11} x^4 (e x^3 + d)^{1/2} + \frac{6}{55} d e x (e x^3 + d)^{1/2} + \frac{4}{55} I d^2 e^{-2} 3^{1/2} (-d e^2)^{1/3} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2}) \cdot \left((x - (-d e^2)^{1/3} / e) / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) \right)^{1/2} \cdot \left(-I (x + 1/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2} \right) / (e x^3 + d)^{1/2} \cdot \text{EllipticF} \left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2}), (I 3^{1/2} (-d e^2)^{1/3} / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) / e \right)^{1/2} \right) + a \cdot \left(\frac{2}{5} x^5 (e x^3 + d)^{1/2} - \frac{2}{5} I d^3 3^{1/2} (-d e^2)^{1/3} / e (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2}) \cdot \left((x - (-d e^2)^{1/3} / e) / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) \right)^{1/2} \cdot \left(-I (x + 1/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2} \right) / (e x^3 + d)^{1/2} \cdot \text{EllipticF} \left(\frac{1}{3} 3^{1/2} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) * 3^{1/2} / (-d e^2)^{1/3} e^{1/2}), (I 3^{1/2} (-d e^2)^{1/3} / (-3/2 (-d e^2)^{1/3} / e + 1/2 I 3^{1/2} (-d e^2)^{1/3} / e) / e \right)^{1/2} \right) \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^6 + bx^3 + a)\sqrt{ex^3 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{ex^3 + d} (cx^6 + bx^3 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)

[Out] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)

sympy [A] time = 3.45, size = 124, normalized size = 0.39

$$\frac{a\sqrt{d}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{b\sqrt{d}x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{c\sqrt{d}x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{ex^3e^{i\pi}}{d} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a), x)

[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))

$$3.38 \quad \int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}} dx$$

Optimal. Leaf size=278

$$\frac{2\sqrt{2+\sqrt{3}} \left(\sqrt[3]{d} + \sqrt[3]{ex}\right) \sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex}\right)^2}} \left(55ae^2 - 22bde + 16cd^2\right) F\left(\sin^{-1}\left(\frac{\sqrt[3]{ex}+(1-\sqrt{3})\sqrt[3]{d}}{\sqrt[3]{ex}+(1+\sqrt{3})\sqrt[3]{d}}\right) \mid -7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3} \sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{ex}\right)}{\left((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex}\right)^2}} \sqrt{d+ex^3}}$$

[Out] $-2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2/165*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*\text{EllipticF}((e^(1/3)*x+d)^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))), I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^2)^(1/2)*3^(3/4)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^2)^(1/2)$

Rubi [A] time = 0.18, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1411, 388, 218}

$$\frac{2\sqrt{2+\sqrt{3}} \left(\sqrt[3]{d} + \sqrt[3]{ex}\right) \sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2}{\left((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex}\right)^2}} \left(55ae^2 - 22bde + 16cd^2\right) F\left(\sin^{-1}\left(\frac{\sqrt[3]{ex}+(1-\sqrt{3})\sqrt[3]{d}}{\sqrt[3]{ex}+(1+\sqrt{3})\sqrt[3]{d}}\right) \mid -7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3} \sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{ex}\right)}{\left((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex}\right)^2}} \sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3], x]

[Out] $(-2*(8*c*d - 11*b*e)*x*\text{Sqrt}[d + e*x^3])/(55*e^2) + (2*c*x^4*\text{Sqrt}[d + e*x^3])/(11*e) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) + e^(1/3)*x)*\text{Sqrt}[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/((1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x)^2)*\text{EllipticF}[\text{ArcSin}[((1 - \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x)/((1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*\text{Sqrt}[3]])/(55*3^(1/4)*e^(7/3)*\text{Sqrt}[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x)^2]*\text{Sqrt}[d + e*x^3])$

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s

$*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*s + r*x}{(1 + \text{Sqrt}[3])*s + r*x}], -7 - 4*\text{Sqrt}[3]]/(3^{1/4}*r*\text{Sqrt}[a + b*x^3])*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] /; \text{FreeQ}\{a, b\}, x\} \& \& \text{PosQ}[a]$

Rule 388

$\text{Int}[(a + b*x^n)^p*(c + d*x^n), x_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{p+1}/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n*(p+1) + 1, 0]$

Rule 1411

$\text{Int}[(d + e*x^n)^q*(a + b*x^n + c*x^{n2}), x_Symbol] \rightarrow \text{Simp}[(c*x^{n+1}*(d + e*x^n)^{q+1})/(e*(n*(q+2) + 1)), x] + \text{Dist}[1/(e*(n*(q+2) + 1)), \text{Int}[(d + e*x^n)^q*(a*e*(n*(q+2) + 1) - (c*d*(n+1) - b*e*(n*(q+2) + 1))*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, n, q\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{a + bx^3 + cx^6}{\sqrt{d + ex^3}} dx &= \frac{2cx^4\sqrt{d + ex^3}}{11e} + \frac{2 \int \frac{\frac{11ae}{2} - (4cd - \frac{11be}{2})x^3}{\sqrt{d + ex^3}} dx}{11e} \\ &= -\frac{2(8cd - 11be)x\sqrt{d + ex^3}}{55e^2} + \frac{2cx^4\sqrt{d + ex^3}}{11e} - \frac{1}{55} \left(-55a - \frac{2d(8cd - 11be)}{e^2} \right) \int \frac{1}{\sqrt{d + ex^3}} dx \\ &= -\frac{2(8cd - 11be)x\sqrt{d + ex^3}}{55e^2} + \frac{2cx^4\sqrt{d + ex^3}}{11e} + \frac{2\sqrt{2 + \sqrt{3}} (16cd^2 - 22bde + 55ae^2) (\sqrt[3]{d} + 55\sqrt[4]{3}e^{7/12})}{55e^2} \end{aligned}$$

Mathematica [C] time = 0.09, size = 98, normalized size = 0.35

$$\frac{x \left(\sqrt{\frac{ex^3}{d} + 1} {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{ex^3}{d} \right) (11e(5ae - 2bd) + 16cd^2) - 2(d + ex^3)(-11be + 8cd - 5cex^3) \right)}{55e^2\sqrt{d + ex^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]

[Out] (x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d + 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]))/(55*e^2*Sqrt[d + e*x^3])

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)

maple [B] time = 0.03, size = 907, normalized size = 3.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)

[Out] c*(2/11/e*x^4*(e*x^3+d)^(1/2)-16/55*d/e^2*x*(e*x^3+d)^(1/2)-32/165*I*d^2/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))+b*(2/5/e*x*(e*x^3+d)^(1/2)+4/15*I*d/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)

$$\frac{1}{e^{-1/2} I^3 (1/2) (-d e^2)^{1/3} / e} \cdot 3^{1/2} / (-d e^2)^{1/3} e^{1/2} \cdot ((x - (-d e^2)^{1/3} / e) / (-3/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e))^{1/2} \cdot (-I (x + 1/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) \cdot 3^{1/2} / (-d e^2)^{1/3} e)^{1/2} / (e x^3 + d)^{1/2} \cdot \text{EllipticF}(1/3, 3^{1/2} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) \cdot 3^{1/2} / (-d e^2)^{1/3} e)^{1/2}, (I^3 (1/2) (-d e^2)^{1/3} / (-3/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) / e)^{1/2})) - 2/3 I a 3^{1/2} (-d e^2)^{1/3} / e \cdot (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) \cdot 3^{1/2} / (-d e^2)^{1/3} e)^{1/2} \cdot ((x - (-d e^2)^{1/3} / e) / (-3/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e))^{1/2} \cdot (-I (x + 1/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) \cdot 3^{1/2} / (-d e^2)^{1/3} e)^{1/2} / (e x^3 + d)^{1/2} \cdot \text{EllipticF}(1/3, 3^{1/2} (I (x + 1/2 (-d e^2)^{1/3} / e - 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) \cdot 3^{1/2} / (-d e^2)^{1/3} e)^{1/2}, (I^3 (1/2) (-d e^2)^{1/3} / (-3/2 (-d e^2)^{1/3} / e + 1/2 I^3 (1/2) (-d e^2)^{1/3} / e) / e)^{1/2}))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)

[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)

sympy [A] time = 2.90, size = 119, normalized size = 0.43

$$\frac{ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{2} \left| \frac{ex^3 e^{i\pi}}{d} \right. \right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{4}{3} \left| \frac{ex^3 e^{i\pi}}{d} \right. \right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{1}{2}, \frac{7}{3} \left| \frac{ex^3 e^{i\pi}}{d} \right. \right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

$$3.39 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}} dx$$

Optimal. Leaf size=289

$$\frac{2x(ae^2 - bde + cd^2)}{3de^2\sqrt{d+ex^3}} \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{e}x) \sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex+e^{2/3}x^2}}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}} (16cd^2 - 5e(ae + 2bd)) F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+(1-\sqrt{3})\sqrt[3]{d}}{\sqrt[3]{e}x+(1+\sqrt{3})\sqrt[3]{d}}\right)\right)}{15\sqrt[4]{3}de^{7/3} \sqrt{\frac{\sqrt[3]{d}(\sqrt[3]{d}+\sqrt[3]{ex})}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}} \sqrt{d+ex^3}}$$

[Out] $2/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(1/2)+2/5*c*x*(e*x^3+d)^(1/2)/e^2-2/45*(16*c*d^2-5*e*(a*e+2*b*d))*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^2)^(1/2)*3^(3/4)/d/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))))^(1/2)$

Rubi [A] time = 0.19, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1409, 388, 218}

$$\frac{2x(ae^2 - bde + cd^2)}{3de^2\sqrt{d+ex^3}} \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{e}x) \sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex+e^{2/3}x^2}}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}} (16cd^2 - 5e(ae + 2bd)) F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+(1-\sqrt{3})\sqrt[3]{d}}{\sqrt[3]{e}x+(1+\sqrt{3})\sqrt[3]{d}}\right)\right)}{15\sqrt[4]{3}de^{7/3} \sqrt{\frac{\sqrt[3]{d}(\sqrt[3]{d}+\sqrt[3]{ex})}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}} \sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x]

[Out] $(2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*\text{Sqrt}[d + e*x^3]) + (2*c*x*\text{Sqrt}[d + e*x^3])/(5*e^2) - (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/3) + e^(1/3)*x)*\text{Sqrt}[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/((1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x)^2)*\text{EllipticF}[\text{ArcSin}[\frac{(1 - \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x}{(1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x}], -7 - 4*\text{Sqrt}[3]]/(15*3^(1/4)*d*e^(7/3)*\text{Sqrt}[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/(1 + \text{Sqrt}[3])*d^(1/3) + e^(1/3)*x]^2)*\text{Sqrt}[d + e*x^3]$

Rule 218

Int[1/Sqrt[(a_) + (b_)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1409

```
Int[((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + bx^3 + cx^6}{(d + ex^3)^{3/2}} dx &= \frac{2(cd^2 - bde + ae^2)x}{3de^2\sqrt{d + ex^3}} - \frac{2 \int \frac{\frac{1}{2}(2cd^2 - e(2bd + ae)) - \frac{3}{2}cdex^3}{\sqrt{d + ex^3}} dx}{3de^2} \\ &= \frac{2(cd^2 - bde + ae^2)x}{3de^2\sqrt{d + ex^3}} + \frac{2cx\sqrt{d + ex^3}}{5e^2} - \frac{(16cd^2 - 5e(2bd + ae)) \int \frac{1}{\sqrt{d + ex^3}} dx}{15de^2} \\ &= \frac{2(cd^2 - bde + ae^2)x}{3de^2\sqrt{d + ex^3}} + \frac{2cx\sqrt{d + ex^3}}{5e^2} - \frac{2\sqrt{2 + \sqrt{3}} (16cd^2 - 5e(2bd + ae)) (\sqrt[3]{d} + \sqrt[3]{ex})}{15\sqrt[4]{3} de^{7/3} \sqrt{\left(\frac{16cd^2 - 5e(2bd + ae)}{15de^2}\right)^2 + \frac{4cx^2}{3d}}} \end{aligned}$$

Mathematica [C] time = 0.10, size = 102, normalized size = 0.35

$$\frac{x \left(\sqrt{\frac{ex^3}{d} + 1} {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{ex^3}{d} \right) (5e(ae + 2bd) - 16cd^2) + 2(5e(ae - bd) + cd(8d + 3ex^3)) \right)}{15de^2\sqrt{d + ex^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x]

[Out] (x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d + a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(15*d*e^2*Sqrt[d + e*x^3])

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^2x^6 + 2dex^3 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2), x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^2*x^6 + 2*d*e*x^3 + d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2), x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)

maple [B] time = 0.05, size = 934, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2), x)

[Out] c*(2/3/e^2*d*x/((x^3+d/e)*e)^(1/2)+2/5/e^2*x*(e*x^3+d)^(1/2)+32/45*I*d/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2

$$\begin{aligned}
 & *(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e/e)^{(1/2)})))+b*(-2/3/e*x/((\\
 & x^3+d/e)*e)^{(1/2)}-4/9*I/e^2*3^{(1/2)}*(-d*e^2)^{(1/3)}*(I*(x+1/2*(-d*e^2)^{(1/3)} \\
 & /e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)}*((x-(-d* \\
 & e^2)^{(1/3)}/e)/(-3/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e))^{(1/2)} \\
 & *(-I*(x+1/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^ \\
 & 2)^{(1/3)*e)^{(1/2)}/(e*x^3+d)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2*(-d*e^2)^ \\
 & (1/3)}/e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)},(I* \\
 & 3^{(1/2)}*(-d*e^2)^{(1/3)}/(-3/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/ \\
 & e)/e)^{(1/2)})))+a*(2/3/d*x/((x^3+d/e)*e)^{(1/2)}-2/9*I/d*3^{(1/2)}*(-d*e^2)^{(1/3)} \\
 & /e*(I*(x+1/2*(-d*e^2)^{(1/3)}/e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e \\
 & ^2)^{(1/3)*e)^{(1/2)}*((x-(-d*e^2)^{(1/3)}/e)/(-3/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/ \\
 & 2)*(-d*e^2)^{(1/3)}/e))^{(1/2)}*(-I*(x+1/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e \\
 & ^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)}/(e*x^3+d)^{(1/2)}*EllipticF(1/3* \\
 & 3^{(1/2)}*(I*(x+1/2*(-d*e^2)^{(1/3)}/e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/ \\
 & (-d*e^2)^{(1/3)*e)^{(1/2)},(I*3^{(1/2)}*(-d*e^2)^{(1/3)}/(-3/2*(-d*e^2)^{(1/3)}/e+1/ \\
 & 2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)/e)^{(1/2)}))
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x)

[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x)

sympy [A] time = 15.07, size = 119, normalized size = 0.41

$$\frac{ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{3}{2} \middle| \frac{ex^3 e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{3}{2} \middle| \frac{ex^3 e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{7}{3}, \frac{3}{2} \middle| \frac{ex^3 e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

$$3.40 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}} dx$$

Optimal. Leaf size=309

$$\frac{2x(-7ae^2 - 2bde + 11cd^2)}{27d^2e^2\sqrt{d+ex^3}} + \frac{2x(ae^2 - bde + cd^2)}{9de^2(d+ex^3)^{3/2}} + \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{ex})\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}}(e(7ae+2bd))}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}(\sqrt[3]{d}+\sqrt[3]{ex})}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}}}$$

[Out] $2/9*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^{(3/2)}-2/27*(-7*a*e^2-2*b*d*e+11*c*d^2)*x/d^2/e^2/(e*x^3+d)^{(1/2)}+2/81*(16*c*d^2+e*(7*a*e+2*b*d))*(d^{(1/3)}+e^{(1/3)}*x)*\text{EllipticF}((e^{(1/3)}*x+d^{(1/3)}*(1-3^{(1/2)}))/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})), I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*((d^{(2/3)}-d^{(1/3)}*e^{(1/3)}*x+e^{(2/3)}*x^2)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^{(1/2)}*3^{(3/4)}/d^2/e^{(7/3)}/(e*x^3+d)^{(1/2)}/(d^{(1/3)}*(d^{(1/3)}+e^{(1/3)}*x)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1409, 385, 218}

$$\frac{2x(-7ae^2 - 2bde + 11cd^2)}{27d^2e^2\sqrt{d+ex^3}} + \frac{2x(ae^2 - bde + cd^2)}{9de^2(d+ex^3)^{3/2}} + \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{ex})\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{ex}+e^{2/3}x^2}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}}(e(7ae+2bd))}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}(\sqrt[3]{d}+\sqrt[3]{ex})}{((1+\sqrt{3})\sqrt[3]{d}+\sqrt[3]{ex})^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x]

[Out] $(2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^{(3/2)}) - (2*(11*c*d^2 - 2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*\text{Sqrt}[d + e*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(16*c*d^2 + e*(2*b*d + 7*a*e))*(d^{(1/3)} + e^{(1/3)}*x)*\text{Sqrt}[(d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}(((1 - \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x))], -7 - 4*\text{Sqrt}[3])]/(27*3^{(1/4)}*d^2*e^{(7/3)}*\text{Sqrt}[(d^{(1/3)}*(d^{(1/3)} + e^{(1/3)}*x))/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{Sqrt}[d + e*x^3])$

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{5/2}} dx = \frac{2(cd^2 - bde + ae^2)x}{9de^2(d + ex^3)^{3/2}} - \frac{2 \int \frac{\frac{1}{2}(2cd^2 - e(2bd + 7ae)) - \frac{9}{2}cdex^3}{(d + ex^3)^{3/2}} dx}{9de^2}$$

$$= \frac{2(cd^2 - bde + ae^2)x}{9de^2(d + ex^3)^{3/2}} - \frac{2(11cd^2 - 2bde - 7ae^2)x}{27d^2e^2\sqrt{d + ex^3}} - \frac{4\left(-\frac{9}{2}cd^2e + \frac{1}{4}e(2cd^2 - e(2bd + 7ae))\right)}{27d^2e^3}$$

$$= \frac{2(cd^2 - bde + ae^2)x}{9de^2(d + ex^3)^{3/2}} - \frac{2(11cd^2 - 2bde - 7ae^2)x}{27d^2e^2\sqrt{d + ex^3}} + \frac{2\sqrt{2 + \sqrt{3}}(16cd^2 + e(2bd + 7ae))}{27\sqrt[4]{3}}$$

Mathematica [C] time = 0.14, size = 129, normalized size = 0.42

$$\frac{x(d + ex^3) \sqrt{\frac{ex^3}{d} + 1} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{ex^3}{d}\right) (e(7ae + 2bd) + 16cd^2) - 2x(e(bd(d - 2ex^3) - ae(10d + 7ex^3))) + cd^2}{27d^2e^2(d + ex^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x]

[Out] (-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3))) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]/(27*d^2*e^2*(d + e*x^3)^(3/2))

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^3x^9 + 3de^2x^6 + 3d^2ex^3 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2), x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^3*x^9 + 3*d*e^2*x^6 + 3*d^2*e*x^3 + d^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2), x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)

maple [B] time = 0.05, size = 1005, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2), x)

```
[Out] c*(2/9*d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2-22/27/e^2*x/((x^3+d/e)*e)^(1/2)-
32/81*I/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)
*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-
3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*
e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)
)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(
1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)
^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + b*
(-2/9*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+4/27/e/d*x/((x^3+d/e)*e)^(1/2)-4/81
*I/e^2/d*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-
d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)
)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/
(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1
/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + a*(2/
9/d*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+14/27/d^2*x/((x^3+d/e)*e)^(1/2)-14/81
*I/d^2*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-
d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2
*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)
)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/
(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/
2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1
/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x)
```

[Out] $\int ((a + b*x^3 + c*x^6)/(d + e*x^3)^{(5/2}), x)$

sympy [A] time = 86.88, size = 119, normalized size = 0.39

$$\frac{ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{5}{2} \middle| \frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{5}{2} \middle| \frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{7}{3}, \frac{5}{2} \middle| \frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2), x)$

[Out] $a*x*\text{gamma}(1/3)*\text{hyper}((1/3, 5/2), (4/3,), e*x**3*\text{exp_polar}(I*\text{pi})/d)/(3*d**(5/2)*\text{gamma}(4/3)) + b*x**4*\text{gamma}(4/3)*\text{hyper}((4/3, 5/2), (7/3,), e*x**3*\text{exp_polar}(I*\text{pi})/d)/(3*d**(5/2)*\text{gamma}(7/3)) + c*x**7*\text{gamma}(7/3)*\text{hyper}((7/3, 5/2), (10/3,), e*x**3*\text{exp_polar}(I*\text{pi})/d)/(3*d**(5/2)*\text{gamma}(10/3))$

$$3.41 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}} dx$$

Optimal. Leaf size=349

$$-\frac{2x(-13ae^2 - 2bde + 17cd^2)}{135d^2e^2(d+ex^3)^{3/2}} + \frac{2x(ae^2 - bde + cd^2)}{15de^2(d+ex^3)^{5/2}} + \frac{2x(91ae^2 + 14bde + 16cd^2)}{405d^3e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{ex})}{\sqrt{\frac{d^2}{(1+...}}}$$

[Out] 2/15*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(5/2)-2/135*(-13*a*e^2-2*b*d*e+17*c*d^2)*x/d^2/e^2/(e*x^3+d)^(3/2)+2/405*(91*a*e^2+14*b*d*e+16*c*d^2)*x/d^3/e^2/(e*x^3+d)^(1/2)+2/1215*(91*a*e^2+14*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^3/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.32, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1409, 385, 199, 218}

$$\frac{2x(91ae^2 + 14bde + 16cd^2)}{405d^3e^2\sqrt{d+ex^3}} - \frac{2x(-13ae^2 - 2bde + 17cd^2)}{135d^2e^2(d+ex^3)^{3/2}} + \frac{2x(ae^2 - bde + cd^2)}{15de^2(d+ex^3)^{5/2}} + \frac{2\sqrt{2+\sqrt{3}}(\sqrt[3]{d} + \sqrt[3]{ex})}{\sqrt{\frac{d^2}{(1+...}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x]

[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 - 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*sqrt[d + e*x^3]) + (2*sqrt[2 + sqrt[3]]*(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]/((1 + sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*sqrt[d + e*x^3])

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1) / (a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)] / ((1 + Sqrt[3])*s + r*x)^2 * EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x) / ((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]) / (3^(1/4)*r*Sqrt[a + b*x^3] * Sqrt[(s*(s + r*x)) / ((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1)) / (a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1)) / (a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1)) / (d*e^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{7/2}} dx &= \frac{2(cd^2 - bde + ae^2)x}{15de^2(d + ex^3)^{5/2}} - \frac{2 \int \frac{\frac{1}{2}(2cd^2 - c(2bd + 13ae)) - \frac{15}{2}cdex^3}{(d + ex^3)^{5/2}} dx}{15de^2} \\
&= \frac{2(cd^2 - bde + ae^2)x}{15de^2(d + ex^3)^{5/2}} - \frac{2(17cd^2 - 2bde - 13ae^2)x}{135d^2e^2(d + ex^3)^{3/2}} + \frac{(16cd^2 + 14bde + 91ae^2) \int \frac{1}{(d + ex^3)^{3/2}} dx}{135d^2e^2} \\
&= \frac{2(cd^2 - bde + ae^2)x}{15de^2(d + ex^3)^{5/2}} - \frac{2(17cd^2 - 2bde - 13ae^2)x}{135d^2e^2(d + ex^3)^{3/2}} + \frac{2(16cd^2 + 14bde + 91ae^2)x}{405d^3e^2\sqrt{d + ex^3}} + \frac{(16cd^2 + 14bde + 91ae^2) \int \frac{1}{(d + ex^3)^{3/2}} dx}{405d^3e^2} \\
&= \frac{2(cd^2 - bde + ae^2)x}{15de^2(d + ex^3)^{5/2}} - \frac{2(17cd^2 - 2bde - 13ae^2)x}{135d^2e^2(d + ex^3)^{3/2}} + \frac{2(16cd^2 + 14bde + 91ae^2)x}{405d^3e^2\sqrt{d + ex^3}} + \frac{2\sqrt{2} \int \frac{1}{(d + ex^3)^{3/2}} dx}{405d^3e^2}
\end{aligned}$$

Mathematica [C] time = 0.19, size = 166, normalized size = 0.48

$$\frac{2x \left(e \left(ae(157d^2 + 221dex^3 + 91e^2x^6) + bd(-7d^2 + 34dex^3 + 14e^2x^6) \right) + cd^2(-8d^2 - 19dex^3 + 16e^2x^6) \right) + x\sqrt{\frac{ex^3}{d}}}{405d^3e^2(d + ex^3)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x]

[Out] (2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x^3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2 + 7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]/(405*d^3*e^2*(d + e*x^3)^(5/2))

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^4x^{12} + 4de^3x^9 + 6d^2e^2x^6 + 4d^3ex^3 + d^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2), x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^4*x^12 + 4*d*e^3*x^9 + 6*d^2*e^2*x^6 + 4*d^3*e*x^3 + d^4), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)

maple [B] time = 0.05, size = 1095, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x)

[Out] c*(2/15*d*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3-34/135*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/405/e^2/d*x/((x^3+d/e)*e)^(1/2)-32/1215*I/e^3/d^3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I^3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + b*(-2/15*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+4/135/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+28/405/e/d^2*x/((x^3+d/e)*e)^(1/2)-28/1215*I/e^2/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I^3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + a*(2/15/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+26/135/d^2*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+182/405/d^3*x/((x^3+d/e)*e)^(1/2)-182/1215*I/d^3*3^(1/2)*(-d*e^2)^(1/3)/e*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e))^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I^3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)

$$\frac{1}{(ex^3+d)^{1/2}} \text{EllipticF}\left(\frac{1}{3} \sqrt{3} \sqrt{\frac{I(x+1/2)(-de^2)^{1/3}/e-1/2 I \sqrt{3} (-1/2)(-de^2)^{1/3}/e} \sqrt{3} \sqrt{\frac{1}{2}} / (-de^2)^{1/3} e^{1/2}}, \frac{I \sqrt{3} \sqrt{\frac{1}{2}} (-de^2)^{1/3} / (-3/2 (-de^2)^{1/3} / e + 1/2 I \sqrt{3} \sqrt{\frac{1}{2}} (-de^2)^{1/3} / e)}{e^{1/2}}\right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x)

[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)

[Out] Timed out

$$3.42 \quad \int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}} dx$$

Optimal. Leaf size=389

$$-\frac{2x(-19ae^2 - 2bde + 23cd^2)}{315d^2e^2(d+ex^3)^{5/2}} + \frac{2x(ae^2 - bde + cd^2)}{21de^2(d+ex^3)^{7/2}} + \frac{2x(247ae^2 + 26bde + 16cd^2)}{1215d^4e^2\sqrt{d+ex^3}} + \frac{2x(247ae^2 + 26bde + 16cd^2)}{2835d^3e^2(d+ex^3)^{3/2}}$$

[Out] $2/21*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^{(7/2)}-2/315*(-19*a*e^2-2*b*d*e+23*c*d^2)*x/d^2/e^2/(e*x^3+d)^{(5/2)}+2/2835*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^3/e^2/(e*x^3+d)^{(3/2)}+2/1215*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^4/e^2/(e*x^3+d)^{(1/2)}+2/3645*(247*a*e^2+26*b*d*e+16*c*d^2)*(d^{(1/3)}+e^{(1/3)}*x)*\text{EllipticF}((e^{(1/3)}*x+d^{(1/3)}*(1-3^{(1/2)}))/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})), I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*((d^{(2/3)}-d^{(1/3)}*e^{(1/3)}*x+e^{(2/3)}*x^2)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^{(1/2)}*3^{(3/4)}/d^4/e^{(7/3)}/(e*x^3+d)^{(1/2)}/(d^{(1/3)}*(d^{(1/3)}+e^{(1/3)}*x)/(e^{(1/3)}*x+d^{(1/3)}*(1+3^{(1/2)})))^{(1/2)}$

Rubi [A] time = 0.39, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1409, 385, 199, 218}

$$\frac{2x(247ae^2 + 26bde + 16cd^2)}{1215d^4e^2\sqrt{d+ex^3}} + \frac{2x(247ae^2 + 26bde + 16cd^2)}{2835d^3e^2(d+ex^3)^{3/2}} - \frac{2x(-19ae^2 - 2bde + 23cd^2)}{315d^2e^2(d+ex^3)^{5/2}} + \frac{2x(ae^2 - bde + cd^2)}{21de^2(d+ex^3)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x]

[Out] $(2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^{(7/2)}) - (2*(23*c*d^2 - 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^{(5/2)}) + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^{(3/2)}) + (2*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*\text{Sqrt}[d + e*x^3]) + (2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^{(1/3)} + e^{(1/3)}*x)*\text{Sqrt}[(d^{(2/3)} - d^{(1/3)}*e^{(1/3)}*x + e^{(2/3)}*x^2)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{EllipticF}[\text{ArcSin}(((1 - \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x))], -7 - 4*\text{Sqrt}[3]))/(1215*3^{(1/4)}*d^4*e^{(7/3)}*\text{Sqrt}[(d^{(1/3)}*(d^{(1/3)} + e^{(1/3)}*x))/((1 + \text{Sqrt}[3])*d^{(1/3)} + e^{(1/3)}*x)^2]*\text{Sqrt}[d + e*x^3])$

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1) / (a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)] / ((1 + Sqrt[3])*s + r*x)^2 * EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x) / ((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]) / (3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x)) / ((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1)) / (a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1)) / (a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1)) / (d*e^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3 + cx^6}{(d + ex^3)^{9/2}} dx &= \frac{2(cd^2 - bde + ae^2)x}{21de^2(d + ex^3)^{7/2}} - \frac{2 \int \frac{\frac{1}{2}(2cd^2 - e(2bd + 19ae)) - \frac{21}{2}cdex^3}{(d + ex^3)^{7/2}} dx}{21de^2} \\
&= \frac{2(cd^2 - bde + ae^2)x}{21de^2(d + ex^3)^{7/2}} - \frac{2(23cd^2 - 2bde - 19ae^2)x}{315d^2e^2(d + ex^3)^{5/2}} + \frac{(16cd^2 + 26bde + 247ae^2) \int \frac{1}{(d + ex^3)}}{315d^2e^2} \\
&= \frac{2(cd^2 - bde + ae^2)x}{21de^2(d + ex^3)^{7/2}} - \frac{2(23cd^2 - 2bde - 19ae^2)x}{315d^2e^2(d + ex^3)^{5/2}} + \frac{2(16cd^2 + 26bde + 247ae^2)x}{2835d^3e^2(d + ex^3)^{3/2}} + \dots \\
&= \frac{2(cd^2 - bde + ae^2)x}{21de^2(d + ex^3)^{7/2}} - \frac{2(23cd^2 - 2bde - 19ae^2)x}{315d^2e^2(d + ex^3)^{5/2}} + \frac{2(16cd^2 + 26bde + 247ae^2)x}{2835d^3e^2(d + ex^3)^{3/2}} + \dots \\
&= \frac{2(cd^2 - bde + ae^2)x}{21de^2(d + ex^3)^{7/2}} - \frac{2(23cd^2 - 2bde - 19ae^2)x}{315d^2e^2(d + ex^3)^{5/2}} + \frac{2(16cd^2 + 26bde + 247ae^2)x}{2835d^3e^2(d + ex^3)^{3/2}} + \dots
\end{aligned}$$

Mathematica [C] time = 0.24, size = 200, normalized size = 0.51

$$7x\sqrt{\frac{ex^3}{d} + 1} (d + ex^3)^3 {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; -\frac{ex^3}{d}\right) (13e(19ae + 2bd) + 16cd^2) + 2x \left(e \left(ae(3388d^3 + 7182d^2ex^3 + 5928de^2x^6 + 1729e^3x^9) + 7(16cd^2 + 13e(2bd + 19ae)) \right) x (d + ex^3)^3 \text{Sqrt}[1 + (ex^3)/d] \text{Hypergeometric2F1}[1/3, 1/2, 4/3, -(ex^3)/d] \right) / (8505d^4e^2(d + ex^3)^{7/2})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x]

[Out] (2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b*d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3 + 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*(2*b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)]/(8505*d^4*e^2*(d + e*x^3)^(7/2))

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^6 + bx^3 + a)\sqrt{ex^3 + d}}{e^5x^{15} + 5de^4x^{12} + 10d^2e^3x^9 + 10d^3e^2x^6 + 5d^4ex^3 + d^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^5*x^15 + 5*d*e^4*x^12 + 10*d^2*e^3*x^9 + 10*d^3*e^2*x^6 + 5*d^4*e*x^3 + d^5), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)

maple [B] time = 0.06, size = 1182, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x)

[Out] c*(2/21*d*x/e^6*(e*x^3+d)^(1/2)/(x^3+d/e)^4-46/315*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3+32/2835/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/1215/e^2/d^2*x/((x^3+d/e)*e)^(1/2)-32/3645*I/e^3/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + b*(-2/21*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^4+4/315/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+52/2835/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+52/1215/e/d^3*x/((x^3+d/e)*e)^(1/2)-52/3645*I/e^2/d^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)*((x-(-d*e^2)^(1/3)/e)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)^(1/2)*(-I*(x+1/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-d*e^2)^(1/3)/e-1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)*3^(1/2)/(-d*e^2)^(1/3)*e)^(1/2), (I*3^(1/2)*(-d*e^2)^(1/3)/(-3/2*(-d*e^2)^(1/3)/e+1/2*I*3^(1/2)*(-d*e^2)^(1/3)/e)/e)^(1/2))) + a*(2/21/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^4+38/315/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+494/2835/d^3*x/e^2*

$(e*x^3+d)^{(1/2)}/(x^3+d/e)^2+494/1215/d^4*x/((x^3+d/e)*e)^{(1/2)}-494/3645*I/d^4*3^{(1/2)}*(-d*e^2)^{(1/3)}/e*(I*(x+1/2*(-d*e^2)^{(1/3)}/e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)}*((x-(-d*e^2)^{(1/3)}/e)/(-3/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e))^{(1/2)}*(-I*(x+1/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)}/(e*x^3+d)^{(1/2)}*EllipticF(1/3*3^{(1/2)}*(I*(x+1/2*(-d*e^2)^{(1/3)}/e-1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)*3^{(1/2)}/(-d*e^2)^{(1/3)*e)^{(1/2)},(I*3^{(1/2)}*(-d*e^2)^{(1/3)}/(-3/2*(-d*e^2)^{(1/3)}/e+1/2*I*3^{(1/2)}*(-d*e^2)^{(1/3)}/e)/e)^{(1/2))}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{cx^6 + bx^3 + a}{(ex^3 + d)^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x)

[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)

[Out] Timed out

$$3.43 \quad \int \frac{x^4(d+ex^4)}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=433

$$\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac} - b\right)^{3/4}} - \frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac} - b\right)^{3/4}} - \frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac} - b\right)^{3/4}}$$

[Out] $e*x/c - 1/4*\arctan(2^{(1/4)}*c^{(1/4)}*x/(-b - (-4*a*c+b^2)^{(1/2)})^{(1/4)})*(c*d - b*e + (2*a*c*e - b^2*e + b*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(5/4)}/(-b - (-4*a*c+b^2)^{(1/2)})^{(3/4)} - 1/4*\arctanh(2^{(1/4)}*c^{(1/4)}*x/(-b - (-4*a*c+b^2)^{(1/2)})^{(1/4)})*(c*d - b*e + (2*a*c*e - b^2*e + b*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(5/4)}/(-b - (-4*a*c+b^2)^{(1/2)})^{(3/4)} - 1/4*\arctan(2^{(1/4)}*c^{(1/4)}*x/(-b + (-4*a*c+b^2)^{(1/2)})^{(1/4)})*(c*d - b*e + (-2*a*c*e + b^2*e - b*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(5/4)}/(-b + (-4*a*c+b^2)^{(1/2)})^{(3/4)} - 1/4*\arctanh(2^{(1/4)}*c^{(1/4)}*x/(-b + (-4*a*c+b^2)^{(1/2)})^{(1/4)})*(c*d - b*e + (-2*a*c*e + b^2*e - b*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(5/4)}/(-b + (-4*a*c+b^2)^{(1/2)})^{(3/4)}$

Rubi [A] time = 1.13, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1502, 1422, 212, 208, 205}

$$\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac} - b\right)^{3/4}} - \frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(\sqrt{b^2-4ac} - b\right)^{3/4}} - \frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}} - be + cd\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2} c^{5/4} \left(-\sqrt{b^2-4ac} - b\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] $(e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*c^{(5/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*c^{(5/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*c^{(5/4)}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*c^{(5/4)}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)})$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1422

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(2n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rule 1502

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(2n_))^(p_), x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8} dx &= \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8} dx}{c} \\
&= \frac{ex}{c} + \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx}{2c} + \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4} dx}{2c} \\
&= \frac{ex}{c} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2} dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} \\
&= \frac{ex}{c} - \frac{\left(cd-be + \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be - \frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 88, normalized size = 0.20

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4be \log(x-\#1) + \#1^4(-c)d \log(x-\#1) + ae \log(x-\#1)}{2\#1^7c + \#1^3b} \& \right]}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1])*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &]/(4*c)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.01, size = 67, normalized size = 0.15

$$\frac{ex}{c} + \frac{\left((-be + cd) \operatorname{RootOf}\left(-Z^8c + Z^4b + a\right)^4 - ae\right) \ln\left(-\operatorname{RootOf}\left(-Z^8c + Z^4b + a\right) + x\right)}{4c \left(2 \operatorname{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + \operatorname{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)

[Out] 1/c*e*x+1/4/c*sum(((b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-_Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ex}{c} - \frac{\int \frac{(cd-be)x^4-ae}{cx^8+bx^4+a} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c

mupad [B] time = 9.63, size = 50213, normalized size = 115.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)

[Out] atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -

$$\begin{aligned}
& 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)/c*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)} - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)} + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*
\end{aligned}$$

$$\begin{aligned}
& a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3 \\
& *e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a \\
& *b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3* \\
& b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c \\
& ^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i + (((4*x* \\
& (4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b \\
& ^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e \\
& - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c \\
& ^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3 \\
& *e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3 \\
& *c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a* \\
& b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c \\
& ^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4* \\
& b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c \\
& ^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4 \\
& d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c \\
& - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c \\
& ^7 - 256*a^3*b^2*c^8)))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a \\
& ^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4* \\
& (-4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d \\
& ^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5* \\
& d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^ \\
& 2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8 \\
& *c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b \\
& ^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^ \\
& 2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d \\
& ^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& /((512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2* \\
& c^8)))^{(3/4)} + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b \\
& ^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2* \\
& c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^ \\
& 2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 2 \\
& 0*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2* \\
& c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^ \\
& 5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6* \\
& d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3* \\
& b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13 \\
& *a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2* \\
& (-4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^
\end{aligned}$$

$$\begin{aligned}
& 4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^ \\
& 2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c \\
& ^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4* \\
& a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^ \\
& 4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^ \\
& 6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d \\
& ^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16 \\
& *a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10* \\
& a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5 \\
& *e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)* \\
& (-b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 \\
& + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2 \\
& *e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c \\
& ^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b \\
& ^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(\\
& 1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e \\
& ^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 \\
& + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b* \\
& c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c \\
& ^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*ii)/((((4*x*(4096*a^4*b*c^ \\
& 7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 2 \\
& 56*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^ \\
& 4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e \\
& ^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^ \\
& 5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c \\
& ^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2 \\
& *c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4* \\
& b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48* \\
& a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(- \\
& (4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200 \\
& *a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^ \\
& 3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b \\
& ^2*c^8)))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - \\
& 12288*a^4*b^2*c^7*d))/c)*(-b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b* \\
& c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6* \\
& c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240 \\
& *a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2 \\
& *c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 -
\end{aligned}$$

$$\begin{aligned}
& 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 \\
& - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4 \\
& *c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3/4)} - \\
& (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2 \\
& *b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5 \\
& *c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4 \\
& *c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d \\
& e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32 \\
& *a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4 \\
& *e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3 \\
& *c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4 \\
& *c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + \\
& a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - \\
& 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + \\
& 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - \\
& 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a \\
& *c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}) \\
& /((512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3 \\
& *b^2*c^8)))^{(1/4)} + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4* \\
& a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4 \\
& *c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d \\
& ^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5* \\
& e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5 \\
& *c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b \\
& ^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6 \\
& *d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3 \\
& *b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 1 \\
& 3*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b \\
& ^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b \\
& ^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2* \\
& c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4 \\
& *a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b \\
& ^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} - (((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5* \\
& b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^ \\
& 2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192* \\
& a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b \\
& c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*
\end{aligned}$$

$$\begin{aligned}
& c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^3e^4 - 4b^8c^3d^3e^3 + 240 \\
& a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 3ab^2c^3e^4(-4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 - \\
& 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - \\
& 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 - 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^4 \\
& c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(1/4)}(\\
& 16384a^5c^8d - 256a^2b^6c^5d + 3072a^3b^4c^6d - 12288a^4b^2c^7d)/c*(-(b^9e^4 + b^5c^4d^4 + b^4e^4(-4ac - b^2)^5)^{(1/2)} + c^4d^4 \\
& (-4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2 \\
& b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^3e^4 - 4b^8c^3d^3e^3 + 240a^2b^3c^4d^2e^2 \\
& + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 3ab^2c^3e^4(-4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 - 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} \\
& - 4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5 \\
& d^2e^2 + 320a^3b^2c^4d^3e^3 - 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^4c^9 + b^8c^5 - \\
& 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(3/4)} + (16(a^3b^6e^5 - 4a^6c^3e^5 + 4a^3b^5c^5d^5 - 7a^4b^4c^5e^5 - a^2b^7d^5e^4 + 12a^4 \\
& c^5d^4e - a^2b^3c^4d^5 + 13a^5b^2c^2e^5 + 8a^5c^4d^2e^3 - 6a^2b^5c^2d^3e^2 + 32a^3b^3c^3d^3e^2 - 22a^3b^4c^2d^2e^3 + 22 \\
& a^4b^2c^3d^2e^3 + 4a^3b^5c^3d^3e^4 - 20a^5b^3c^3d^3e^4 + 4a^2b^4c^3d^4e + 4a^2b^6c^3d^2e^3 - 19a^3b^2c^4d^4e - 32a^4b^3c^4d^3e^2 \\
& + 5a^4b^3c^2d^4e)/c*(-(b^9e^4 + b^5c^4d^4 + b^4e^4(-4ac - b^2)^5)^{(1/2)} + c^4d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2 \\
& b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} \\
& + 6b^7c^2d^2e^2 - 13ab^7c^3e^4 - 4b^8c^3d^3e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 3ab^2c^3e^4(-4ac - b^2)^5)^{(1/2)} \\
& + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 - 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - \\
& 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 - 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^4c^9 + b^8c^5 - \\
& 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(1/4)} + (4x*(a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^5e^6 - 2 \\
& a^3b^5d^5e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3 \\
& c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^3d^5e^5 + 2a^5b^3c^2d^5e^5 - 4a^2b^3c^3d^5e^3 + 2a^3b^4c^3d^2e^4 + 12a^4b^3c^3d^3e^3))/c*(-(b^9e^4 + b^5c^4d^4 + b^4e^4
\end{aligned}$$

$$\begin{aligned}
& e^4 * (-4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c \\
& ^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c \\
& ^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4* \\
& (-4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4 \\
& *b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48 \\
& *a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(\\
& -(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20 \\
& 0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2* \\
& (-4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1 \\
& /2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3* \\
& b^2*c^8)))^{(1/4))*(-b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1 \\
& /2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 \\
& + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3 \\
& *e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^ \\
& 3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4* \\
& (-4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^ \\
& 3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288 \\
& *a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2))/(512*(256*a^4*c^9 + \\
& b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*2i + \text{ata} \\
& n(((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - \\
& 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384* \\
& a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e)))/c - (16*(-(b^9* \\
& e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128* \\
& a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - \\
& 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2* \\
& e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2* \\
& d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^ \\
& ^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 12 \\
& 8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a \\
& ^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e \\
& ^3*(-(4*a*c - b^2)^5)^{(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9 \\
& 6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5 \\
& *d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-b^9*e^4 + b^5*c^4*d^4 \\
& - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8* \\
& a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 1 \\
& 28*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e \\
& ^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c* \\
& e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3
\end{aligned}$$

$$\begin{aligned}
& *e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c* \\
& d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3 \\
& *e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 \\
& + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2 \\
&)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 2 \\
& 56*a^3*b^2*c^8)))^{(3/4)} - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^ \\
& 5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + \\
& 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3 \\
& *c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5* \\
& c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - \\
& 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c*(-(b \\
& ^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1 \\
& 28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 \\
& - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d \\
& ^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c \\
& ^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - \\
& 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32 \\
& 0*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2* \\
& d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 \\
& + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5 \\
& *d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 \\
& + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d \\
& ^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^ \\
& 2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2* \\
& b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^ \\
& 3*e^3))/c*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^ \\
& 4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^ \\
& 4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61* \\
& a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^ \\
& 2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e* \\
& (- (4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^ \\
& 5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c \\
& ^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 \\
& - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*i + (((((4*x*(40 \\
& 96*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3* \\
& c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - \\
& 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4* \\
& d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
\end{aligned}$$

$$\begin{aligned}
& - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 3ab^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} + 4b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 + 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8ab^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)} / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(1/4)} * (16384a^5c^8d - 256a^2b^6c^5d + 3072a^3b^4c^6d - 12288a^4b^2c^7d) / c * (-b^9e^4 + b^5c^4d^4 - b^4e^4(-4ac - b^2)^5)^{(1/2)} - c^4d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^2e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 3ab^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} + 4b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 + 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8ab^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)} / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(3/4)} + (16(a^3b^6e^5 - 4a^6c^3e^5 + 4a^3b^5c^5d^5 - 7a^4b^4c^5e^5 - a^2b^7d^5e^4 + 12a^4c^5d^4e - a^2b^3c^4d^5 + 13a^5b^2c^2e^5 + 8a^5c^4d^2e^3 - 6a^2b^5c^2d^3e^2 + 32a^3b^3c^3d^3e^2 - 22a^3b^4c^2d^2e^3 + 22a^4b^2c^3d^2e^3 + 4a^3b^5c^2d^2e^4 - 20a^5b^3c^3d^2e^4 + 4a^2b^4c^3d^4e + 4a^2b^6c^2d^2e^3 - 19a^3b^2c^4d^4e - 32a^4b^3c^4d^3e^2 + 5a^4b^3c^2d^2e^4) / c * (-b^9e^4 + b^5c^4d^4 - b^4e^4(-4ac - b^2)^5)^{(1/2)} - c^4d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^2e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 3ab^2c^2e^4(-4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} + 4b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 + 6ac^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8ab^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)} / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8)))^{(1/4)} + (4x*(a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^2e^6 - 2a^3b^5d^5e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^2d^5e + 2a^5b^3c^2d^5e - 4a^2b^3c^3d^5e - 4a^2b^5c^2d^3e^3 + 2a^3b^4c^2d^2e^4 + 12a^4b^3c^3d^3e^3) / c * (-
\end{aligned}$$

$$\begin{aligned}
& b^9 e^4 + b^5 c^4 d^4 - b^4 e^4 (-4ac - b^2)^5)^{1/2} - c^4 d^4 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^5 d^4 + 16a^2 b^3 c^6 d^4 + 80a^4 b^3 c^4 e^4 + \\
& 128a^3 c^6 d^3 e - 128a^4 c^5 d^3 e^3 - 4b^6 c^3 d^3 e + 61a^2 b^5 c^2 e^4 - 120a^3 b^3 c^3 e^4 - a^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + 6b^7 c^2 d^2 e^2 - 13a^2 b^7 c^2 e^4 - 4b^8 c^2 d^2 e^3 + 240a^2 b^3 c^4 d^2 e^2 - 6b^2 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 3a^2 b^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + \\
& 40a^2 b^4 c^4 d^3 e + 48a^2 b^6 c^2 d^2 e^3 + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} - 66a^2 b^5 c^3 d^2 e^2 - 128a^2 b^2 c^5 d^3 e - 200a^2 b^4 c^3 d^2 e^3 - 288a^3 b^3 c^5 d^2 e^2 + 320a^3 b^2 c^4 d^2 e^3 + 6a^2 c^3 d^2 e^2 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^2 d^2 e^3 (-4ac - b^2)^5)^{1/2} / (512(256a^4 c^9 + b^8 c^5 - 16a^2 b^6 c^6 + 96a^2 b^4 c^7 - 256a^3 b^2 c^8))^{1/4} * i) / (((((4x*(4096a^4 b^3 c^7 d^2 + 4096a^5 b^3 c^6 e^2 + 256a^2 b^5 c^5 d^2 - 2048a^3 b^3 c^6 d^2 + 256a^3 b^5 c^4 e^2 - 2048a^4 b^3 c^5 e^2 - 16384a^5 c^7 d^2 e - 1024a^3 b^4 c^5 d^2 e + 8192a^4 b^2 c^6 d^2 e)) / c - (16*(-(b^9 e^4 + b^5 c^4 d^4 - b^4 e^4 (-4ac - b^2)^5)^{1/2} - c^4 d^4 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^5 d^4 + 16a^2 b^3 c^6 d^4 + 80a^4 b^3 c^4 e^4 + 128a^3 c^6 d^3 e - 128a^4 c^5 d^3 e^3 - 4b^6 c^3 d^3 e + 61a^2 b^5 c^2 e^4 - 120a^3 b^3 c^3 e^4 - a^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + 6b^7 c^2 d^2 e^2 - 13a^2 b^7 c^2 e^4 - 4b^8 c^2 d^2 e^3 + 240a^2 b^3 c^4 d^2 e^2 - 6b^2 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 3a^2 b^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + 40a^2 b^4 c^4 d^3 e + 48a^2 b^6 c^2 d^2 e^3 + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} - 66a^2 b^5 c^3 d^2 e^2 - 128a^2 b^2 c^5 d^3 e - 200a^2 b^4 c^3 d^2 e^3 - 288a^3 b^3 c^5 d^2 e^2 + 320a^3 b^2 c^4 d^2 e^3 + 6a^2 c^3 d^2 e^2 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^2 d^2 e^3 (-4ac - b^2)^5)^{1/2} / (512(256a^4 c^9 + b^8 c^5 - 16a^2 b^6 c^6 + 96a^2 b^4 c^7 - 256a^3 b^2 c^8))^{1/4} * (16384a^5 c^8 d - 256a^2 b^6 c^5 d + 3072a^3 b^4 c^6 d - 12288a^4 b^2 c^7 d) / c) * (- (b^9 e^4 + b^5 c^4 d^4 - b^4 e^4 (-4ac - b^2)^5)^{1/2} - c^4 d^4 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^5 d^4 + 16a^2 b^3 c^6 d^4 + 80a^4 b^3 c^4 e^4 + 128a^3 c^6 d^3 e - 128a^4 c^5 d^3 e^3 - 4b^6 c^3 d^3 e + 61a^2 b^5 c^2 e^4 - 120a^3 b^3 c^3 e^4 - a^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + 6b^7 c^2 d^2 e^2 - 13a^2 b^7 c^2 e^4 - 4b^8 c^2 d^2 e^3 + 240a^2 b^3 c^4 d^2 e^2 - 6b^2 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 3a^2 b^2 c^2 e^4 (-4ac - b^2)^5)^{1/2} + 40a^2 b^4 c^4 d^3 e + 48a^2 b^6 c^2 d^2 e^3 + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} + 4b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} - 66a^2 b^5 c^3 d^2 e^2 - 128a^2 b^2 c^5 d^3 e - 200a^2 b^4 c^3 d^2 e^3 - 288a^3 b^3 c^5 d^2 e^2 + 320a^3 b^2 c^4 d^2 e^3 + 6a^2 c^3 d^2 e^2 (-4ac - b^2)^5)^{1/2} - 8a^2 b^3 c^2 d^2 e^3 (-4ac - b^2)^5)^{1/2} / (512(256a^4 c^9 + b^8 c^5 - 16a^2 b^6 c^6 + 96a^2 b^4 c^7 - 256a^3 b^2 c^8))^{3/4} - (16*(a^3 b^6 e^5 - 4a^6 c^3 e^5 + 4a^3 b^3 c^5 d^5 - 7a^4 b^4 c^2 e^5 - a^2 b^7 d^2 e^4 + 12a^4 c^5 d^4 e - a^2 b^3 c^4 d^5 + 13a^5 b^2 c^2 e^5 + 8a^5 c^4 d^2 e^3 - 6a^2 b^5 c^2 d^3 e^2 + 32a^3 b^3 c^3 d^3 e^2 - 22a^3 b^4 c^2 d^2 e^3 + 22a^4 b^2 c^3 d^2 e^3 + 4a^3 b^5 c^2 d^2 e^4 - 20a^5 b^3 c^3 d^2 e^4 + 4a^2 b^4 c^3 d^4 e + 4a^2 b^6 c^2 d^2 e^3 - 19a^3 b^2 c^4 d^4 e - 32a^4 b^3 c^4 d^3 e^2 + 5a^4 b^3 c^2 d^2 e^4)) / c) * (- (b^9 e^4 + b^5 c^4 d^4 - b^4 e^4
\end{aligned}$$

$$\begin{aligned}
&^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)} + (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)} - (((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4
\end{aligned}$$

$$\begin{aligned}
& *(- (4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4 * (- (4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4 * (- (4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4b^3c^3d^3e^3 * (- (4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 + 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8ab^3c^2d^3e^3 * (- (4ac - b^2)^5)^{1/2} / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{3/4} + (16 * (a^3b^6e^5 - 4a^6c^3e^5 + 4a^3b^3c^5d^5 - 7a^4b^4c^3e^5 - a^2b^7d^4e^4 + 12a^4c^5d^4e - a^2b^3c^4d^5 + 13a^5b^2c^2e^5 + 8a^5c^4d^2e^3 - 6a^2b^5c^2d^3e^2 + 32a^3b^3c^3d^3e^2 - 22a^3b^4c^2d^2e^3 + 22a^4b^2c^3d^2e^3 + 4a^3b^5c^3d^3e^4 - 20a^5b^3c^3d^3e^4 + 4a^2b^4c^3d^4e + 4a^2b^6c^3d^2e^3 - 19a^3b^2c^4d^4e - 32a^4b^3c^4d^3e^2 + 5a^4b^3c^2d^4e^4)) / c * (- (b^9e^4 + b^5c^4d^4 - b^4e^4 * (- (4ac - b^2)^5)^{1/2} - c^4d^4 * (- (4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4 * (- (4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4 * (- (4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4b^3c^3d^3e^3 * (- (4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 + 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8ab^3c^2d^3e^3 * (- (4ac - b^2)^5)^{1/2} / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} + (4 * x * (a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^3e^6 - 2a^3b^5d^5e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^3d^5e + 2a^5b^3c^2d^5e - 4a^2b^3c^3d^5e - 4a^2b^5c^3d^3e^3 + 2a^3b^4c^3d^2e^4 + 12a^4b^3c^3d^3e^3)) / c * (- (b^9e^4 + b^5c^4d^4 - b^4e^4 * (- (4ac - b^2)^5)^{1/2} - c^4d^4 * (- (4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4 * (- (4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4 * (- (4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4b^3c^3d^3e^3 * (- (4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 + 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8ab^3c^2d^3e^3 * (- (4ac - b^2)^5)^{1/2} / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4}
\end{aligned}$$

$$\begin{aligned}
& *c^8)))^{(1/4)}) * (- (b^9 * e^4 + b^5 * c^4 * d^4 - b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& - c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + \\
& 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e \\
& + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 - a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 \\
& - 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 + 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& + 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e \\
& - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 + 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& - 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)) / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8)))^{(1/4)} * 2i + 2 * \operatorname{atan} \\
& (((((4 * x * (4096 * a^4 * b * c^7 * d^2 + 4096 * a^5 * b * c^6 * e^2 + 256 * a^2 * b^5 * c^5 * d^2 - 2048 * a^3 * b^3 * c^6 * d^2 + 256 * a^3 * b^5 * c^4 * e^2 - 2048 * a^4 * b^3 * c^5 * e^2 - 16384 * a^5 * c^7 * d * e - 1024 * a^3 * b^4 * c^5 * d * e + 8192 * a^4 * b^2 * c^6 * d * e)) / c - ((- (b^9 * e^4 + b^5 * c^4 * d^4 + b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 + a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 + 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 - 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 - 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)) / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8)))^{(1/4)} * (16384 * a^5 * c^8 * d - 256 * a^2 * b^6 * c^5 * d + 3072 * a^3 * b^4 * c^6 * d - 12288 * a^4 * b^2 * c^7 * d) * 16i) / c) * (- (b^9 * e^4 + b^5 * c^4 * d^4 + b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 + a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 + 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 - 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 - 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)) / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8)))^{(3/4)} * 1i + (16 * (a^3 * b^6 * e^5 - 4 * a^6 * c^3 * e^5 + 4 * a^3 * b * c^5 * d^5 - 7 * a^4 * b^4 * c * e^5 - a^2 * b^7 * d * e^4 + 12 * a^4 * c^5 * d^4 * e - a^2 * b^3 * c^4 * d^5 + 13 * a^5 * b^2 * c^2 * e^5 + 8 * a^5 * c^4 * d^2 * e^3 - 6 * a^2 * b^5 * c^2 * d^3 * e^2 + 32 * a^3 * b^3 * c^3 * d^3 * e^2 - 22 * a^3 * b^4 * c^2 * d^2 * e^3 + 22 * a^4 * b^2 * c^3 * d^2 * e^3 + 4 * a^3 * b^5 * c * d * e^4 - 20 * a^5 * b * c^3 * d * e^4 + 4 * a^2 * b^4 * c^3 * d^4 * e + 4 * a^2 * b^6 * c * d^2 * e^3 - 19 * a^3 * b^2 * c^4 * d^4 * e - 32 * a^4 * b * c^4 * d^3 * e^2 + 5 * a^4 * b^3 * c^2 * d * e^4)) / c) * (
\end{aligned}$$

$$\begin{aligned}
& -(b^9e^4 + b^5c^4d^4 + b^4e^4(-4ac - b^2)^5)^{1/2} + c^4d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 \\
& + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 \\
& - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} - 3ab^2c^2e^4(-4ac - b^2)^5)^{1/2} \\
& + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 \\
& - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 - 6ac^3d^2e^2(-4ac - b^2)^5)^{1/2} + 8ab^2c^2d^2e^3 \\
& (-4ac - b^2)^5)^{1/2} / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} * i - (4x(a^4b^4e^6 - 2a^3c^5d^6 \\
& + 2a^6c^2e^6 - 4a^5b^2c^2e^6 - 2a^3b^5d^2e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 \\
& - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^2d^2e^5 + 2a^5b^2c^2d^2e^5 - 4a^2b^3c^3d^5e \\
& - 4a^2b^5c^2d^3e^3 + 2a^3b^4c^2d^2e^4 + 12a^4b^2c^3d^3e^3) / c * (-b^9e^4 + b^5c^4d^4 + b^4e^4(-4ac - b^2)^5)^{1/2} \\
& + c^4d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e \\
& + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 \\
& + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} - 3ab^2c^2e^4(-4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} \\
& - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 \\
& - 6ac^3d^2e^2(-4ac - b^2)^5)^{1/2} + 8ab^2c^2d^2e^3(-4ac - b^2)^5)^{1/2} / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} \\
& + (((4x(4096a^4b^3c^7d^2 + 4096a^5b^3c^6e^2 + 256a^2b^5c^5d^2 - 2048a^3b^3c^6d^2 + 256a^3b^5c^4e^2 - 2048a^4b^3c^5e^2 - 16384a^5c^7d^2e \\
& - 1024a^3b^4c^5d^2e + 8192a^4b^2c^6d^2e)) / c + ((-b^9e^4 + b^5c^4d^4 + b^4e^4(-4ac - b^2)^5)^{1/2} + c^4d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^5d^4 \\
& + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4(-4ac - b^2)^5)^{1/2} \\
& + 6b^7c^2d^2e^2 - 13ab^7c^2e^4 - 4b^8c^2d^2e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} - 3ab^2c^2e^4(-4ac - b^2)^5)^{1/2} \\
& + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e \\
& - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 - 6ac^3d^2e^2(-4ac - b^2)^5)^{1/2} + 8ab^2c^2d^2e^3(-4ac - b^2)^5)^{1/2} \\
& / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} * (16384a^5c^8d - 256a^2b^6c^5d + 3072a^3b^4c^6d - 12288a^4b^2c^7d) * 16i) / c * (-b^9e^4 + b^5c^4d^4 + b^4e^4
\end{aligned}$$

$$\begin{aligned}
& *(- (4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(- (4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d^3*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(- (4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c^4 - 4*b^8*c*d^3*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c^4*(- (4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d^3*e^3 - 4*b*c^3*d^3*e*(- (4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d^3*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e^3 - 6*a*c^3*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(3/4)}*i - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c^5 - a^2*b^7*d^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d^4 - 20*a^5*b*c^3*d^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d^4)))/c*(- (b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(- (4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(- (4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d^3*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(- (4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c^4 - 4*b^8*c*d^3*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c^4*(- (4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d^3*e^3 - 4*b*c^3*d^3*e*(- (4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d^3*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e^3 - 6*a*c^3*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8))^{(1/4)}*i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c^4*e^6 - 2*a^3*b^5*d^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d^5 + 2*a^5*b*c^2*d^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c*(- (b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(- (4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(- (4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d^3*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(- (4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c^4 - 4*b^8*c*d^3*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} - 3*a*b^2*c^4*(- (4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d^3*e^3 - 4*b*c^3*d^3*e*(- (4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d^3*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d^3*e^3 - 6*a*c^3*d^2*e^2*(- (4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d^3*e^3*(- (4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16
\end{aligned}$$

$$\begin{aligned}
& *a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)})/((((4*x*(4096*a^4* \\
& b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b^3*c^6*d^2 \\
& + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e - 1024*a^ \\
& 3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - ((-(b^9*e^4 + b^5*c^4*d^4 + b^4* \\
& e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c \\
& ^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4* \\
& c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^ \\
& 2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4 \\
& *b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48 \\
& *a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(\\
& -(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 20 \\
& 0*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c \\
& ^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1 \\
& /2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3* \\
& b^2*c^8)))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5*d + 3072*a^3*b^4*c^6*d \\
& - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a \\
& ^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4 \\
& *b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 \\
& + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 3* \\
& a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d* \\
& e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3*c*d*e^3*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3 \\
& *d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(25 \\
& 6*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(3 \\
& /4)}*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e \\
& ^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^ \\
& 5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22 \\
& *a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5* \\
& b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^ \\
& 4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4* \\
& d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e \\
& - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^ \\
& 3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7 \\
& *c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4* \\
& d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*b^3 \\
& *c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5* \\
& d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e \\
& ^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a*b*c^2*d*e^3*(-(4*a*c - \\
& b^2)^5)^{(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
\end{aligned}$$

$$\begin{aligned}
& - 256a^3b^2c^8))^{(1/4)} * i - (4x*(a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^e^6 - 2a^3b^5d^e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^2c^4d^5e + 6a^4b^3c^d^e^5 + 2a^5b^2c^d^e^5 - 4a^2b^3c^3d^5e - 4a^2b^5c^d^3e^3 + 2a^3b^4c^d^2e^4 + 12a^4b^2c^3d^3e^3))/c * (- (b^9e^4 + b^5c^4d^4 + b^4e^4 * (- (4ac - b^2)^5)^{(1/2)} + c^4d^4 * (- (4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^2c^6d^4 + 80a^4b^2c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4 * (- (4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^e^4 - 4b^8c^d^e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} - 3ab^2c^e^4 * (- (4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^e^3 - 4b^2c^3d^3e * (- (4ac - b^2)^5)^{(1/2)} - 4b^3c^d^e^3 * (- (4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^e^3 - 288a^3b^2c^5d^2e^2 + 320a^3b^2c^4d^e^3 - 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^e^3 * (- (4ac - b^2)^5)^{(1/2)})) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(1/4)} * i - (((4x*(4096a^4b^2c^7d^2 + 4096a^5b^2c^6e^2 + 256a^2b^5c^5d^2 - 2048a^3b^3c^6d^2 + 256a^3b^5c^4e^2 - 2048a^4b^3c^5e^2 - 16384a^5c^7d^e - 1024a^3b^4c^5d^e + 8192a^4b^2c^6d^e))/c + ((- (b^9e^4 + b^5c^4d^4 + b^4e^4 * (- (4ac - b^2)^5)^{(1/2)} + c^4d^4 * (- (4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^2c^6d^4 + 80a^4b^2c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4 * (- (4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^e^4 - 4b^8c^d^e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} - 3ab^2c^e^4 * (- (4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^e^3 - 4b^2c^3d^3e * (- (4ac - b^2)^5)^{(1/2)} - 4b^3c^d^e^3 * (- (4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^e^3 - 288a^3b^2c^5d^2e^2 + 320a^3b^2c^4d^e^3 - 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^e^3 * (- (4ac - b^2)^5)^{(1/2)})) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(1/4)} * (16384a^5c^8d - 256a^2b^6c^5d + 3072a^3b^4c^6d - 12288a^4b^2c^7d) * 16i) / c * (- (b^9e^4 + b^5c^4d^4 + b^4e^4 * (- (4ac - b^2)^5)^{(1/2)} + c^4d^4 * (- (4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^2c^6d^4 + 80a^4b^2c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 + a^2c^2e^4 * (- (4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^e^4 - 4b^8c^d^e^3 + 240a^2b^3c^4d^2e^2 + 6b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} - 3ab^2c^e^4 * (- (4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^e^3 - 4b^2c^3d^3e * (- (4ac - b^2)^5)^{(1/2)} - 4b^3c^d^e^3 * (- (4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^e^3 - 288a^3b^2c^5d^2e^2 + 320a^3b^2c^4d^e^3 - 6ac^3d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 8ab^2c^2d^e^3 * (- (4ac - b^2)^5)^{(1/2)})) / (512 * (256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(3/4)} * i
\end{aligned}$$

$$\begin{aligned}
& - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2)))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i))*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))
\end{aligned}$$

$$\begin{aligned}
 & 2) / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8))^{(1/4)} + 2 * \text{atan}(\left(\frac{((4 * x * (4096 * a^4 * b * c^7 * d^2 + 4096 * a^5 * b * c^6 * e^2 + 256 * a^2 * b^5 * c^5 * d^2 - 2048 * a^3 * b^3 * c^6 * d^2 + 256 * a^3 * b^5 * c^4 * e^2 - 2048 * a^4 * b^3 * c^5 * e^2 - 16384 * a^5 * c^7 * d * e - 1024 * a^3 * b^4 * c^5 * d * e + 8192 * a^4 * b^2 * c^6 * d * e)) / c - ((- (b^9 * e^4 + b^5 * c^4 * d^4 - b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 - a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 - 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 + 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 + 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2))} / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8))^{(1/4)} * (16384 * a^5 * c^8 * d - 256 * a^2 * b^6 * c^5 * d + 3072 * a^3 * b^4 * c^6 * d - 12288 * a^4 * b^2 * c^7 * d) * 16i) / c * (- (b^9 * e^4 + b^5 * c^4 * d^4 - b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 - a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 - 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 + 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 + 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2))} / (512 * (256 * a^4 * c^9 + b^8 * c^5 - 16 * a * b^6 * c^6 + 96 * a^2 * b^4 * c^7 - 256 * a^3 * b^2 * c^8))^{(3/4)} * 1i + (16 * (a^3 * b^6 * e^5 - 4 * a^6 * c^3 * e^5 + 4 * a^3 * b * c^5 * d^5 - 7 * a^4 * b^4 * c * e^5 - a^2 * b^7 * d * e^4 + 12 * a^4 * c^5 * d^4 * e - a^2 * b^3 * c^4 * d^5 + 13 * a^5 * b^2 * c^2 * e^5 + 8 * a^5 * c^4 * d^2 * e^3 - 6 * a^2 * b^5 * c^2 * d^3 * e^2 + 32 * a^3 * b^3 * c^3 * d^3 * e^2 - 22 * a^3 * b^4 * c^2 * d^2 * e^3 + 22 * a^4 * b^2 * c^3 * d^2 * e^3 + 4 * a^3 * b^5 * c * d * e^4 - 20 * a^5 * b * c^3 * d * e^4 + 4 * a^2 * b^4 * c^3 * d^4 * e + 4 * a^2 * b^6 * c * d^2 * e^3 - 19 * a^3 * b^2 * c^4 * d^4 * e - 32 * a^4 * b * c^4 * d^3 * e^2 + 5 * a^4 * b^3 * c^2 * d * e^4)) / c * (- (b^9 * e^4 + b^5 * c^4 * d^4 - b^4 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - c^4 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^5 * d^4 + 16 * a^2 * b * c^6 * d^4 + 80 * a^4 * b * c^4 * e^4 + 128 * a^3 * c^6 * d^3 * e - 128 * a^4 * c^5 * d * e^3 - 4 * b^6 * c^3 * d^3 * e + 61 * a^2 * b^5 * c^2 * e^4 - 120 * a^3 * b^3 * c^3 * e^4 - a^2 * c^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 6 * b^7 * c^2 * d^2 * e^2 - 13 * a * b^7 * c * e^4 - 4 * b^8 * c * d * e^3 + 240 * a^2 * b^3 * c^4 * d^2 * e^2 - 6 * b^2 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 3 * a * b^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a * b^4 * c^4 * d^3 * e + 48 * a * b^6 * c^2 * d * e^3 + 4 * b * c^3 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * b^3 * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 66 * a * b^5 * c^3 * d^2 * e^2 - 128 * a^2 * b^2 * c^5 * d^3 * e - 200 * a^2 * b^4 * c^3 * d * e^3 - 288 * a^3 * b * c^5 * d^2 * e^2 + 320 * a^3 * b^2 * c^4 * d * e^3 + 6 * a * c^3 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b * c^2 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2))} / (512 * (256 * a^4 * c^9 +
 \end{aligned}$$

$$\begin{aligned}
& 9 + b^8c^5 - 16a^2b^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(1/4)} * i - \\
& (4*x*(a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^5e^6 - 2a^3b^5d^5e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^3d^5e^5 + 2a^5b^3c^2d^5e^5 - 4a^2b^3c^3d^5e^5 - 4a^2b^5c^3d^3e^3 + 2a^3b^4c^3d^2e^4 + 12a^4b^3c^3d^3e^3))/c)*(-(b^9e^4 + b^5c^4d^4 - b^4e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - c^4d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3c^5d^4 + 16*a^2b^6c^3d^3e + 61*a^2b^5c^2e^4 - 120*a^3b^3c^3e^4 - a^2c^2e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7c^2d^2e^2 - 13*a*b^7c^4e^4 - 4*b^8c^3d^2e^3 + 240*a^2b^3c^4d^2e^2 - 6*b^2c^2d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2c^4e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4c^4d^3e + 48*a*b^6c^2d^2e^3 + 4*b^3c^3d^3e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3c^3d^3e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5c^3d^2e^2 - 128*a^2b^2c^5d^3e - 200*a^2b^4c^3d^2e^3 - 288*a^3b^5c^5d^2e^2 + 320*a^3b^2c^4d^2e^3 + 6*a*c^3d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^2c^2d^2e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4c^9 + b^8c^5 - 16*a^2b^6c^6 + 96*a^2b^4c^7 - 256*a^3b^2c^8))^{(1/4)} + (((4*x*(4096*a^4b^3c^7d^2 + 4096*a^5b^3c^6e^2 + 256*a^2b^5c^5d^2 - 2048*a^3b^3c^6d^2 + 256*a^3b^5c^4e^2 - 2048*a^4b^3c^5e^2 - 16384*a^5c^7d^2e - 1024*a^3b^4c^5d^2e + 8192*a^4b^2c^6d^2e))/c + ((- (b^9e^4 + b^5c^4d^4 - b^4e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3c^5d^4 + 16*a^2b^6c^3d^3e + 61*a^2b^5c^2e^4 - 120*a^3b^3c^3e^4 - a^2c^2e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7c^2d^2e^2 - 13*a*b^7c^4e^4 - 4*b^8c^3d^2e^3 + 240*a^2b^3c^4d^2e^2 - 6*b^2c^2d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2c^4e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4c^4d^3e + 48*a*b^6c^2d^2e^3 + 4*b^3c^3d^3e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3c^3d^3e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5c^3d^2e^2 - 128*a^2b^2c^5d^3e - 200*a^2b^4c^3d^2e^3 - 288*a^3b^5c^5d^2e^2 + 320*a^3b^2c^4d^2e^3 + 6*a*c^3d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^2c^2d^2e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4c^9 + b^8c^5 - 16*a^2b^6c^6 + 96*a^2b^4c^7 - 256*a^3b^2c^8))^{(1/4)}*(16384*a^5c^8d - 256*a^2b^6c^5d + 3072*a^3b^4c^6d - 12288*a^4b^2c^7d)*16i)/c)*(-(b^9e^4 + b^5c^4d^4 - b^4e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3c^5d^4 + 16*a^2b^6c^3d^3e + 80*a^4b^3c^4e^4 + 128*a^3c^6d^3e - 128*a^4c^5d^2e^3 - 4*b^6c^3d^3e + 61*a^2b^5c^2e^4 - 120*a^3b^3c^3e^4 - a^2c^2e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7c^2d^2e^2 - 13*a*b^7c^4e^4 - 4*b^8c^3d^2e^3 + 240*a^2b^3c^4d^2e^2 - 6*b^2c^2d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2c^4e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4c^4d^3e + 48*a*b^6c^2d^2e^3 + 4*b^3c^3d^3e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3c^3d^3e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5c^3d^2e^2 - 128*a^2b^2c^5d^3e - 200*a^2b^4c^3d^2e^3 - 288*a^3b^5c^5d^2e^2 + 320*a^3b^2c^4d^2e^3 + 6*a*c^3d^2e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^2c^2d^2e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4c^9 + b^8c^5 - 16*a^2b^6c^6 + 96*a
\end{aligned}$$

$$\begin{aligned}
& \left(2b^4c^7 - 256a^3b^2c^8 \right)^{3/4} i - \left(16(a^3b^6e^5 - 4a^6c^3e^5 + 4a^3b^5c^5d^5 - 7a^4b^4c^5e^5 - a^2b^7d^5e^4 + 12a^4c^5d^4e - a^2b^3c^4d^5 + 13a^5b^2c^2e^5 + 8a^5c^4d^2e^3 - 6a^2b^5c^2d^3e^2 + 32a^3b^3c^3d^3e^2 - 22a^3b^4c^2d^2e^3 + 22a^4b^2c^3d^2e^3 + 4a^3b^5c^4d^2e^4 - 20a^5b^3c^3d^2e^4 + 4a^2b^4c^3d^4e + 4a^2b^6c^4d^2e^3 - 19a^3b^2c^4d^4e - 32a^4b^3c^4d^3e^2 + 5a^4b^3c^2d^2e^4) / c \right) \cdot \left(-(b^9e^4 + b^5c^4d^4 - b^4e^4(-4ac - b^2)^5)^{1/2} - c^4d^4(-4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^6c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^4d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4(-4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} + 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 + 6ac^3d^2e^2(-4ac - b^2)^5)^{1/2} - 8ab^3c^2d^3e^3(-4ac - b^2)^5)^{1/2} \right) / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} i - (4x^*(a^4b^4e^6 - 2a^3c^5d^6 + 2a^6c^2e^6 - 4a^5b^2c^5e^6 - 2a^3b^5d^5e^5 + a^2b^2c^4d^6 + a^2b^6d^2e^4 - 2a^4c^4d^4e^2 + 2a^5c^3d^2e^4 + 6a^2b^4c^2d^4e^2 - 16a^3b^2c^3d^4e^2 + 8a^3b^3c^2d^3e^3 - 17a^4b^2c^2d^2e^4 + 10a^3b^3c^4d^5e + 6a^4b^3c^4d^5e + 2a^5b^3c^2d^5e - 4a^2b^3c^3d^5e - 4a^2b^5c^4d^3e^3 + 2a^3b^4c^4d^2e^4 + 12a^4b^3c^3d^3e^3) / c) \cdot \left(-(b^9e^4 + b^5c^4d^4 - b^4e^4(-4ac - b^2)^5)^{1/2} - c^4d^4(-4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^6c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^4d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4(-4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} + 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^3e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^3e^3 + 6ac^3d^2e^2(-4ac - b^2)^5)^{1/2} - 8ab^3c^2d^3e^3(-4ac - b^2)^5)^{1/2} \right) / (512(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{1/4} / (((((4x^*(4096a^4b^3c^7d^2 + 4096a^5b^3c^6e^2 + 256a^2b^5c^5d^2 - 2048a^3b^3c^6d^2 + 256a^3b^5c^4e^2 - 2048a^4b^3c^5e^2 - 16384a^5c^7d^2e - 1024a^3b^4c^5d^2e + 8192a^4b^2c^6d^2e)) / c - ((-(b^9e^4 + b^5c^4d^4 - b^4e^4(-4ac - b^2)^5)^{1/2} - c^4d^4(-4ac - b^2)^5)^{1/2} - 8ab^3c^5d^4 + 16a^2b^6c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^3e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{1/2} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^4d^3e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 3ab^2c^4e^4(-4ac - b^2)^5)^{1/2} + 40ab^4c^4d^3e + 48ab^6c^2d^3e^3 + 4b^3c^3d^3e(-4ac - b^2)^5)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& ^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 12 \\
& 8*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a \\
& ^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e \\
& ^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 9 \\
& 6*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*(16384*a^5*c^8*d - 256*a^2*b^6*c^5 \\
& *d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d)*16i)/c)*(-(b^9*e^4 + b^5*c^4 \\
& *d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e \\
& - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c \\
& ^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7 \\
& *c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4* \\
& a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4 \\
& *d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^ \\
& 3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5 \\
& *d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d* \\
& e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - \\
& b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 \\
& - 256*a^3*b^2*c^8)))^{(3/4)}*1i + (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b \\
& *c^5*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4 \\
& *d^5 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32* \\
& a^3*b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a \\
& ^3*b^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2 \\
& *e^3 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/ \\
& c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(- \\
& (4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4* \\
& e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5* \\
& c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^ \\
& 7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - \\
& 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^ \\
& 2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e \\
& ^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a \\
& *b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b \\
& ^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i - (4*x*(a^4*b^4*e^6 - \\
& 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^ \\
& 2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2 \\
& *b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4* \\
& b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^ \\
& 5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^ \\
& 4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^ \\
& (1/2) - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d \\
& ^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d \\
& ^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^ \\
& 2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^3c^4d^2e^2 - 6b^2c^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 3ab^2c^4e^4*(-(4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)} \\
& - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 + 6ac^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} \\
& - 8ab^2c^2d^2e^3*(-(4ac - b^2)^5)^{(1/2)}/(512*(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(1/4)}*1i - (\\
& ((4x*(4096a^4b^3c^7d^2 + 4096a^5b^3c^6e^2 + 256a^2b^5c^5d^2 - 2048a^3b^3c^6d^2 + 256a^3b^5c^4e^2 - 2048a^4b^3c^5e^2 - 16384a^5c^7d^2e \\
& - 1024a^3b^4c^5d^2e + 8192a^4b^2c^6d^2e))/c + ((-b^9e^4 + b^5c^4d^4 - b^4e^4*(-(4ac - b^2)^5)^{(1/2)} - c^4d^4*(-(4ac - b^2)^5)^{(1/2)} \\
& - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^2e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 \\
& - a^2c^2e^4*(-(4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} \\
& + 3ab^2c^4e^4*(-(4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)} \\
& + 4b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 \\
& + 6ac^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8ab^2c^2d^2e^3*(-(4ac - b^2)^5)^{(1/2)}/(512*(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(1/4)} \\
& *(16384a^5c^8d - 256a^2b^6c^5d + 3072a^3b^4c^6d - 12288a^4b^2c^7d)*16i)/c*(-(b^9e^4 + b^5c^4d^4 - b^4e^4*(-(4ac - b^2)^5)^{(1/2)} - c^4d^4*(-(4ac - b^2)^5)^{(1/2)} \\
& - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^2e^3 - 4b^6c^3d^3e + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 \\
& - a^2c^2e^4*(-(4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} \\
& + 3ab^2c^4e^4*(-(4ac - b^2)^5)^{(1/2)} + 40ab^4c^4d^3e + 48ab^6c^2d^2e^3 + 4b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 4b^3c^3d^3e \\
& ^3*(-(4ac - b^2)^5)^{(1/2)} - 66ab^5c^3d^2e^2 - 128a^2b^2c^5d^3e - 200a^2b^4c^3d^2e^3 - 288a^3b^3c^5d^2e^2 + 320a^3b^2c^4d^2e^3 + 6 \\
& ac^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8ab^2c^2d^2e^3*(-(4ac - b^2)^5)^{(1/2)}/(512*(256a^4c^9 + b^8c^5 - 16ab^6c^6 + 96a^2b^4c^7 - 256a^3b^2c^8))^{(3/4)} \\
& *1i - (16*(a^3b^6e^5 - 4a^6c^3e^5 + 4a^3b^5c^5d^5 - 7a^4b^4c^5e^5 - a^2b^7d^5e^4 + 12a^4c^5d^4e - a^2b^3c^4d^5 + 13a^5b^2c^2e^5 \\
& + 8a^5c^4d^2e^3 - 6a^2b^5c^2d^3e^2 + 32a^3b^3c^3d^3e^2 - 22a^3b^4c^2d^2e^3 + 22a^4b^2c^3d^2e^3 + 4a^3b^5c^3d^2e^4 - 20a^5b^3c^3d^2e^4 \\
& + 4a^2b^4c^3d^4e + 4a^2b^6c^3d^2e^3 - 19a^3b^2c^4d^4e - 32a^4b^3c^4d^3e^2 + 5a^4b^3c^2d^2e^4))/c*(-(b^9e^4 + b^5c^4d^4 - b^4e^4*(-(4ac - b^2)^5)^{(1/2)} \\
& - c^4d^4*(-(4ac - b^2)^5)^{(1/2)} - 8ab^3c^5d^4 + 16a^2b^3c^6d^4 + 80a^4b^3c^4e^4 + 128a^3c^6d^3e - 128a^4c^5d^2e^3 - 4b^6c^3d^3e \\
& + 61a^2b^5c^2e^4 - 120a^3b^3c^3e^4 - a^2c^2e^4*(-(4ac - b^2)^5)^{(1/2)} + 6b^7c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^2e^3 + 240a^2b^3c^4d^2e^2 \\
& - 6b^2c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2 - 13ab^7c^4e^4 - 4b^8c^3d^2e^3 + 240a^2b^3c^4d^2e^2 - 6b^2c^2d^2e^2
\end{aligned}$$

$$\begin{aligned} & ^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\ &) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - \\ & 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 \\ & + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i - (4*x*(a^4*b^4*e^6 - 2*a^3*c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)}*1i))*(-(b^9*e^4 + b^5*c^4*d^4 - b^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - c^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 - a^2*c^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 - 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 + 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 + 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^{(1/4)} + (e*x)/c \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.44 \quad \int \frac{x^3(d+ex^4)}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=72

$$\frac{e \log(a + bx^4 + cx^8)}{8c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

[Out] $1/8*e*\ln(c*x^8+b*x^4+a)/c-1/4*(-b*e+2*c*d)*\operatorname{arctanh}((2*c*x^4+b)/(-4*a*c+b^2)^{(1/2)})/c/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e \log(a + bx^4 + cx^8)}{8c} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8), x]$

[Out] $-((2*c*d - b*e)*\operatorname{ArcTanh}[(b + 2*c*x^4)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(4*c*\operatorname{Sqrt}[b^2 - 4*a*c]) + (e*\operatorname{Log}[a + b*x^4 + c*x^8])/(8*c)$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt} Q[a, 0] \parallel \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\operatorname{Int}[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \operatorname{Simp}[(d*\operatorname{Log}[\operatorname{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x] \&\& \operatorname{Eq} Q[2*c*d - b*e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3 (d + ex^4)}{a + bx^4 + cx^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{d + ex}{a + bx + cx^2} dx, x, x^4 \right) \\ &= \frac{e \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^4 \right)}{8c} + \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^4 \right)}{8c} \\ &= \frac{e \log(a + bx^4 + cx^8)}{8c} - \frac{(2cd - be) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^4 \right)}{4c} \\ &= -\frac{(2cd - be) \tanh^{-1} \left(\frac{b+2cx^4}{\sqrt{b^2 - 4ac}} \right)}{4c\sqrt{b^2 - 4ac}} + \frac{e \log(a + bx^4 + cx^8)}{8c} \end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.99

$$\frac{e \log(a + bx^4 + cx^8) - \frac{2(be - 2cd) \tan^{-1} \left(\frac{b+2cx^4}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}}}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8), x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

fricas [A] time = 1.02, size = 216, normalized size = 3.00

$$\left[\frac{(b^2 - 4ac)e \log(cx^8 + bx^4 + a) - \sqrt{b^2 - 4ac} (2cd - be) \log\left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac + (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right)}{8(b^2c - 4ac^2)}, \frac{(b^2 - 4ac)e \log(cx^8 + bx^4 + a) - \sqrt{b^2 - 4ac} (2cd - be) \arctan\left(\frac{2cx^4 + b}{\sqrt{b^2 - 4ac}}\right)}{8(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]

giac [A] time = 20.74, size = 70, normalized size = 0.97

$$\frac{e \log(cx^8 + bx^4 + a)}{8c} + \frac{(2cd - be) \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right)}{4\sqrt{-b^2 + 4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)

maple [A] time = 0.00, size = 99, normalized size = 1.38

$$-\frac{be \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{4\sqrt{4ac - b^2}c} + \frac{d \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}} + \frac{e \ln(cx^8 + bx^4 + a)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)

[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*d-1/4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*e*b/c

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.21, size = 3704, normalized size = 51.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] - (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) -
(atan((8*x^4*((a*c - b^2)*(((4*b^2*e - 16*a*c*e)*((b*e - 2*c*d)*(448*b
^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 1
6*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*
e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16
*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^
3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2
*c))))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*
(4*a*c - b^2)^(1/2))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) - (((
(b*e - 2*c*d)*((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4
*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)) +
(32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a
*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))*(b*e - 2*c*d))/
(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4
*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*
(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c))))/(2*(64*a*c^2 - 16*b^2*c)) + 1
44*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 +
20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*(4*a*c - b^2)^(1/2)
) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*(64*a*c^2 - 16*b^2*c)*(
4*a*c - b^2)^(3/2)))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^3*e^4 + (b^3*(b*e -
2*c*d)^4)/(8*(4*a*c - b^2)^2) - c^3*d^3*e - (((b*e - 2*c*d)*((b*e - 2*c*
d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*
a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*
a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*
a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a
*c^2 - 16*b^2*c)*(4*a*c - b^2))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^
2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4*d^2 + ((4*b^
2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16
```

$$\begin{aligned}
& a*c*e)) / (64*a*c^2 - 16*b^2*c)) / (2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 \\
& - 240*b^2*c^3*d*e) / (2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 + 20*b^3*c*e^3 \\
& - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e) / (2*(64*a*c^2 - 16*b^2*c)) + 3*b*c^2*d \\
& ^2*e^2 - (((4*b^2*e - 16*a*c*e) * ((b*e - 2*c*d) * (448*b^3*c^3*e - 384*b^2*c \\
& ^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a \\
& *c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e) * (b*e - 2*c*d)) / ((64*a*c \\
& ^2 - 16*b^2*c) * (4*a*c - b^2)^(1/2))) / (2*(64*a*c^2 - 16*b^2*c)) + ((b*e - 2 \\
& *c*d) * (96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e) * (448*b^3*c^3*e - 384*b^2*c^4*d \\
& + (256*b^3*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (2*(64*a*c^2 - \\
& 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e) / (8*c*(4*a*c - b^2)^(1/2)) \\
&) * (b*e - 2*c*d) / (8*c*(4*a*c - b^2)^(1/2)) - 3*b^2*c*d*e^3) / (8*a^3*c^2*(4* \\
& a*c - b^2)^(1/2)) * (4*a*c - b^2)^2 / (b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2* \\
& e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*e^3) + ((a*c - b^2) * (4*a*c - b^2)^2 * ((4*b \\
& ^2*e - 16*a*c*e) * (((b*e - 2*c*d) * (768*a*b^2*c^3*e - 512*a*b*c^4*d + (512* \\
& a*b^2*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^(\\
& 1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e) * (b*e - 2*c*d)) / ((64*a*c^2 - 16* \\
& b^2*c) * (4*a*c - b^2)^(1/2))) * (4*b^2*e - 16*a*c*e) / (2*(64*a*c^2 - 16*b^2*c) \\
&) + ((b*e - 2*c*d) * (64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e) * (768*a*b^2*c^3*e - \\
& 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c) \\
&)) / (2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e) / (8*c*(\\
& 4*a*c - b^2)^(1/2))) / (2*(64*a*c^2 - 16*b^2*c)) - ((b*e - 2*c*d) * (((b*e - \\
& 2*c*d) * (768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c \\
& *e)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b \\
& ^2*e - 16*a*c*e) * (b*e - 2*c*d)) / ((64*a*c^2 - 16*b^2*c) * (4*a*c - b^2)^(1/2)) \\
&) * (b*e - 2*c*d) / (8*c*(4*a*c - b^2)^(1/2)) + (8*a*b^2*c^2*(4*b^2*e - 16*a*c \\
& *e) * (b*e - 2*c*d)^2) / ((64*a*c^2 - 16*b^2*c) * (4*a*c - b^2))) / (8*c*(4*a*c - \\
& b^2)^(1/2)) + ((b*e - 2*c*d) * (((4*b^2*e - 16*a*c*e) * (64*a*c^4*d^2 + ((4*b^2 \\
& *e - 16*a*c*e) * (768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - \\
& 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c \\
& ^2*e^2 - 256*a*b*c^3*d*e) / (2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c*e^3 + 16* \\
& a*c^3*d^2*e - 40*a*b*c^2*d*e^2) / (8*c*(4*a*c - b^2)^(1/2)) - (a*b^2*c*(4*b^ \\
& 2*e - 16*a*c*e) * (b*e - 2*c*d)^3) / ((64*a*c^2 - 16*b^2*c) * (4*a*c - b^2)^(3/2) \\
&)) / (a^3*c^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e - \\
& 8*b^3*c*d*e^3) + ((4*a*c - b^2)^(3/2) * (b^3 - 3*a*b*c) * (a*b^2*e^4 - ((4*b^2 \\
& *e - 16*a*c*e) * (((b*e - 2*c*d) * (768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a* \\
& b^2*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (8*c*(4*a*c - b^2)^(1 \\
& /2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e) * (b*e - 2*c*d)) / ((64*a*c^2 - 16*b^ \\
& 2*c) * (4*a*c - b^2)^(1/2))) * (b*e - 2*c*d)) / (8*c*(4*a*c - b^2)^(1/2)) + (8*a* \\
& b^2*c^2*(4*b^2*e - 16*a*c*e) * (b*e - 2*c*d)^2) / ((64*a*c^2 - 16*b^2*c) * (4*a*c \\
& - b^2))) / (2*(64*a*c^2 - 16*b^2*c)) + ((4*b^2*e - 16*a*c*e) * ((4*b^2*e - 1 \\
& 6*a*c*e) * (64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e) * (768*a*b^2*c^3*e - 512*a*b*c \\
& ^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e)) / (64*a*c^2 - 16*b^2*c))) / (2*(64* \\
& a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e) / (2*(64*a*c^2 - 1 \\
& 6*b^2*c)) + 24*a*b^2*c*e^3 + 16*a*c^3*d^2*e - 40*a*b*c^2*d*e^2) / (2*(64*a*c \\
& ^2 - 16*b^2*c)) + a*c^2*d^2*e^2 - ((((((b*e - 2*c*d) * (768*a*b^2*c^3*e - 512
\end{aligned}$$

$$\begin{aligned} & *a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(\\ & 8*c*(4*a*c - b^2)^{(1/2)}) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d) \\ &))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^{(1/2)}))* (4*b^2*e - 16*a*c*e))/(2*(64 \\ & *a*c^2 - 16*b^2*c)) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)* \\ & (768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64 \\ & *a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a* \\ & b*c^3*d*e))/(8*c*(4*a*c - b^2)^{(1/2)}))* (b*e - 2*c*d))/(8*c*(4*a*c - b^2)^{(1 \\ & /2)}) + (a*b^2*(b*e - 2*c*d)^4)/(4*(4*a*c - b^2)^2) - 2*a*b*c*d*e^3)/(a^3*c \\ & ^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d* \\ & e^3)))*(b*e - 2*c*d))/(4*c*(4*a*c - b^2)^{(1/2)}) \end{aligned}$$

sympy [B] time = 18.30, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{8c(4ac - b^2)} \right) \log \left(x^4 + \frac{-16ac \left(\frac{e}{8c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{8c(4ac - b^2)} \right) + 2ae + 4b^2 \left(\frac{e}{8c} - \frac{\sqrt{-4ac + b^2} (be - 2cd)}{8c(4ac - b^2)} \right) - bd}{be - 2cd} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a), x)

[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))

$$3.45 \quad \int \frac{x^2(d+ex^4)}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=375

$$\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) - \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} c^{3/4} \sqrt[4]{-\sqrt{b^2-4ac}-b} + 2^{3/4} c^{3/4} \sqrt[4]{\sqrt{b^2-4ac}-b} - 2^{3/4} c^{3/4} \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

[Out] $\frac{1}{4} \arctan(2^{1/4} c^{1/4} x / (-b - (-4ac + b^2)^{1/2}))^{1/4} (e + (b^2 - 2cd) / (-4ac + b^2)^{1/2})^{1/4} / c^{3/4} / (-b - (-4ac + b^2)^{1/2})^{1/4} - \frac{1}{4} \operatorname{arctanh}(2^{1/4} c^{1/4} x / (-b - (-4ac + b^2)^{1/2}))^{1/4} (e + (b^2 - 2cd) / (-4ac + b^2)^{1/2})^{1/4} / c^{3/4} / (-b - (-4ac + b^2)^{1/2})^{1/4} + \frac{1}{4} \arctan(2^{1/4} c^{1/4} x / (-b + (-4ac + b^2)^{1/2}))^{1/4} (e - (b^2 - 2cd) / (-4ac + b^2)^{1/2})^{1/4} / c^{3/4} / (-b + (-4ac + b^2)^{1/2})^{1/4} - \frac{1}{4} \operatorname{arctanh}(2^{1/4} c^{1/4} x / (-b + (-4ac + b^2)^{1/2}))^{1/4} (e - (b^2 - 2cd) / (-4ac + b^2)^{1/2})^{1/4} / c^{3/4} / (-b + (-4ac + b^2)^{1/2})^{1/4}$

Rubi [A] time = 0.46, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1510, 298, 205, 208}

$$\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) + \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) - \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{cx}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4} c^{3/4} \sqrt[4]{-\sqrt{b^2-4ac}-b} + 2^{3/4} c^{3/4} \sqrt[4]{\sqrt{b^2-4ac}-b} - 2^{3/4} c^{3/4} \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] $((e - (2cd - b^2) / \sqrt{b^2 - 4ac}) \operatorname{ArcTan}[(2^{1/4} c^{1/4} x) / (-b - \sqrt{b^2 - 4ac})^{1/4}]) / (2^{3/4} c^{3/4} (-b - \sqrt{b^2 - 4ac})^{1/4}) + ((e + (2cd - b^2) / \sqrt{b^2 - 4ac}) \operatorname{ArcTan}[(2^{1/4} c^{1/4} x) / (-b + \sqrt{b^2 - 4ac})^{1/4}]) / (2^{3/4} c^{3/4} (-b + \sqrt{b^2 - 4ac})^{1/4}) - ((e - (2cd - b^2) / \sqrt{b^2 - 4ac}) \operatorname{ArcTanh}[(2^{1/4} c^{1/4} x) / (-b - \sqrt{b^2 - 4ac})^{1/4}]) / (2^{3/4} c^{3/4} (-b - \sqrt{b^2 - 4ac})^{1/4}) - ((e + (2cd - b^2) / \sqrt{b^2 - 4ac}) \operatorname{ArcTanh}[(2^{1/4} c^{1/4} x) / (-b + \sqrt{b^2 - 4ac})^{1/4}]) / (2^{3/4} c^{3/4} (-b + \sqrt{b^2 - 4ac})^{1/4})$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1510

Int[(((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2(d+ex^4)}{a+bx^4+cx^8} dx &= \frac{1}{2} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx + \frac{1}{2} \left(e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac}} dx \\ &= \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}\sqrt{c}} + \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{2}\sqrt{c}} - \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}} \right)}{2 \cdot 2^{3/4} c^{3/4} \sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}} \right)}{2 \cdot 2^{3/4} c^{3/4} \sqrt[4]{-b+\sqrt{b^2-4ac}}} - \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right)}{2 \cdot 2^{3/4} c^{3/4}} \end{aligned}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.16

$$\frac{1}{4} \text{RootSum} \left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 e \log(x - \#1) + d \log(x - \#1)}{2\#1^5 c + \#1 b} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/4

fricas [B] time = 48.37, size = 13521, normalized size = 36.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out]
$$-\sqrt{\sqrt{1/2}}\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9))}/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))*\arctan(1/2*((2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*e)*x*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)} + ((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x - \sqrt{1/2}*((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 + (2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*e)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*\sqrt{(2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - \sqrt{1/2}*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 + ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2$$

$$\begin{aligned}
& 2e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9))/((a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)))*\sqrt{\sqrt{1/2}*\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^4*d^6 - b*c^3*d^5*e - 5*a*c^3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2*c^2)*d^2*e^4 + (a*b^3 + 3*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)} + \sqrt{\sqrt{1/2}*\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*\arctan(1/2*(\sqrt{1/2}*((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 - (2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*e)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*\sqrt{\sqrt{1/2}*\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*\sqrt{((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - \sqrt{1/2}*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 +
\end{aligned}$$

$$\begin{aligned}
& 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c \\
& + 4*a^4*b*c^2)*e^6 - ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e \\
& ^2)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d \\
& ^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9))*\sqrt{ \\
& -(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d \\
& *e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{ \\
& (c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 \\
& - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 \\
& - 8*a^2*b^2*c^4 + 16*a^3*c^5))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5* \\
& e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6 \\
& *(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 \\
& + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8) + ((2*(a*b^4*c^4 \\
& - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5) \\
&)*e)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3) \\
&)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)) - (\\
& (b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c \\
& - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*\sqrt{\sqrt{ \\
& 1/2)*\sqrt{-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2* \\
& a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16* \\
& a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3) \\
& ^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/ \\
& (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^4*d^6 - b*c^3*d^5*e - 5*a*c^3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2*c^2)*d^2*e^4 + (a*b^3 + 3 \\
& *a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) - 1/4*\sqrt{\sqrt{1/2)*\sqrt{-(b*c^3 \\
& *d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + \\
& (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*\sqrt{(c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a \\
& b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(\\
& a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2* \\
& b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2* \\
& b^2*c^4 + 16*a^3*c^5))*\log(1/2*\sqrt{1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 \\
& - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + \\
& 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + \\
& 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - \\
& 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)
\end{aligned}$$

$$\begin{aligned}
& *e^7 - ((a^7b^7c^5 - 12a^2b^5c^6 + 48a^3b^3c^7 - 64a^4b^2c^8)d^3 - \\
& 6(a^2b^6c^5 - 12a^3b^4c^6 + 48a^4b^2c^7 - 64a^5c^8)d^2e + 3(a \\
& ^2b^7c^4 - 12a^3b^5c^5 + 48a^4b^3c^6 - 64a^5b^2c^7)d^2e^2 - (a^2b \\
& ^8c^3 - 14a^3b^6c^4 + 72a^4b^4c^5 - 160a^5b^2c^6 + 128a^6c^7)*e \\
& ^3)*\sqrt{(c^6d^8 - 12a^5c^6d^6e^2 + 8a^4b^4d^5e^3 - 48a^2b^2c^3d^3 \\
& *e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d \\
& ^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^4e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2 \\
&)e^8)/(a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)))*\sqrt{(\\
& \sqrt{1/2})*\sqrt{-(b^2c^3d^4 - 8a^3c^3d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c \\
& - 2a^2c^2)d^4e^3 + (a^2b^3 - 3a^2b^2c)*e^4 + (a^2b^4c^3 - 8a^2b^2c^4 \\
& + 16a^3c^5))*\sqrt{(c^6d^8 - 12a^5c^6d^6e^2 + 8a^4b^4d^5e^3 - 48a^2 \\
& *b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a \\
& ^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^4e^7 + (a^2b^4 - 2a^3b^2c \\
& + a^4c^2)e^8)/(a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9 \\
&))/(a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)))*\sqrt{-(b^2c^3d^4 - 8a^3c^3 \\
& d^3e + 6a^2b^2c^2d^2e^2 - 4(a^2b^2c - 2a^2c^2)d^4e^3 + (a^2b^3 - 3a^2* \\
& b^2c)*e^4 + (a^2b^4c^3 - 8a^2b^2c^4 + 16a^3c^5))*\sqrt{(c^6d^8 - 12a^5c^ \\
& 6d^6e^2 + 8a^4b^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^ \\
& 2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3 \\
& *b^2c^2)d^4e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8)/(a^2b^6c^6 - 12a^ \\
& 3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)))/(a^2b^6c^6 - 12a^3b^4c^7 + 16 \\
& a^3c^5)) + (c^6d^{10} - 3b^2c^5d^9e + 3(b^2c^4 - a^5c^5)d^8e^2 - (b^3c \\
& ^3 - 16a^2b^2c^4)d^7e^3 - 14(2a^2b^2c^3 + a^2c^4)d^6e^4 + 21(a^2b^3c \\
& ^2 + 2a^2b^2c^3)d^5e^5 - 7(a^2b^4c + 6a^2b^2c^2 + 2a^3c^3)d^4e^ \\
& 6 + (a^2b^5 + 17a^2b^3c + 24a^3b^2c^2)d^3e^7 - 3(a^2b^4 + 4a^3b^2* \\
& c + a^4c^2)d^2e^8 + (3a^3b^3 + a^4b^2c)*d^2e^9 - (a^4b^2 - a^5c)*e^{10} \\
&)*x) + 1/4*\sqrt{\sqrt{1/2})*\sqrt{-(b^2c^3d^4 - 8a^3c^3d^3e + 6a^2b^2c^2d^2* \\
& e^2 - 4(a^2b^2c - 2a^2c^2)d^4e^3 + (a^2b^3 - 3a^2b^2c)*e^4 + (a^2b^4c^3 \\
& - 8a^2b^2c^4 + 16a^3c^5))*\sqrt{(c^6d^8 - 12a^5c^6d^6e^2 + 8a^4b^4d^5e^3 - 48a^2 \\
& *b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4e^4 + 4(7a^2b^2c^2 - 3a^3c^3) \\
& d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^4e^7 + (a^2b^4 - 2a^3b^2c + a^4c^2)e^8) / (a^2 \\
& b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9)))/(a^2b^6c^6 - 12a^3b^4c^7 + 16 \\
& a^3c^5)))*\log(-1/2*\sqrt{\sqrt{1/2})*((b^4c^5 - 8a^2b^2c^6 + 16a^2c^7)d^7 - 9(a^2b^4c^4 - 8a^2b^ \\
& 2c^5 + 16a^3c^6)d^5e^2 + 5(a^2b^5c^3 - 8a^2b^3c^4 + 16a^3b^2c^5)* \\
& d^4e^3 - (a^2b^6c^2 - 27a^2b^4c^3 + 168a^3b^2c^4 - 304a^4c^5)d^3* \\
& e^4 - 18(a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^2c^4)d^2e^5 + (7a^2b^6c \\
& - 59a^3b^4c^2 + 136a^4b^2c^3 - 48a^5c^4)d^2e^6 - (a^2b^7 - 9a^3 \\
& *b^5c + 24a^4b^3c^2 - 16a^5b^2c^3)*e^7 - ((a^7b^7c^5 - 12a^2b^5c^6 \\
& + 48a^3b^3c^7 - 64a^4b^2c^8)d^3 - 6(a^2b^6c^5 - 12a^3b^4c^6 + 48 \\
& *a^4b^2c^7 - 64a^5c^8)d^2e + 3(a^2b^7c^4 - 12a^3b^5c^5 + 48a^4 \\
& *b^3c^6 - 64a^5b^2c^7)d^2e^2 - (a^2b^8c^3 - 14a^3b^6c^4 + 72a^4b^4 \\
& *c^5 - 160a^5b^2c^6 + 128a^6c^7)*e^3)*\sqrt{(c^6d^8 - 12a^5c^6d^6e^2 \\
& + 8a^4b^4d^5e^3 - 48a^2b^2c^3d^3e^5 - 2(a^2b^2c^3 - 19a^2c^4)d^4 \\
& e^4 + 4(7a^2b^2c^2 - 3a^3c^3)d^2e^6 - 8(a^2b^3c - a^3b^2c^2)d^4e^7 +}
\end{aligned}$$

$$\frac{(a^2c + a^4c^2)e^8)/(a^2b^6c^6 - 12a^3b^4c^7 + 48a^4b^2c^8 - 64a^5c^9))}{(a^2b^4c^3 - 8a^2b^2c^4 + 16a^3c^5)} + (c^6d^{10} - 3b^5c^5d^9e + 3(b^2c^4 - ac^5)d^8e^2 - (b^3c^3 - 16ab^2c^4)d^7e^3 - 14(2ab^2c^3 + a^2c^4)d^6e^4 + 21(a^2b^3c^2 + 2a^2b^2c^3)d^5e^5 - 7(a^2b^4c + 6a^2b^2c^2 + 2a^3c^3)d^4e^6 + (ab^5 + 17a^2b^3c + 24a^3b^2c^2)d^3e^7 - 3(a^2b^4 + 4a^3b^2c + a^4c^2)d^2e^8 + (3a^3b^3 + a^4b^2c)d^2e^9 - (a^4b^2 - a^5c)e^{10})x$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x); OUTPUT:Evaluation time: 8.38Unable to divide, perhaps due to rounding error%{-512, [0,10,0,3,5,2,7]}+%{-1152, [0,10,0,3,4,4,6]}+%{-512, [0,10,0,3,3,6,5]}+%{-64, [0,10,0,3,2,8,4]}+%{-1024, [0,9,1,3,6,1,7]}+%{-4352, [0,9,1,3,5,3,6]}+%{-512, [0,9,1,3,4,5,5]}+%{-640, [0,9,1,3,3,7,4]}+%{-128, [0,9,1,3,2,9,3]}+%{-512, [0,8,2,3,7,0,7]}+%{-7296, [0,8,2,3,6,2,6]}+%{-6144, [0,8,2,3,5,4,5]}+%{-4544, [0,8,2,3,4,6,4]}+%{-384, [0,8,2,3,3,8,3]}+%{-64, [0,8,2,3,2,10,2]}+%{-6144, [0,7,3,3,7,1,6]}+%{-22016, [0,7,3,3,6,3,5]}+%{-9472, [0,7,3,3,5,5,4]}+%{-1152, [0,7,3,3,4,7,3]}+%{-512, [0,7,3,3,3,9,2]}+%{-2048, [0,6,4,3,8,0,6]}+%{-31232, [0,6,4,3,7,2,5]}+%{-4352, [0,6,4,3,6,4,4]}+%{-8064, [0,6,4,3,5,6,3]}+%{-1792, [0,6,4,3,4,8,2]}+%{-1048576, [0,6,0,7,9,1,8]}+%{-3670016, [0,6,0,7,8,3,7]}+%{-2424832, [0,6,0,7,7,5,6]}+%{-589824, [0,6,0,7,6,7,5]}+%{-49152, [0,6,0,7,5,9,4]}+%{-20480, [0,5,5,3,8,1,5]}+%{-11264, [0,5,5,3,7,3,4]}+%{-18432, [0,5,5,3,6,5,3]}+%{-3584, [0,5,5,3,5,7,2]}+%{-7340032, [0,5,1,7,9,2,7]}+%{-2097152, [0,5,1,7,8,4,6]}+%{-1376256, [0,5,1,7,7,6,5]}+%{-622592, [0,5,1,7,6,8,4]}+%{-65536, [0,5,1,7,5,10,3]}+%{-5120, [0,4,6,3,9,0,5]}+%{-18176, [0,4,6,3,8,2,4]}+%{-23040, [0,4,6,3,7,4,3]}+%{-4544, [0,4,6,3,6,6,2]}+%{-8388608, [0,4,2,7,10,1,7]}+%{-2359296, [0,4,2,7,9,3,6]}+%{-3801088, [0,4,2,7,8,5,5]}+%{-409600, [0,4,2,7,7,7,4]}+%{-196608, [0,4,2,7,6,9,3]}+%{-32768, [0,4,2,7,5,11,2]}+%{-10240, [0,3,7,3,9,1,4]}+%{-17920, [0,3,7,3,8,3,3]}+%{-3840, [0,3,7,3,7,5,2]}+%{-4194304, [0,3,3,7,10,2,6]}+%{-2621440, [0,3,3,7,9,4,5]}+%{-4194304, [0,3,3,7,8,6,4]}+%{-1343488, [0,3,3,7,7,8,3]}+%{-131072, [0,3,3,7,6,10,2]}+%{-2048, [0,2,8,3,10,0,4]}+%{-9216, [0,2,8,3,9,2,3]}+%{-2176, [0,2,8,3,8,4,2]}+%{-5242880, [0,2,4,7,11,1,6]}+%{-12582912, [0,2,4,7,10,3,5]}+%{-8454144, [0,2,4,7,9,5,4]}+%{-2195456, [0,2,4,7,8,7,3]}+%{-196608, [0,2,4,7,7,9,2]}+%{-2147483648, [0,2,0,11,12,2,8]}+%{-2147483648, [0,2,0,11,11,4,7]}+%{-8053

06368, [0,2,0,11,10,6,6]%%}+%%{-134217728, [0,2,0,11,9,8,5]%%}+%%{8388608, [0,2,0,11,8,10,4]%%}+%%{3072, [0,1,9,3,10,1,3]%%}+%%{-768, [0,1,9,3,9,3,2]%%}+%%{5242880, [0,1,5,7,11,2,5]%%}+%%{-4718592, [0,1,5,7,10,4,4]%%}+%%{1376256, [0,1,5,7,9,6,3]%%}+%%{-131072, [0,1,5,7,8,8,2]%%}+%%{-2147483648, [0,1,1,11,12,3,7]%%}+%%{2147483648, [0,1,1,11,11,5,6]%%}+%%{-805306368, [0,1,1,11,10,7,5]%%}+%%{134217728, [0,1,1,11,9,9,4]%%}+%%{-8388608, [0,1,1,11,8,11,3]%%}+%%{-512, [0,0,10,3,11,0,3]%%}+%%{128, [0,0,10,3,10,2,2]%%}+%%{-2097152, [0,0,6,7,12,1,5]%%}+%%{1835008, [0,0,6,7,11,3,4]%%}+%%{-524288, [0,0,6,7,10,5,3]%%}+%%{49152, [0,0,6,7,9,7,2]%%}+%%{-2147483648, [0,0,2,11,13,2,7]%%}+%%{3221225472, [0,0,2,11,12,4,6]%%}+%%{-1879048192, [0,0,2,11,11,6,5]%%}+%%{536870912, [0,0,2,11,10,8,4]%%}+%%{-75497472, [0,0,2,11,9,10,3]%%}+%%{4194304, [0,0,2,11,8,12,2]%%} / %%{1, [0,6,0,0,1,2,2]%%}+%%{-2, [0,5,1,0,2,1,2]%%}+%%{-2, [0,5,1,0,1,3,1]%%}+%%{1, [0,4,2,0,3,0,2]%%}+%%{6, [0,4,2,0,2,2,1]%%}+%%{1, [0,4,2,0,1,4,0]%%}+%%{-6, [0,3,3,0,3,1,1]%%}+%%{-4, [0,3,3,0,2,3,0]%%}+%%{2, [0,2,4,0,4,0,1]%%}+%%{6, [0,2,4,0,3,2,0]%%}+%%{-2048, [0,2,0,4,5,1,3]%%}+%%{512, [0,2,0,4,4,3,2]%%}+%%{-4, [0,1,5,0,4,1,0]%%}+%%{2048, [0,1,1,4,5,2,2]%%}+%%{-512, [0,1,1,4,4,4,1]%%}+%%{1, [0,0,6,0,5,0,0]%%}+%%{2048, [0,0,2,4,6,1,2]%%}+%%{-1536, [0,0,2,4,5,3,1]%%}+%%{256, [0,0,2,4,4,5,0]%%} Error: Bad Argument Value

maple [C] time = 0.00, size = 51, normalized size = 0.14

$$\frac{\left(\text{RootOf}\left(-Z^8c + Z^4b + a\right)^6 e + \text{RootOf}\left(-Z^8c + Z^4b + a\right)^2 d\right) \ln\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right) + x\right)}{8 \text{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + 4 \text{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(-_R+x), _R=RootOf(-Z^8*c+_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^4 + d)x^2}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)

mupad [B] time = 9.57, size = 29445, normalized size = 78.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8), x)$

[Out] $2*\text{atan}\left(\frac{(x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{1/2} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{1/2} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{1/2} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{1/2} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{1/2})}{(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6))^{3/4}}\right)*(x*(-a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{1/2} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{1/2} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{1/2} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{1/2} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{1/2})}{(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6))^{1/4}}\right)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{1/2} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{1/2} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{1/2} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{1/2} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{1/2})}{(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6))^{1/4}}\right) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{1/2} - 8*$

$$\begin{aligned}
& a^2 b^3 c^4 d^4 + 16 a^2 b^2 c^5 d^4 - a^2 b^2 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 11 a^2 b^5 c^4 e^4 - 48 a^4 b^2 c^3 e^4 + a^2 c^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 128 a^3 c^5 d^3 e + 128 a^4 c^4 d^3 e^3 + 40 a^3 b^3 c^2 e^4 - 4 a^2 b^6 c^2 d^3 e - 48 a^2 b^3 c^3 d^2 e^2 - 8 a^2 b^4 c^3 d^3 e + 6 a^2 b^5 c^2 d^2 e^2 + 64 a^2 b^2 c^4 d^3 e + 40 a^2 b^4 c^2 d^3 e^3 + 96 a^3 b^2 c^4 d^2 e^2 - 128 a^3 b^2 c^3 d^2 e^3 - 6 a^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 4 a^2 b^2 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)) / (512 (256 a^5 c^7 + a^2 b^8 c^3 - 16 a^2 b^6 c^4 + 96 a^3 b^4 c^5 - 256 a^4 b^2 c^6))^{(3/4)} * (x (-a^2 b^7 e^4 + b^5 c^3 d^4 + c^3 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 8 a^2 b^3 c^4 d^4 + 16 a^2 b^2 c^5 d^4 - a^2 b^2 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 11 a^2 b^5 c^4 e^4 - 48 a^4 b^2 c^3 e^4 + a^2 c^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 128 a^3 c^5 d^3 e + 128 a^4 c^4 d^3 e^3 + 40 a^3 b^3 c^2 e^4 - 4 a^2 b^6 c^2 d^3 e - 48 a^2 b^3 c^3 d^2 e^2 - 8 a^2 b^4 c^3 d^3 e + 6 a^2 b^5 c^2 d^2 e^2 + 64 a^2 b^2 c^4 d^3 e + 40 a^2 b^4 c^2 d^3 e^3 + 96 a^3 b^2 c^4 d^2 e^2 - 128 a^3 b^2 c^3 d^2 e^3 - 6 a^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 4 a^2 b^2 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)) / (512 (256 a^5 c^7 + a^2 b^8 c^3 - 16 a^2 b^6 c^4 + 96 a^3 b^4 c^5 - 256 a^4 b^2 c^6))^{(1/4)} * (32768 a^4 c^7 d^2 - 32768 a^5 c^6 e^2 - 1024 a^2 b^6 c^4 d^2 + 10240 a^2 b^4 c^5 d^2 - 32768 a^3 b^2 c^6 d^2 - 2048 a^3 b^4 c^4 e^2 + 16384 a^4 b^2 c^5 e^2 + 32768 a^4 b^2 c^6 d^2 + 2048 a^2 b^5 c^4 d^2 e - 16384 a^3 b^3 c^5 d^2 e) * i + 4096 a^5 c^5 e^3 + 256 a^2 b^5 c^4 d^3 + 4096 a^3 b^2 c^6 d^3 - 12288 a^4 c^6 d^2 e - 2048 a^2 b^3 c^5 d^3 + 256 a^3 b^4 c^3 e^3 - 2048 a^4 b^2 c^4 e^3 - 768 a^2 b^4 c^4 d^2 e + 6144 a^3 b^2 c^5 d^2 e) * i) * (-a^2 b^7 e^4 + b^5 c^3 d^4 + c^3 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 8 a^2 b^3 c^4 d^4 + 16 a^2 b^2 c^5 d^4 - a^2 b^2 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 11 a^2 b^5 c^4 e^4 - 48 a^4 b^2 c^3 e^4 + a^2 c^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 128 a^3 c^5 d^3 e + 128 a^4 c^4 d^3 e^3 + 40 a^3 b^3 c^2 e^4 - 4 a^2 b^6 c^2 d^3 e - 48 a^2 b^3 c^3 d^2 e^2 - 8 a^2 b^4 c^3 d^3 e + 6 a^2 b^5 c^2 d^2 e^2 + 64 a^2 b^2 c^4 d^3 e + 40 a^2 b^4 c^2 d^3 e^3 + 96 a^3 b^2 c^4 d^2 e^2 - 128 a^3 b^2 c^3 d^2 e^3 - 6 a^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 4 a^2 b^2 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)) / (512 (256 a^5 c^7 + a^2 b^8 c^3 - 16 a^2 b^6 c^4 + 96 a^3 b^4 c^5 - 256 a^4 b^2 c^6))^{(1/4)} / ((x (4 a^3 b^3 c^4 e^6 - 12 a^4 b^2 c^2 e^6 + 16 a^2 c^5 d^5 e + 16 a^4 c^3 d^5 e^5 + 32 a^3 c^4 d^3 e^3 + 4 a^2 b^2 c^5 d^6 + 16 a^2 b^2 c^3 d^3 e^3 + 12 a^2 b^3 c^2 d^2 e^4 - 16 a^2 b^2 c^4 d^5 e + 4 a^2 b^5 c^2 d^2 e^4 - 8 a^2 b^4 c^3 d^2 e^5 + 24 a^2 b^3 c^3 d^4 e^2 - 16 a^2 b^4 c^2 d^3 e^3 - 36 a^2 b^2 c^4 d^4 e^2 - 52 a^3 b^2 c^3 d^2 e^4 + 16 a^3 b^2 c^2 d^2 e^5) + (-a^2 b^7 e^4 + b^5 c^3 d^4 + c^3 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 8 a^2 b^3 c^4 d^4 + 16 a^2 b^2 c^5 d^4 - a^2 b^2 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 11 a^2 b^5 c^4 e^4 - 48 a^4 b^2 c^3 e^4 + a^2 c^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 128 a^3 c^5 d^3 e + 128 a^4 c^4 d^3 e^3 + 40 a^3 b^3 c^2 e^4 - 4 a^2 b^6 c^2 d^3 e - 48 a^2 b^3 c^3 d^2 e^2 - 8 a^2 b^4 c^3 d^3 e + 6 a^2 b^5 c^2 d^2 e^2 + 64 a^2 b^2 c^4 d^3 e + 40 a^2 b^4 c^2 d^3 e^3 + 96 a^3 b^2 c^4 d^2 e^2 - 128 a^3 b^2 c^3 d^2 e^3 - 6 a^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 4 a^2 b^2 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)) / (512 (256 a^5 c^7 + a^2 b^8 c^3 - 16 a^2 b^6 c^4 + 96 a^3 b^4 c^5 - 256 a^4 b^2 c^6))^{(3/4)} * (x (-a^2 b^7 e^4 + b^5 c^3 d^4 + c^3 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 8 a^2 b^3 c^4 d^4 + 16 a^2 b^2 c^5 d^4 - a^2 b^2 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 11 a^2 b^5 c^4 e^4
\end{aligned}$$

$$\begin{aligned}
& - 48a^4b^3c^3e^4 + a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e \\
& + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 4ab^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} * (32768a^4c^7d^2 - 32768a^5c^6e^2 - 1024ab^6c^4d^2 \\
& + 10240a^2b^4c^5d^2 - 32768a^3b^2c^6d^2 - 2048a^3b^4c^4e^2 + 16384a^4b^2c^5e^2 + 32768a^4b^3c^6d^2e + 2048a^2b^5c^4d^2e - 1 \\
& 6384a^3b^3c^5d^2e) * 1i - 4096a^5c^5e^3 - 256ab^5c^4d^3 - 4096a^3b^3c^6d^3 + 12288a^4c^6d^2e + 2048a^2b^3c^5d^3 - 256a^3b^4c^3e^3 \\
& + 2048a^4b^2c^4e^3 + 768a^2b^4c^4d^2e - 6144a^3b^2c^5d^2e) * 1i) * (-ab^7e^4 + b^5c^3d^4 + c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 \\
& + 16a^2b^3c^5d^4 - ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^4e^4 - 48a^4b^3c^3e^4 + a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e \\
& + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 4ab^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} * 1i - (x(4a^3b^3c^3e^6 - 12a^4b^3c^2e^6 + 16a^2c^5d^5e \\
& + 16a^4c^3d^5e + 32a^3c^4d^3e^3 + 4ab^3c^5d^6 + 16a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16ab^2c^4d^5e + 4ab^5c^4d^2e^4 \\
& - 8a^2b^4c^3d^5e + 24ab^3c^3d^4e^2 - 16ab^4c^2d^3e^3 - 36a^2b^3c^4d^4e^2 - 52a^3b^3c^3d^2e^4 + 16a^3b^2c^2d^5e) \\
& + (-ab^7e^4 + b^5c^3d^4 + c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 - ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^4e^4 \\
& - 48a^4b^3c^3e^4 + a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 \\
& - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} \\
& + 4ab^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(3/4)} * (x(-ab^7e^4 + b^5c^3d^4 + c^3d^4(-4ac \\
& - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 - ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^4e^4 - 48a^4b^3c^3e^4 + a^2c^4e^4(-4ac \\
& - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 \\
& + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} \\
& + 4ab^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} * (32768a^4c^7d^2 \\
& - 32768a^5c^6e^2 - 1024ab^6c^4d^2 + 10240a^2b^4c^5d^2 - 32768a^3b^2c^6d^2 - 2048a^3b^4c^4e^2 + 16384a^4b^2c^5e^2 + 32768a^4b^3c^6d^2e \\
& + 2048a^2b^5c^4d^2e - 16384a^3b^3c^5d^2e) * 1i + 4096a^5c^5
\end{aligned}$$

$$\begin{aligned}
& *e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048* \\
& a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4* \\
& c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3* \\
& b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e \\
& + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^ \\
& 3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^5*c^7 + a*b^8 \\
& *c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)}*1i + 2*a* \\
& c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b \\
& *c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a \\
& ^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3* \\
& d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5))*(-(a*b^7*e^4 + b^5*c \\
& ^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5* \\
& d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3* \\
& e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4* \\
& d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a \\
& *b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^ \\
& 2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256* \\
& a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)} - \operatorname{atan}(-((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*b*c^2*e^6 + 16*a^2*c^5*d^5*e + \\
& 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3* \\
& e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a \\
& ^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4 \\
& *d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^ \\
& 5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c \\
& ^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c \\
& ^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c \\
& ^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - \\
& 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4 \\
& *c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(2 \\
& 56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6) \\
&))^{(3/4)}*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^ \\
& 3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a \\
& ^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^ \\
& 2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4 \\
& *a*c - b^2)^5)^{(1/2)})/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a \\
& ^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)}*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^
\end{aligned}$$

$$\begin{aligned}
& 2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)) * (- (a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)) / (512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) * 1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (- (a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)) / (512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4) * (x*(- (a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2)) / (512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) * (32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)) * (- (a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*d^
\end{aligned}$$

$$\begin{aligned}
& 3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^2e^3 + 96a^3b^2c^4d^2e^2 - 128a^3b^2c^3d^2e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4a^2b^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)) / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} * i) / \\
& (x(4a^3b^3c^2e^6 - 12a^4b^2c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^2e^5 + 32a^3c^4d^3e^3 + 4a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16a^2b^2c^4d^5e + 4a^2b^5c^2d^2e^4 - 8a^2b^4c^3d^2e^5 + 24a^2b^3c^3d^4e^2 - 16a^2b^4c^2d^3e^3 - 36a^2b^2c^4d^4e^2 - 52a^3b^2c^3d^2e^4 + 16a^3b^2c^2d^2e^5) - ((a^2b^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^4d^4 + 16a^2b^2c^5d^4 + a^2b^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^2e^4 - 48a^4b^2c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^2e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^2d^2e^3 - 48a^2b^3c^3d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^2e^3 + 96a^3b^2c^4d^2e^2 - 128a^3b^2c^3d^2e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4a^2b^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)) / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(3/4)} * (x(-a^2b^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^4d^4 + 16a^2b^2c^5d^4 + a^2b^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^2e^4 - 48a^4b^2c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^2e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^2d^2e^3 - 48a^2b^3c^3d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^2e^3 + 96a^3b^2c^4d^2e^2 - 128a^3b^2c^3d^2e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4a^2b^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)) / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} * (32768a^4c^7d^2 - 32768a^5c^6e^2 - 1024a^2b^6c^4d^2 + 10240a^2b^4c^5d^2 - 32768a^3b^2c^6d^2 - 2048a^3b^4c^4e^2 + 16384a^4b^2c^5e^2 + 32768a^4b^2c^6d^2e + 2048a^2b^5c^4d^2e - 16384a^3b^3c^5d^2e) - 4096a^5c^5e^3 - 256a^2b^5c^4d^3 - 4096a^3b^2c^6d^3 + 12288a^4c^6d^2e + 2048a^2b^3c^5d^3 - 256a^3b^4c^3e^3 + 2048a^4b^2c^4e^3 + 768a^2b^4c^4d^2e - 6144a^3b^2c^5d^2e) * (-a^2b^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^4d^4 + 16a^2b^2c^5d^4 + a^2b^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^2e^4 - 48a^4b^2c^3e^4 - a^2c^2e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^2e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^2d^2e^3 - 48a^2b^3c^3d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^2e^3 + 96a^3b^2c^4d^2e^2 - 128a^3b^2c^3d^2e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4a^2b^2c^2d^2e^3(-4ac - b^2)^5)^{(1/2)) / (512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} - (x(4a^3b^3c^2e^6 - 12a^4b^2c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^2e^5 + 32a^3c^4d^3e^3 + 4a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16a^2b^2c^4d^5e + 4a^2b^5c^2d^2e^4 - 8a^2b^4c^3d^2e^5 + 24a^2b^3c^3d^4e^2 - 16a^2b^4c^2d^3e^3 - 36a^2b^2c^4d^4e^2 - 52a^3b^2c^3d^2e^4 + 16a^3b^2c^2d^2e^5) - ((a^2b^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^4d^4
\end{aligned}$$

$$\begin{aligned}
& + 16a^2b^5c^5d^4 + a^2b^5e^4(-4ac - b^2)^5)^{1/2} - 11a^2b^5c^5e^4 \\
& - 48a^4b^3c^3e^4 - a^2c^5e^4(-4ac - b^2)^5)^{1/2} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^3d^3e^3 - 48a^2b^3c^3 \\
& d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e \\
& + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6a^2 \\
& c^2d^2e^2(-4ac - b^2)^5)^{1/2} - 4a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{1/2} \\
& / (512(256a^5c^7 + a^2b^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6))^{3/4} \\
& (x^2(-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^4d^4 \\
& + 16a^2b^5c^5d^4 + a^2b^5e^4(-4ac - b^2)^5)^{1/2} - 11a^2b^5c^5e^4 \\
& - 48a^4b^3c^3e^4 - a^2c^5e^4(-4ac - b^2)^5)^{1/2} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^3d^3e^3 - 48a^2b^3c^3 \\
& d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2 \\
& d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} \\
& - 4a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{1/2} / (512(256a^5c^7 + a^2b^8c^3 - 16a^2b^6c^4 \\
& + 96a^3b^4c^5 - 256a^4b^2c^6))^{1/4} (32768a^4c^7d^2 - 32768a^5c^6e^2 \\
& - 1024a^2b^6c^4d^2 + 10240a^2b^4c^5d^2 - 32768a^3b^2c^6d^2 - 2048a^3b^4c^4e^2 \\
& + 16384a^4b^2c^5e^2 + 32768a^4b^3c^6d^2e + 2048a^2b^5c^4d^2e - 16384a^3b^3c^5d^2e \\
& + 4096a^5c^5e^3 + 256a^2b^5c^4d^3 + 4096a^3b^3c^6d^3 - 12288a^4c^6d^2e \\
& - 2048a^2b^3c^5d^3 + 256a^3b^4c^3e^3 - 2048a^4b^2c^4e^3 - 768a^2b^4c^4d^2e \\
& + 6144a^3b^2c^5d^2e) * (-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{1/2} \\
& - 8a^2b^3c^4d^4 + 16a^2b^5c^5d^4 + a^2b^5e^4(-4ac - b^2)^5)^{1/2} - 11a^2b^5c^5e^4 \\
& - 48a^4b^3c^3e^4 - a^2c^5e^4(-4ac - b^2)^5)^{1/2} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 \\
& - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 \\
& + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} \\
& - 4a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{1/2} / (512(256a^5c^7 + a^2b^8c^3 - 16a^2b^6c^4 \\
& + 96a^3b^4c^5 - 256a^4b^2c^6))^{1/4} + 2a^2c^5d^7 + 2a^4c^2d^6e^6 \\
& + 6a^2c^4d^5e^2 + 6a^3c^3d^3e^4 - 2a^4b^3c^5e^7 - 8a^2b^3c^4d^6e \\
& + 18a^2b^2c^2d^3e^4 + 2a^2b^4c^3d^3e^4 + 6a^3b^2c^3d^6e^6 + 12a^2b^2c^3d^5e^2 \\
& - 8a^2b^3c^2d^4e^3 - 18a^2b^3c^3d^4e^3 - 6a^2b^3c^3d^2e^5 - 12a^3b^3c^2d^2e^5) \\
& * (-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^4d^4 \\
& + 16a^2b^5c^5d^4 + a^2b^5e^4(-4ac - b^2)^5)^{1/2} - 11a^2b^5c^5e^4 \\
& - 48a^4b^3c^3e^4 - a^2c^5e^4(-4ac - b^2)^5)^{1/2} - 128a^3c^5d^3e \\
& + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 \\
& - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 \\
& + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} \\
& - 4a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{1/2} / (512(256a^5c^7 + a^2b^8c^3 - 16a^2b^6c^4 \\
& + 96a^3b^4c^5 - 256a^4b^2c^6))^{1/4} * 2i - \operatorname{atan}\left(\frac{x(4a^3b^3c^5e^6 - 12a^4b^3c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^5e^5 + 32a^3c^4d^3e^3 + 4a^2b^3c^5d^6 + 16a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16a^2b^2c^4d^5e + 4a^2b^5c^3d^2e^4 - 8a^2b^4c^3d^5e^5)}{x^2(-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{1/2} - 8a^2b^3c^4d^4 + 16a^2b^5c^5d^4 + a^2b^5e^4(-4ac - b^2)^5)^{1/2} - 11a^2b^5c^5e^4 - 48a^4b^3c^3e^4 - a^2c^5e^4(-4ac - b^2)^5)^{1/2} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4a^2b^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8a^2b^4c^3d^3e + 6a^2b^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} - 4a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{1/2}}{512(256a^5c^7 + a^2b^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)}\right)
\end{aligned}$$

$$\begin{aligned}
& + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52* \\
& a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - ((a*b^7*e^4 + b^5*c^3*d^4 + c^ \\
& 3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2 \\
& *e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c \\
& *e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40* \\
& a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^ \\
& 3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 9 \\
& 6*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^ \\
& 2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^5*c^7 + a \\
& *b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(3/4)}*(x*(- \\
& (a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d \\
& ^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c* \\
& e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^ \\
& 3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3 \\
& *c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3 \\
& *e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - \\
& 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5) \\
& ^{(1/2)))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 2 \\
& 56*a^4*b^2*c^6)))^{(1/4)}*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6 \\
& *c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4 \\
& *e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - \\
& 16384*a^3*b^3*c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b \\
& *c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 \\
& + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))* \\
& (- (a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4 \\
& *d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c \\
& *e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5* \\
& d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b \\
& ^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^ \\
& ^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 \\
& - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - \\
& 256*a^4*b^2*c^6)))^{(1/4)}*1i + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16* \\
& a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16* \\
& a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5 \\
& *c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^ \\
& 3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (\\
& -(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4* \\
& d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c* \\
& e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d \\
& ^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^ \\
& 3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^ \\
& 3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - \\
& 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5) \\
& ^{(1/2)))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 -
\end{aligned}$$

$$\begin{aligned}
& 256a^4b^2c^6))^{(3/4)}(x*(-(ab^7e^4 + b^5c^3d^4 + c^3d^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^5c^7d^4 - ab^2e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 11a^2b^5c^7e^4 - 48a^4b^3c^3e^4 + a^2c^7e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 \\
& - 4ab^6c^4d^3e - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2 \\
& d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2 \\
& e^2 - 128a^3b^2c^3d^3e^3 - 6ac^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 4 \\
& ab^3c^4d^3e^3*(-(4ac - b^2)^5)^{(1/2)})/(512*(256a^5c^7 + ab^8c^3 - 16a^2 \\
& b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6))^{(1/4)}(32768a^4c^7d^2 - \\
& 32768a^5c^6e^2 - 1024ab^6c^4d^2 + 10240a^2b^4c^5d^2 - 32768a^3 \\
& b^2c^6d^2 - 2048a^3b^4c^4e^2 + 16384a^4b^2c^5e^2 + 32768a^4b^3c^6 \\
& d^2e + 2048a^2b^5c^4d^2e - 16384a^3b^3c^5d^2e) + 4096a^5c^5e^3 + \\
& 256ab^5c^4d^3 + 4096a^3b^3c^6d^3 - 12288a^4c^6d^2e - 2048a^2b^3 \\
& c^5d^3 + 256a^3b^4c^3e^3 - 2048a^4b^2c^4e^3 - 768a^2b^4c^4d^2 \\
& e + 6144a^3b^2c^5d^2e))(-(ab^7e^4 + b^5c^3d^4 + c^3d^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^5c^7d^4 - ab^2e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 11a^2b^5c^7e^4 - 48a^4b^3c^3e^4 + a^2c^7e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 \\
& - 4ab^6c^4d^3e - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2 \\
& d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2 \\
& e^2 - 128a^3b^2c^3d^3e^3 - 6ac^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + \\
& 4ab^3c^4d^3e^3*(-(4ac - b^2)^5)^{(1/2)})/(512*(256a^5c^7 + ab^8c^3 - 16a^2 \\
& b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6))^{(1/4)}*ii)/((x*(4a^3b^3c^6 \\
& e^6 - 12a^4b^3c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^5e^5 + 32a^3c^4 \\
& d^3e^3 + 4ab^3c^5d^6 + 16a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 \\
& - 16ab^2c^4d^5e + 4ab^5c^4d^2e^4 - 8a^2b^4c^3d^5e^5 + 24ab^3c^3 \\
& d^4e^2 - 16ab^4c^2d^3e^3 - 36a^2b^3c^4d^4e^2 - 52a^3b^3c^3d^2e^4 \\
& + 16a^3b^2c^2d^5e) - (-(ab^7e^4 + b^5c^3d^4 + c^3d^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^5c^7d^4 - ab^2e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 11a^2b^5c^7e^4 - 48a^4b^3c^3e^4 + a^2c^7e^4*(-(4ac - \\
& b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 \\
& - 4ab^6c^4d^3e - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2 \\
& d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2 \\
& e^2 - 128a^3b^2c^3d^3e^3 - 6ac^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 4 \\
& ab^3c^4d^3e^3*(-(4ac - b^2)^5)^{(1/2)})/(512*(256a^5c^7 + ab^8c^3 - 16a^2 \\
& b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6))^{(3/4)}(x*(-(ab^7e^4 + b^5 \\
& c^3d^4 + c^3d^4*(-(4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^5c^7 \\
& d^4 - ab^2e^4*(-(4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^7e^4 - 48a^4b^3c^3 \\
& e^4 + a^2c^7e^4*(-(4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4 \\
& d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^4d^3e - 48a^2b^3c^3d^2e^2 - \\
& 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4 \\
& c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 - 6ac^2d^2e^2 \\
& *(-(4ac - b^2)^5)^{(1/2)} + 4ab^3c^4d^3e^3*(-(4ac - b^2)^5)^{(1/2)})/(512*(2 \\
& 56a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6) \\
&))^{(1/4)}(32768a^4c^7d^2 - 32768a^5c^6e^2 - 1024ab^6c^4d^2 + 1024
\end{aligned}$$

$$\begin{aligned}
 & 0*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4 \\
 & 4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3* \\
 & c^5*d*e) - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 1228 \\
 & 8*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2 \\
 & *c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e))*(-(a*b^7*e^4 + \\
 & b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b \\
 & *c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b \\
 & *c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4 \\
 & *c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 \\
 & - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b \\
 & ^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e \\
 & ^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512* \\
 & (256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^ \\
 & 6)))^(1/4) - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16 \\
 & *a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^ \\
 & 3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2 \\
 & *b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d \\
 & ^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) - (-(a*b^7*e^4 + b^5* \\
 & c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5 \\
 & *d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3 \\
 & *e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4 \\
 & *d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8* \\
 & a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c \\
 & ^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(\\
 & -(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256 \\
 & *a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6))) \\
 & ^{(3/4)}*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8 \\
 & *a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11 \\
 & *a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 12 \\
 & 8*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 \\
 & - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2 \\
 & *b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2* \\
 & c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a \\
 & *c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3 \\
 & *b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 \\
 & - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048 \\
 & *a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b \\
 & ^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e) + 4096*a^5*c^5*e^3 + 256*a*b^5*c^4*d^3 \\
 & + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e - 2048*a^2*b^3*c^5*d^3 + 256*a^3 \\
 & *b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^2*b^4*c^4*d^2*e + 6144*a^3*b^2* \\
 & c^5*d^2*e))*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - \\
 & 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - \\
 & 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - \\
 & 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^ \\
 & 3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a
 \end{aligned}$$

$$\begin{aligned}
& 2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)} + 2*a*c^5*d^7 + 2*a^4*c^2*d*e^6 + 6*a^2*c^4*d^5*e^2 + 6*a^3*c^3*d^3*e^4 - 2*a^4*b*c*e^7 - 8*a*b*c^4*d^6*e + 18*a^2*b^2*c^2*d^3*e^4 + 2*a*b^4*c*d^3*e^4 + 6*a^3*b^2*c*d*e^6 + 12*a*b^2*c^3*d^5*e^2 - 8*a*b^3*c^2*d^4*e^3 - 18*a^2*b*c^3*d^4*e^3 - 6*a^2*b^3*c*d^2*e^5 - 12*a^3*b*c^2*d^2*e^5)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)}*2i + 2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(3/4)}*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)}*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 -
\end{aligned}$$

$$\begin{aligned}
& 48a^4b^3c^3e^4 - a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + \\
& 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + \\
& 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6ac^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
&)/(512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)} + (x(4a^3b^3c^3e^6 - 12a^4b^3c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^3e^5 + 32a^3c^4d^3e^3 + 4ab^6c^5d^6 + 16a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16ab^2c^4d^5e + 4ab^5c^3d^2e^4 - 8a^2b^4c^3d^5e + 24ab^3c^3d^4e^2 - 16ab^4c^2d^3e^3 - 36a^2b^3c^4d^4e^2 - 52a^3b^3c^3d^2e^4 + 16a^3b^2c^2d^3e^5) + (-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 + ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^3e^4 - 48a^4b^3c^3e^4 - a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6ac^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(3/4)}(x(-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 + ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^3e^4 - 48a^4b^3c^3e^4 - a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6ac^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)}(32768a^4c^7d^2 - 32768a^5c^6e^2 - 1024ab^6c^4d^2 + 10240a^2b^4c^5d^2 - 32768a^3b^2c^6d^2 - 2048a^3b^4c^4e^2 + 16384a^4b^2c^5e^2 + 32768a^4b^3c^6d^2e + 2048a^2b^5c^4d^2e - 16384a^3b^3c^5d^2e)*1i + 4096a^5c^5e^3 + 256ab^5c^4d^3 + 4096a^3b^3c^6d^3 - 12288a^4c^6d^2e - 2048a^2b^3c^5d^3 + 256a^3b^4c^3e^3 - 2048a^4b^2c^4e^3 - 768a^2b^4c^4d^2e + 6144a^3b^2c^5d^2e)*1i)*(-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 + ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^3e^4 - 48a^4b^3c^3e^4 - a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6ac^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)})/((x(4a^3b^3c^3e^6 - 12a^4b^3c^2e^6 + 16a^2c^5d^5e + 16a^4c^3d^3e^5 + 32a^3c^4d^3e^3 + 4ab^6c^5d^6 + 16a^2b^2c^3d^3e^3 + 12a^2b^3c^2d^2e^4 - 16ab^2c^4d^5e + 4ab^5c^3d^2e^4 - 8a^2b^4c^3d^5e + 24ab^3c^3d^4e^2 - 16ab^4c^2d^3e^3 - 36a^2b^3c^4d^4e^2 - 52a^3b^3c^3d^2e^4 + 16a^3b^2c^2d^3e^5) + (-ab^7e^4 + b^5c^3d^4 - c^3d^4(-4ac - b^2)^5)^{(1/2)} - 8ab^3c^4d^4 + 16a^2b^3c^5d^4 + ab^2e^4(-4ac - b^2)^5)^{(1/2)} - 11a^2b^5c^3e^4 - 48a^4b^3c^3e^4 - a^2c^4e^4(-4ac - b^2)^5)^{(1/2)} - 128a^3c^5d^3e + 128a^4c^4d^3e^3 + 40a^3b^3c^2e^4 - 4ab^6c^3d^3e^3 - 48a^2b^3c^3d^2e^2 - 8ab^4c^3d^3e + 6ab^5c^2d^2e^2 + 64a^2b^2c^4d^3e + 40a^2b^4c^2d^3e^3 + 96a^3b^3c^4d^2e^2 - 128a^3b^2c^3d^3e^3 + 6ac^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4ac - b^2)^5)^{(1/2)})/(512(256a^5c^7 + ab^8c^3 - 16a^2b^6c^4 + 96a^3b^4c^5 - 256a^4b^2c^6)))^{(1/4)}).
\end{aligned}$$

$$\begin{aligned}
& 2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16 \\
& *a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a* \\
& b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2* \\
& e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - \\
& 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c* \\
& d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6* \\
& c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(3/4)}*(x*(-(a*b^7*e^4 + b^5*c^3*d \\
& ^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 \\
& + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 \\
& - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^ \\
& 3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4* \\
& c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d* \\
& e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^5* \\
& c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^{(1/4)} \\
& *(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b \\
& ^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c \\
& ^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e \\
&)*i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 - 4096*a^3*b*c^6*d^3 + 12288*a^ \\
& 4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b^4*c^3*e^3 + 2048*a^4*b^2*c^4 \\
& *e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^5*d^2*e)*i)*(-(a*b^7*e^4 + b \\
& ^5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b* \\
& c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b* \\
& c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4* \\
& c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - \\
& 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^ \\
& 4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^ \\
& 2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(\\
& 256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6 \\
&)))^{(1/4)}*i - (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + \\
& 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3* \\
& e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a \\
& ^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4 \\
& *d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^ \\
& 5*c^3*d^4 - c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a*b^3*c^4*d^4 + 16*a^2*b* \\
& c^5*d^4 + a*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a^2*b^5*c*e^4 - 48*a^4*b* \\
& c^3*e^4 - a^2*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 128*a^3*c^5*d^3*e + 128*a^4* \\
& c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - \\
& 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4 \\
& *c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 + 6*a*c^2*d^2*e^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(2 \\
& 56*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)
\end{aligned}$$

$$\begin{aligned} &))^{(3/4)} * (x * (- (a * b^7 * e^4 + b^5 * c^3 * d^4 - c^3 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - \\ & 8 * a * b^3 * c^4 * d^4 + 16 * a^2 * b * c^5 * d^4 + a * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - \\ & 11 * a^2 * b^5 * c * e^4 - 48 * a^4 * b * c^3 * e^4 - a^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - \\ & 128 * a^3 * c^5 * d^3 * e + 128 * a^4 * c^4 * d * e^3 + 40 * a^3 * b^3 * c^2 * e^4 - 4 * a * b^6 * c * d * e^3 \\ & - 48 * a^2 * b^3 * c^3 * d^2 * e^2 - 8 * a * b^4 * c^3 * d^3 * e + 6 * a * b^5 * c^2 * d^2 * e^2 + 64 * a^2 * b^2 * c^4 * d^3 * e \\ & + 40 * a^2 * b^4 * c^2 * d * e^3 + 96 * a^3 * b * c^4 * d^2 * e^2 - 128 * a^3 * b^2 * c^3 * d * e^3 + 6 * a * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} \\ & - 4 * a * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^5 * c^7 + a * b^8 * c^3 - 16 * a^2 * b^6 * c^4 + 96 * a^3 * b^4 * c^5 \\ & - 256 * a^4 * b^2 * c^6)))^{(1/4)} * (32768 * a^4 * c^7 * d^2 - 32768 * a^5 * c^6 * e^2 - 1024 * a * b^6 * c^4 * d^2 + 10240 * a^2 * b^4 * c^5 * d^2 \\ & - 32768 * a^3 * b^2 * c^6 * d^2 - 2048 * a^3 * b^4 * c^4 * e^2 + 16384 * a^4 * b^2 * c^5 * e^2 + 32768 * a^4 * b * c^6 * d * e + 2048 * a^2 * b^5 * c^4 * d * e \\ & - 16384 * a^3 * b^3 * c^5 * d * e) * 1i + 4096 * a^5 * c^5 * e^3 + 256 * a * b^5 * c^4 * d^3 + 4096 * a^3 * b * c^6 * d^3 - 12288 * a^4 * c^6 * d^2 * e \\ & - 2048 * a^2 * b^3 * c^5 * d^3 + 256 * a^3 * b^4 * c^3 * e^3 - 2048 * a^4 * b^2 * c^4 * e^3 - 768 * a^2 * b^4 * c^4 * d^2 * e + 6144 * a^3 * b^2 * c^5 * d^2 * e) * 1i) \\ & * (- (a * b^7 * e^4 + b^5 * c^3 * d^4 - c^3 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^4 * d^4 + 16 * a^2 * b * c^5 * d^4 \\ & + a * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 11 * a^2 * b^5 * c * e^4 - 48 * a^4 * b * c^3 * e^4 - a^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\ & - 128 * a^3 * c^5 * d^3 * e + 128 * a^4 * c^4 * d * e^3 + 40 * a^3 * b^3 * c^2 * e^4 - 4 * a * b^6 * c * d * e^3 - 48 * a^2 * b^3 * c^3 * d^2 * e^2 \\ & - 8 * a * b^4 * c^3 * d^3 * e + 6 * a * b^5 * c^2 * d^2 * e^2 + 64 * a^2 * b^2 * c^4 * d^3 * e + 40 * a^2 * b^4 * c^2 * d * e^3 + 96 * a^3 * b * c^4 * d^2 * e^2 \\ & - 128 * a^3 * b^2 * c^3 * d * e^3 + 6 * a * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * a * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^5 * c^7 \\ & + a * b^8 * c^3 - 16 * a^2 * b^6 * c^4 + 96 * a^3 * b^4 * c^5 - 256 * a^4 * b^2 * c^6)))^{(1/4)} * 1i + 2 * a * c^5 * d^7 + 2 * a^4 * c^2 * d * e^6 \\ & + 6 * a^2 * c^4 * d^5 * e^2 + 6 * a^3 * c^3 * d^3 * e^4 - 2 * a^4 * b * c * e^7 - 8 * a * b * c^4 * d^6 * e + 18 * a^2 * b^2 * c^2 * d^3 * e^4 \\ & + 2 * a * b^4 * c * d^3 * e^4 + 6 * a^3 * b^2 * c * d * e^6 + 12 * a * b^2 * c^3 * d^5 * e^2 - 8 * a * b^3 * c^2 * d^4 * e^3 - 18 * a^2 * b * c^3 * d^4 * e^3 - 6 * a^2 * b^3 * c * d^2 * e^5 \\ & - 12 * a^3 * b * c^2 * d^2 * e^5) * (- (a * b^7 * e^4 + b^5 * c^3 * d^4 - c^3 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a * b^3 * c^4 * d^4 \\ & + 16 * a^2 * b * c^5 * d^4 + a * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 11 * a^2 * b^5 * c * e^4 - 48 * a^4 * b * c^3 * e^4 - a^2 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\ & - 128 * a^3 * c^5 * d^3 * e + 128 * a^4 * c^4 * d * e^3 + 40 * a^3 * b^3 * c^2 * e^4 - 4 * a * b^6 * c * d * e^3 - 48 * a^2 * b^3 * c^3 * d^2 * e^2 \\ & - 8 * a * b^4 * c^3 * d^3 * e + 6 * a * b^5 * c^2 * d^2 * e^2 + 64 * a^2 * b^2 * c^4 * d^3 * e + 40 * a^2 * b^4 * c^2 * d * e^3 + 96 * a^3 * b * c^4 * d^2 * e^2 \\ & - 128 * a^3 * b^2 * c^3 * d * e^3 + 6 * a * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * a * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^5 * c^7 \\ & + a * b^8 * c^3 - 16 * a^2 * b^6 * c^4 + 96 * a^3 * b^4 * c^5 - 256 * a^4 * b^2 * c^6)))^{(1/4)} \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.46 \quad \int \frac{x(d+ex^4)}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=184

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] $\frac{1}{4} \arctan(x^2 \cdot 2^{(1/2)} \cdot c^{(1/2)} / (b - (-4 \cdot a \cdot c + b^2)^{(1/2)})^{(1/2)}) \cdot (e + (-b \cdot e + 2 \cdot c \cdot d) / (-4 \cdot a \cdot c + b^2)^{(1/2)}) \cdot 2^{(1/2)} / c^{(1/2)} / (b - (-4 \cdot a \cdot c + b^2)^{(1/2)})^{(1/2)} + \frac{1}{4} \arctan(x^2 \cdot 2^{(1/2)} \cdot c^{(1/2)} / (b + (-4 \cdot a \cdot c + b^2)^{(1/2)})^{(1/2)}) \cdot (e + (b \cdot e - 2 \cdot c \cdot d) / (-4 \cdot a \cdot c + b^2)^{(1/2)}) \cdot 2^{(1/2)} / c^{(1/2)} / (b + (-4 \cdot a \cdot c + b^2)^{(1/2)})^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1490, 1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] $\frac{(e + (2 \cdot c \cdot d - b \cdot e) / \text{Sqrt}[b^2 - 4 \cdot a \cdot c]) \cdot \text{ArcTan}[(\text{Sqrt}[2] \cdot \text{Sqrt}[c] \cdot x^2) / \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 \cdot a \cdot c]]]}{(2 \cdot \text{Sqrt}[2] \cdot \text{Sqrt}[c] \cdot \text{Sqrt}[b - \text{Sqrt}[b^2 - 4 \cdot a \cdot c]])} + \frac{(e - (2 \cdot c \cdot d - b \cdot e) / \text{Sqrt}[b^2 - 4 \cdot a \cdot c]) \cdot \text{ArcTan}[(\text{Sqrt}[2] \cdot \text{Sqrt}[c] \cdot x^2) / \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 \cdot a \cdot c]]]}{(2 \cdot \text{Sqrt}[2] \cdot \text{Sqrt}[c] \cdot \text{Sqrt}[b + \text{Sqrt}[b^2 - 4 \cdot a \cdot c]])}$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1490

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_)*((d_) + (e_.)*(x_)^(n_.))^q_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x(d + ex^4)}{a + bx^4 + cx^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d + ex^2}{a + bx^2 + cx^4} dx, x, x^2 \right) \\ &= \frac{1}{4} \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx, x, x^2 \right) + \frac{1}{4} \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \text{Subst} \\ &\quad \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) - \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right) \\ &= \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}\sqrt{c}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}\sqrt{c}\sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.15, size = 179, normalized size = 0.97

$$\frac{\left(e(\sqrt{b^2 - 4ac} - b) + 2cd \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(e(\sqrt{b^2 - 4ac} + b) - 2cd \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{\sqrt{b + \sqrt{b^2 - 4ac}}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8), x]

[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

fricas [B] time = 1.47, size = 1535, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] $\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 + (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2))\log(-(c^2d^4 - b^2cd^3e + a^2bd^3e^3 - a^2e^4)x^2 + \frac{1}{2}\sqrt{\frac{1}{2}}((b^2c - 4a^2c^2)d^3 - (ab^2 - 4a^2c)d^2e^2 - ((ab^3c - 4a^2b^2c^2)d - 2(a^2b^2c - 4a^3c^2)e))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))})\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 + (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2)) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 + (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2))\log(-(c^2d^4 - b^2cd^3e + a^2bd^3e^3 - a^2e^4)x^2 - \frac{1}{2}\sqrt{\frac{1}{2}}((b^2c - 4a^2c^2)d^3 - (ab^2 - 4a^2c)d^2e^2 - ((ab^3c - 4a^2b^2c^2)d - 2(a^2b^2c - 4a^3c^2)e))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))})\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 + (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2)) + \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 - (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2))\log(-(c^2d^4 - b^2cd^3e + a^2bd^3e^3 - a^2e^4)x^2 + \frac{1}{2}\sqrt{\frac{1}{2}}((b^2c - 4a^2c^2)d^3 - (ab^2 - 4a^2c)d^2e^2 + ((ab^3c - 4a^2b^2c^2)d - 2(a^2b^2c - 4a^3c^2)e))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))})\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 - (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2)) - \frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 - (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2))\log(-(c^2d^4 - b^2cd^3e + a^2bd^3e^3 - a^2e^4)x^2 - \frac{1}{2}\sqrt{\frac{1}{2}}((b^2c - 4a^2c^2)d^3 - (ab^2 - 4a^2c)d^2e^2 + ((ab^3c - 4a^2b^2c^2)d - 2(a^2b^2c - 4a^3c^2)e))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))})\sqrt{-(b^2cd^2 - 4a^2cd^2e + a^2b^2e^2 - (ab^2c - 4a^2c^2))\sqrt{((c^2d^4 - 2a^2cd^2e^2 + a^2e^4)/(a^2b^2c^2 - 4a^3c^3))}}/(ab^2c - 4a^2c^2))$

giac [B] time = 20.31, size = 1406, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] $\frac{1}{8}((\sqrt{2})\sqrt{b^2c + \sqrt{b^2 - 4ac}})b^4 - 8\sqrt{2}\sqrt{b^2c + \sqrt{b^2 - 4ac}})a^2b^2c - 2\sqrt{2}\sqrt{b^2c + \sqrt{b^2 - 4ac}})b^3c - 2b^4c + 16\sqrt{2}\sqrt{b^2c + \sqrt{b^2 - 4ac}})a^2c^2 + 8\sqrt{2}\sqrt{b^2c + \sqrt{b^2 - 4ac}})a^2b^2c^2 + \sqrt{2}\sqrt{b^2c + \sqrt{b^2 - 4ac}})$

```

*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^
2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^
2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*
b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*
(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c
- 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))

```

maple [B] time = 0.02, size = 340, normalized size = 1.85

$$\frac{\sqrt{2} b e \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2} b e \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c}} - \frac{\sqrt{2} c d \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a), x)

[Out] $-1/4*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x^2)*e+1/4/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*c*x^2)$

$$\begin{aligned}
& 2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64* \\
& a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 \\
& 2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2)} + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3 \\
& *c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2 \\
& *b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d* \\
& e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(3/2)} + \\
& 2048*a^4*b*c^3*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12 \\
& *a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12 \\
& *a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c \\
& *d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(3/2)} - 48*a^3*b^2*c*e \\
& ^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12 \\
& *a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512* \\
& a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2)} + 16*a^2*b^2*c^2*d^2*(-(a*b^3 \\
& *e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a* \\
& b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - \\
& 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2)} - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c \\
& *d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2 \\
& *(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - \\
& 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2* \\
& c^2 + 32*a*b^4*c)^{(1/2)} + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2 \\
& *(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64* \\
& a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 \\
& 2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2)))*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2))*2i + atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i + a^2*e^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)}*1i - a^3*b*c*e^3*x^2*4i - a*b^4*d*e^2*x^2*1i + b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)}*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i - a*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)}*1i - a*c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)}*1i + a^2*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(1/2)} - 1024*a^3*b^3*c^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c)^{(3/2)} - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^3*c*d^2 - a
\end{aligned}$$

$$\begin{aligned}
& *e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - \\
& 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c \\
& *e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32* \\
& a*b^4*c))^{(1/2)} + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64 \\
& *a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 4 \\
& 8*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2* \\
& c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^{(1/2)} \\
&) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a \\
& ^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - \\
& 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b \\
& ^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^{(3/2)} + 2048*a^4*b* \\
& c^3*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1 \\
& 2*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(\\
& 1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512 \\
& *a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^{(3/2)} - 48*a^3*b^2*c*e^2*(-(a*b^3 \\
& *e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(\\
& 1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b \\
& *c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 2 \\
& 56*a^2*b^2*c^2 + 32*a*b^4*c))^{(1/2)} + 16*a^2*b^2*c^2*d^2*(-(a*b^3*e^2 + b^3 \\
& *c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d \\
& ^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - \\
& 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2 \\
& *c^2 + 32*a*b^4*c))^{(1/2)} - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e \\
& ^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64 \\
& *a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e \\
& ^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a* \\
& b^4*c))^{(1/2)} + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64 \\
& *a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 4 \\
& 8*a^2*b^2*c^2 - 12*a*b^4*c)^{(1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2* \\
& c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^{(1/2)} \\
&))*(-(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1 \\
& 2*a*b^4*c)^{(1/2)} + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^{(\\
& 1/2)} - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512 \\
& *a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^{(1/2)}*2i
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a), x)

[Out] Timed out

$$3.47 \quad \int \frac{d+ex^4}{a+bx^4+cx^8} dx$$

Optimal. Leaf size=375

$$\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \quad 2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4} \quad 2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

[Out] $-1/4*\arctan(2^{(1/4)*c^{(1/4)*x}/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(e+(b*e-2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(1/4)}/(-b-(-4*a*c+b^2)^{(1/2)})^{(3/4)}-1/4*\arctanh(2^{(1/4)*c^{(1/4)*x}/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(e+(b*e-2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(1/4)}/(-b-(-4*a*c+b^2)^{(1/2)})^{(3/4)}-1/4*\arctan(2^{(1/4)*c^{(1/4)*x}/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(1/4)}/(-b+(-4*a*c+b^2)^{(1/2)})^{(3/4)}-1/4*\arctanh(2^{(1/4)*c^{(1/4)*x}/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/c^{(1/4)}/(-b+(-4*a*c+b^2)^{(1/2)})^{(3/4)}$

Rubi [A] time = 0.35, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1422, 212, 208, 205}

$$\frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right) \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right) \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \quad 2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4} \quad 2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8), x]

[Out] $-((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)*c^{(1/4)*x}})/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)*c^{(1/4)*x}}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)*c^{(1/4)*x}})/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)*c^{(1/4)*x}}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((e - (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)*c^{(1/4)*x}})/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)*c^{(1/4)*x}}*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) - ((e + (2*c*d - b*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(2^{(1/4)*c^{(1/4)*x}})/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)*c^{(1/4)*x}}*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)})$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1422

Int[((d_) + (e_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(2n_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rubi steps

$$\begin{aligned} \int \frac{d + ex^4}{a + bx^4 + cx^8} dx &= \frac{1}{2} \left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx + \frac{1}{2} \left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\sqrt{-b - \sqrt{b^2 - 4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{-b - \sqrt{b^2 - 4ac}}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\sqrt{-b - \sqrt{b^2 - 4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{-b - \sqrt{b^2 - 4ac}}} \\ &= \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt[4]{2}\sqrt[4]{c} \left(-b - \sqrt{b^2 - 4ac} \right)^{3/4}} - \frac{\left(e + \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt[4]{2}\sqrt[4]{c} \left(-b + \sqrt{b^2 - 4ac} \right)^{3/4}} - \frac{\left(e - \frac{2cd - be}{\sqrt{b^2 - 4ac}} \right) \int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2\sqrt{-b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.16

$$\frac{1}{4} \text{RootSum} \left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 e \log(x - \#1) + d \log(x - \#1)}{2\#1^7 c + \#1^3 b} \& \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/4
```

```
fricas [B] time = 9.67, size = 13304, normalized size = 35.48
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*arctan(1/4*(2*sqrt(1/2)*((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*x*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) + ((b^7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 5*9*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) - ((b^7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5
```


$$\begin{aligned}
& *b*c^3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5 \\
& *b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^7 + ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72* \\
& a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5* \\
& b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^ \\
& 4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + \\
& 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c \\
& *d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d \\
& ^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^ \\
& 2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48* \\
& a^8*b^2*c^4 - 64*a^9*c^5))*\sqrt{-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3* \\
& b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4* \\
& c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3 \\
& *e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + \\
& 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + \\
& 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8* \\
& b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*\sqrt{(2*(\\
& 14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b \\
& ^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a* \\
& b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b \\
& ^2 - 4*a^4*c)*d^2*e^6)*x^2 - \sqrt{1/2))*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c \\
& ^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e + \\
& 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4* \\
& a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4 \\
& *b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 + ((a^3*b^7*c - 12*a^4* \\
& b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^ \\
& 2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b \\
& *c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4) \\
& *d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6* \\
& e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4 \\
& 8*a^8*b^2*c^4 - 64*a^9*c^5))*\sqrt{-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^ \\
& 3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^ \\
& 4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d \\
& ^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 \\
& + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 \\
& + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^ \\
& 8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^ \\
& 3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^ \\
& 2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c \\
& + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - \\
& 4*a^4*c)*d^2*e^6))*\sqrt{\sqrt{1/2))*\sqrt{-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 \\
& + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a \\
& ^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4* \\
& b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4) \\
&)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6 \\
& *e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
\end{aligned}$$

$$\begin{aligned}
& (48a^8b^2c^4 - 64a^9c^5)) / (a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)) / (\\
& 3a^5b^2d^9 - a^6e^{10} + (b^2c^4 - ac^5)d^{10} - (3b^3c^3 + abc^4)d \\
& ^9e + 3(b^4c^2 + 4ab^2c^3 + a^2c^4)d^8e^2 - (b^5c + 17ab^3c^2 \\
& + 24a^2b^2c^3)d^7e^3 + 7(ab^4c + 6a^2b^2c^2 + 2a^3c^3)d^6e^4 - \\
& 21(a^2b^3c + 2a^3b^2c^2)d^5e^5 + 14(2a^3b^2c + a^4c^2)d^4e^6 \\
& + (a^3b^3 - 16a^4b^2c)d^3e^7 - 3(a^4b^2 - a^5c)d^2e^8) - \text{sqrt}(\text{sqrt} \\
& (1/2) * \text{sqrt}(-(6a^2b^2cd^2e^2 - 8a^3c^2de^3 + a^3b^2e^4 + (b^3c - 3ab \\
& c^2)d^4 - 4(ab^2c - 2a^2c^2)d^3e + (a^3b^4c - 8a^4b^2c^2 + 1 \\
& 6a^5c^3) * \text{sqrt}(-(48a^3b^2cd^5e^3 - 8a^4b^2cd^3e^5 + 12a^5c^2d^2e \\
& ^6 - a^6e^8 - (b^4c^2 - 2ab^2c^3 + a^2c^4)d^8 + 8(ab^3c^2 - a^2b \\
& c^3)d^7e - 4(7a^2b^2c^2 - 3a^3c^3)d^6e^2 + 2(a^3b^2c - 19a^4 \\
& c^2)d^4e^4) / (a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5) \\
&)) / (a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)) * \arctan(1/4 * (2 * \text{sqrt}(1/2) * ((a^ \\
& 3b^8c^2 - 14a^4b^6c^3 + 72a^5b^4c^4 - 160a^6b^2c^5 + 128a^7c^6 \\
&)d^3 - 3(a^4b^7c^2 - 12a^5b^5c^3 + 48a^6b^3c^4 - 64a^7b^2c^5)d^ \\
& 2e + 6(a^5b^6c^2 - 12a^6b^4c^3 + 48a^7b^2c^4 - 64a^8c^5)d^2e^2 \\
& - (a^5b^7c - 12a^6b^5c^2 + 48a^7b^3c^3 - 64a^8b^2c^4)e^3) * x * \text{sqrt} \\
& (- (48a^3b^2cd^5e^3 - 8a^4b^2cd^3e^5 + 12a^5c^2d^2e^6 - a^6e^8 - (\\
& b^4c^2 - 2ab^2c^3 + a^2c^4)d^8 + 8(ab^3c^2 - a^2b^2c^3)d^7e - 4 * \\
& (7a^2b^2c^2 - 3a^3c^3)d^6e^2 + 2(a^3b^2c - 19a^4c^2)d^4e^4) / (\\
& a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5)) - ((b^7c^2 - \\
& 9ab^5c^3 + 24a^2b^3c^4 - 16a^3b^2c^5)d^7 - (7ab^6c^2 - 59a^2b^ \\
& 4c^3 + 136a^3b^2c^4 - 48a^4c^5)d^6e + 18(a^2b^5c^2 - 8a^3b^3c^ \\
& ^3 + 16a^4b^2c^4)d^5e^2 + (a^2b^6c - 27a^3b^4c^2 + 168a^4b^2c^3 \\
& - 304a^5c^4)d^4e^3 - 5(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^3e^4 + \\
& 9(a^4b^4c - 8a^5b^2c^2 + 16a^6c^3)d^2e^5 - (a^5b^4 - 8a^6b^ \\
& ^2c + 16a^7c^2)e^7) * x) * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(-(6a^2b^2cd^2e^2 - 8a^3 \\
& c^2de^3 + a^3b^2e^4 + (b^3c - 3ab^2c^2)d^4 - 4(ab^2c - 2a^2c^2)d^ \\
& 3e + (a^3b^4c - 8a^4b^2c^2 + 16a^5c^3) * \text{sqrt}(-(48a^3b^2cd^5e^3 \\
& - 8a^4b^2cd^3e^5 + 12a^5c^2d^2e^6 - a^6e^8 - (b^4c^2 - 2ab^2c^3 + \\
& a^2c^4)d^8 + 8(ab^3c^2 - a^2b^2c^3)d^7e - 4(7a^2b^2c^2 - 3a^3c^ \\
& ^3)d^6e^2 + 2(a^3b^2c - 19a^4c^2)d^4e^4) / (a^6b^6c^2 - 12a^7b^ \\
& 4c^3 + 48a^8b^2c^4 - 64a^9c^5)) / (a^3b^4c - 8a^4b^2c^2 + 16a^5c^ \\
& ^3)) * \text{sqrt}(-(6a^2b^2cd^2e^2 - 8a^3c^2de^3 + a^3b^2e^4 + (b^3c - 3ab \\
& c^2)d^4 - 4(ab^2c - 2a^2c^2)d^3e + (a^3b^4c - 8a^4b^2c^2 + 1 \\
& 6a^5c^3) * \text{sqrt}(-(48a^3b^2cd^5e^3 - 8a^4b^2cd^3e^5 + 12a^5c^2d^2e \\
& ^6 - a^6e^8 - (b^4c^2 - 2ab^2c^3 + a^2c^4)d^8 + 8(ab^3c^2 - a^2b \\
& c^3)d^7e - 4(7a^2b^2c^2 - 3a^3c^3)d^6e^2 + 2(a^3b^2c - 19a^4 \\
& c^2)d^4e^4) / (a^6b^6c^2 - 12a^7b^4c^3 + 48a^8b^2c^4 - 64a^9c^5) \\
&)) / (a^3b^4c - 8a^4b^2c^2 + 16a^5c^3)) + ((b^7c^2 - 9ab^5c^3 + 24 \\
& a^2b^3c^4 - 16a^3b^2c^5)d^7 - (7ab^6c^2 - 59a^2b^4c^3 + 136a^3b^ \\
& ^2c^4 - 48a^4c^5)d^6e + 18(a^2b^5c^2 - 8a^3b^3c^3 + 16a^4b^2c^ \\
& ^4)d^5e^2 + (a^2b^6c - 27a^3b^4c^2 + 168a^4b^2c^3 - 304a^5c^4)d^ \\
& ^4e^3 - 5(a^3b^5c - 8a^4b^3c^2 + 16a^5b^2c^3)d^3e^4 + 9(a^4b^4c \\
& - 8a^5b^2c^2 + 16a^6c^3)d^2e^5 - (a^5b^4 - 8a^6b^2c + 16a^7c
\end{aligned}$$

$$\begin{aligned}
&^2) * e^7 - ((a^3 * b^8 * c^2 - 14 * a^4 * b^6 * c^3 + 72 * a^5 * b^4 * c^4 - 160 * a^6 * b^2 * c^5 \\
&+ 128 * a^7 * c^6) * d^3 - 3 * (a^4 * b^7 * c^2 - 12 * a^5 * b^5 * c^3 + 48 * a^6 * b^3 * c^4 - 64 \\
&* a^7 * b * c^5) * d^2 * e + 6 * (a^5 * b^6 * c^2 - 12 * a^6 * b^4 * c^3 + 48 * a^7 * b^2 * c^4 - 64 * a \\
&^8 * c^5) * d * e^2 - (a^5 * b^7 * c - 12 * a^6 * b^5 * c^2 + 48 * a^7 * b^3 * c^3 - 64 * a^8 * b * c^4 \\
&)* e^3) * \text{sqrt}(-(48 * a^3 * b * c^2 * d^5 * e^3 - 8 * a^4 * b * c * d^3 * e^5 + 12 * a^5 * c * d^2 * e^6 - \\
&a^6 * e^8 - (b^4 * c^2 - 2 * a * b^2 * c^3 + a^2 * c^4) * d^8 + 8 * (a * b^3 * c^2 - a^2 * b * c^3 \\
&)* d^7 * e - 4 * (7 * a^2 * b^2 * c^2 - 3 * a^3 * c^3) * d^6 * e^2 + 2 * (a^3 * b^2 * c - 19 * a^4 * c^2 \\
&)* d^4 * e^4) / (a^6 * b^6 * c^2 - 12 * a^7 * b^4 * c^3 + 48 * a^8 * b^2 * c^4 - 64 * a^9 * c^5))) * \text{sqrt}(\text{sqrt}(1/2) * \text{sqrt}(-(6 * a^2 * b * c * d^2 * e^2 - 8 * a^3 * c * d * e^3 + a^3 * b * e^4 + (b^3 * c \\
&- 3 * a * b * c^2) * d^4 - 4 * (a * b^2 * c - 2 * a^2 * c^2) * d^3 * e + (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3) * \text{sqrt}(-(48 * a^3 * b * c^2 * d^5 * e^3 - 8 * a^4 * b * c * d^3 * e^5 + 12 * a^5 * c * d^2 * e^6 - a^6 * e^8 - (b^4 * c^2 - 2 * a * b^2 * c^3 + a^2 * c^4) * d^8 + 8 * (a * b^3 * c^2 - a^2 * b * c^3) * d^7 * e - 4 * (7 * a^2 * b^2 * c^2 - 3 * a^3 * c^3) * d^6 * e^2 + 2 * (a^3 * b^2 * c - 19 * a^4 * c^2) * d^4 * e^4) / (a^6 * b^6 * c^2 - 12 * a^7 * b^4 * c^3 + 48 * a^8 * b^2 * c^4 - 64 * a^9 * c^5)))) / (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3))) * \text{sqrt}(-(6 * a^2 * b * c * d^2 * e^2 - 8 * a^3 * c * d * e^3 + a^3 * b * e^4 + (b^3 * c - 3 * a * b * c^2) * d^4 - 4 * (a * b^2 * c - 2 * a^2 * c^2) * d^3 * e + (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3) * \text{sqrt}(-(48 * a^3 * b * c^2 * d^5 * e^3 - 8 * a^4 * b * c * d^3 * e^5 + 12 * a^5 * c * d^2 * e^6 - a^6 * e^8 - (b^4 * c^2 - 2 * a * b^2 * c^3 + a^2 * c^4) * d^8 + 8 * (a * b^3 * c^2 - a^2 * b * c^3) * d^7 * e - 4 * (7 * a^2 * b^2 * c^2 - 3 * a^3 * c^3) * d^6 * e^2 + 2 * (a^3 * b^2 * c - 19 * a^4 * c^2) * d^4 * e^4) / (a^6 * b^6 * c^2 - 12 * a^7 * b^4 * c^3 + 48 * a^8 * b^2 * c^4 - 64 * a^9 * c^5)))) / (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3))) * \text{sqrt}((2 * (14 * a^3 * b * c * d^3 * e^5 - 2 * a^4 * b * d * e^7 + a^5 * e^8 - (b^2 * c^3 - a * c^4) * d^8 + 2 * (b^3 * c^2 + a * b * c^3) * d^7 * e - (b^4 * c + 9 * a * b^2 * c^2 + 4 * a^2 * c^3) * d^6 * e^2 + 6 * (a * b^3 * c + 3 * a^2 * b * c^2) * d^5 * e^3 - 5 * (3 * a^2 * b^2 * c + 2 * a^3 * c^2) * d^4 * e^4 + (a^3 * b^2 - 4 * a^4 * c) * d^2 * e^6) * x^2 - \text{sqrt}(1/2) * ((b^6 * c - 7 * a * b^4 * c^2 + 14 * a^2 * b^2 * c^3 - 8 * a^3 * c^4) * d^6 - 2 * (3 * a * b^5 * c - 17 * a^2 * b^3 * c^2 + 20 * a^3 * b * c^3) * d^5 * e + 2 * (8 * a^2 * b^4 * c - 39 * a^3 * b^2 * c^2 + 28 * a^4 * c^3) * d^4 * e^2 - 20 * (a^3 * b^3 * c - 4 * a^4 * b * c^2) * d^3 * e^3 - (a^3 * b^4 - 18 * a^4 * b^2 * c + 56 * a^5 * c^2) * d^2 * e^4 + 2 * (a^4 * b^3 - 4 * a^5 * b * c) * d * e^5 - 2 * (a^5 * b^2 - 4 * a^6 * c) * e^6 - ((a^3 * b^7 * c - 12 * a^4 * b^5 * c^2 + 48 * a^5 * b^3 * c^3 - 64 * a^6 * b * c^4) * d^2 - 2 * (a^4 * b^6 * c - 12 * a^5 * b^4 * c^2 + 48 * a^6 * b^2 * c^3 - 64 * a^7 * c^4) * d * e) * \text{sqrt}(-(48 * a^3 * b * c^2 * d^5 * e^3 - 8 * a^4 * b * c * d^3 * e^5 + 12 * a^5 * c * d^2 * e^6 - a^6 * e^8 - (b^4 * c^2 - 2 * a * b^2 * c^3 + a^2 * c^4) * d^8 + 8 * (a * b^3 * c^2 - a^2 * b * c^3) * d^7 * e - 4 * (7 * a^2 * b^2 * c^2 - 3 * a^3 * c^3) * d^6 * e^2 + 2 * (a^3 * b^2 * c - 19 * a^4 * c^2) * d^4 * e^4) / (a^6 * b^6 * c^2 - 12 * a^7 * b^4 * c^3 + 48 * a^8 * b^2 * c^4 - 64 * a^9 * c^5)))) * \text{sqrt}(-(6 * a^2 * b * c * d^2 * e^2 - 8 * a^3 * c * d * e^3 + a^3 * b * e^4 + (b^3 * c - 3 * a * b * c^2) * d^4 - 4 * (a * b^2 * c - 2 * a^2 * c^2) * d^3 * e + (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3) * \text{sqrt}(-(48 * a^3 * b * c^2 * d^5 * e^3 - 8 * a^4 * b * c * d^3 * e^5 + 12 * a^5 * c * d^2 * e^6 - a^6 * e^8 - (b^4 * c^2 - 2 * a * b^2 * c^3 + a^2 * c^4) * d^8 + 8 * (a * b^3 * c^2 - a^2 * b * c^3) * d^7 * e - 4 * (7 * a^2 * b^2 * c^2 - 3 * a^3 * c^3) * d^6 * e^2 + 2 * (a^3 * b^2 * c - 19 * a^4 * c^2) * d^4 * e^4) / (a^6 * b^6 * c^2 - 12 * a^7 * b^4 * c^3 + 48 * a^8 * b^2 * c^4 - 64 * a^9 * c^5)))) / (a^3 * b^4 * c - 8 * a^4 * b^2 * c^2 + 16 * a^5 * c^3))) / (14 * a^3 * b * c * d^3 * e^5 - 2 * a^4 * b * d * e^7 + a^5 * e^8 - (b^2 * c^3 - a * c^4) * d^8 + 2 * (b^3 * c^2 + a * b * c^3) * d^7 * e - (b^4 * c + 9 * a * b^2 * c^2 + 4 * a^2 * c^3) * d^6 * e^2 + 6 * (a * b^3 * c + 3 * a^2 * b * c^2) * d^5 * e^3 - 5 * (3 * a^2 * b^2 * c + 2 * a^3 * c^2) * d^4 * e^4 + (a^3 * b^2 - 4 * a^4 * c) * d^2 * e^6))) / (3 * a^5 * b * d * e^9 - a^6 * e^10 + (b
\end{aligned}$$

$$\begin{aligned}
& ^2*c^4 - a*c^5)*d^{10} - (3*b^3*c^3 + a*b*c^4)*d^9*e + 3*(b^4*c^2 + 4*a*b^2*c \\
& ^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2 + 24*a^2*b*c^3)*d^7*e^3 + 7*(\\
& a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 - 21*(a^2*b^3*c + 2*a^3*b*c^2) \\
& *d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6 + (a^3*b^3 - 16*a^4*b*c)*d^3* \\
& e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d \\
& ^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - \\
& 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3* \\
& b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - \\
& 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^ \\
& 2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c \\
& ^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2 \\
& *c^2 + 16*a^5*c^3))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^ \\
& 5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2* \\
& c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a \\
& *b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 \\
& - 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b \\
& ^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b \\
& *c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4) \\
& *d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6* \\
& e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4 \\
& 8*a^8*b^2*c^4 - 64*a^9*c^5))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a \\
& ^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)* \\
& d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^ \\
& 3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 \\
& + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^ \\
& 3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7* \\
& b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^ \\
& 5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a \\
& ^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b \\
& ^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c* \\
& d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^ \\
& 8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 \\
& + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a \\
& ^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*log((\\
& 10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a \\
& *c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x - \\
& 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4* \\
& e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b \\
& ^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16* \\
& a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2* \\
& e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2* \\
& b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^ \\
& 4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5 \\
&)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (\\
& b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4
\end{aligned}$$

$$\begin{aligned}
& *b^2*c^2 + 16*a^5*c^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12 \\
& *a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3 \\
& *c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2 \\
& *c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - \\
& 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) + 1/4*\sqrt{\sqrt{(\\
& 1/2)*\sqrt{-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b* \\
& c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16* \\
& a^5*c^3)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 \\
& - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c \\
& ^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c \\
& ^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5))} \\
& / (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*\log((10*a^2*b*c*d^3*e^3 - 5*a^3 \\
& *c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b \\
& *c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 \\
& + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c \\
& ^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16* \\
& a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*\sqrt{-(48*a^3* \\
& b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - \\
& 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^ \\
& 2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c \\
& ^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*\sqrt{\sqrt{(1/2)*\sqrt{-(\\
& 6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4 \\
& *(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*\sqrt{ \\
& -(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - \\
& (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - \\
& 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4} \\
& / (a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c \\
& - 8*a^4*b^2*c^2 + 16*a^5*c^3)) - 1/4*\sqrt{\sqrt{(1/2)*\sqrt{-(6*a^2*b*c*d^2* \\
& e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2* \\
& a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*\sqrt{-(48*a^3*b*c \\
& ^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2* \\
& a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c \\
& ^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 \\
& - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^ \\
& 2 + 16*a^5*c^3)))*\log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + \\
& a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + \\
& a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^ \\
& 3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4 \\
& *a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4* \\
& c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*\sqrt{-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c* \\
& d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^ \\
& 8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 \\
& + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a \\
& ^8*b^2*c^4 - 64*a^9*c^5)))*\sqrt{\sqrt{(1/2)*\sqrt{-(6*a^2*b*c*d^2*e^2 - 8*a^3* \\
& c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3}
\end{aligned}$$

*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.00, size = 47, normalized size = 0.13

$$\frac{\left(\text{RootOf}\left(-Z^8c + Z^4b + a\right)^4 e + d\right) \ln\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right) + x\right)}{8 \text{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + 4 \text{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x)

[Out] 1/4*sum((R^4*e+d)/(2*R^7*c+R^3*b)*ln(-R+x), R=RootOf(-Z^8*c+Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^4 + d}{cx^8 + bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)

mupad [B] time = 8.75, size = 36707, normalized size = 97.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^4)/(a + b*x^4 + c*x^8),x)

$$\begin{aligned}
& [\text{Out}] - \operatorname{atan}\left(\frac{\left(\left(-b^7cd^4 + a^3b^5e^4 + a^3e^4(-4ac - b^2)^5\right)^{1/2} - 11\right.}{512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4)}\right)^{1/4} \left(\left(-b^7cd^4 + a^3b^5e^4 + a^3e^4(-4ac - b^2)^5\right)^{1/2} - 11\right. \\
& *a^5c^2d^4 - 48a^3b^3c^4d^4 + a^3c^2d^4(-4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^3c^2e^4 - b^2c^4d^4(-4ac - b^2)^5)^{1/2} + 128 \\
& *a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4a^2b^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^2d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 4a^2b^3c^3d^3e(-4ac - b^2)^5)^{1/2} \\
& \left. / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))\right)^{1/4} \left(\left(-b^7cd^4 + a^3b^5e^4 + a^3e^4(-4ac - b^2)^5\right)^{1/2} - 11\right. \\
& *a^5c^2d^4 - 48a^3b^3c^4d^4 + a^3c^2d^4(-4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^3c^2e^4 - b^2c^4d^4(-4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4a^2b^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^2d^3e + 4 \\
& 0a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 4a^2b^3c^3d^3e(-4ac - b^2)^5)^{1/2} \\
& \left. / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))\right)^{1/4} (262144a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2b^6c^4e + 4 \\
& 9152a^3b^4c^5e - 196608a^4b^2c^6e + 4096a^2b^7c^4d - 262144a^4b^3c^7d) + x(1024b^7c^4d^2 - 11264a^2b^5c^5d^2 - 49152a^3b^3c^7d^2 + \\
& 16384a^4b^3c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 - 8192a^3b^3c^5e^2 + 65536a^4c^7d^2e - 2048a^2b^6c^4d^2e + 20480a^2b^4c^5d^2e - 65536a^3b^2c^6d^2e) \\
& \left. \left(-b^7cd^4 + a^3b^5e^4 + a^3e^4(-4ac - b^2)^5\right)^{1/2} - 11\right) *a^5c^2d^4 - 48a^3b^3c^4d^4 + a^3c^2d^4(-4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^3c^2e^4 - b^2c^4d^4(-4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4a^2b^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^2d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 4a^2b^3c^3d^3e(-4ac - b^2)^5)^{1/2} \\
& \left. / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))\right)^{3/4} + 64a^2c^7d^5 - 16b^2c^6d^5 + 64a^3b^3c^4e^5 - 192a^3c^5d^4e^4 + 16b^3c^5d^4e - 16a^2b^3c^3e^5 - 128a^2c^6d^3e^2 - 64a^2b^3c^4d^2e^3 + 256a^2b^3c^5d^2e^3 - 16a^2b^2c^4d^2e^4 + 32a^2b^2c^5d^3e^2 - 64a^2b^3c^4d^2e^3 + 256a^2b^3c^5d^2e^3 - 16a^2b^2c^4d^2e^4) + x(8c^7d^6 - 8a^3c^4e^6 + 8a^2c^6d^4e^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - 16b^3c^4d^3e^3 + 4b^4c^3d^2e^4 - 24b^3c^6d^5e - 16a^2b^3c^5d^3e^3 - 8a^2b^3c^3d^3e^5 + 8a^2b^3c^4d^2e^5 + 16a^2b^2c^4d^2e^4) \left(-b^7cd^4 + a^3b^5e^4 + a^3e^4(-4ac - b^2)^5\right)^{1/2} - 11 *a^5c^2d^4 - 48a^3b^3c^4d^4 + a^3c^2d^4(-4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^3c^2e^4 - b^2c^4d^4(-4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4a^2b^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^2d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2c^2d^2e^2(-4ac - b^2)^5)^{1/2} + 4a^2b^3c^3d^3e(-4ac - b^2)^5)^{1/2} \\
& \left. / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))\right)^{1/4}
\end{aligned}$$

$$\begin{aligned}
& \left(a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4 \right) \left(\frac{1}{4} \right) i - \left(\left(- (b^7 c d^4 + a^3 b^5 e^4 + a^3 e^4 (- (4 a c - b^2)^5)^{1/2}) - \right. \right. \\
& 11 a b^5 c^2 d^4 - 48 a^3 b c^4 d^4 + a c^2 d^4 (- (4 a c - b^2)^5)^{1/2} - \\
& 8 a^4 b^3 c e^4 + 16 a^5 b c^2 e^4 - b^2 c d^4 (- (4 a c - b^2)^5)^{1/2} + 1 \\
& 28 a^4 c^4 d^3 e - 128 a^5 c^3 d e^3 + 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e \\
& - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 \\
& 2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b c^3 d^2 e^2 + 64 a^4 b^2 \\
& c^2 d e^3 - 6 a^2 c d^2 e^2 (- (4 a c - b^2)^5)^{1/2} + 4 a b c d^3 e (- (4 a \\
& a c - b^2)^5)^{1/2} \left. \right) / \left(512 (256 a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - \right. \\
& 256 a^6 b^2 c^4) \left. \right) \left(\frac{1}{4} \right) * \left(\left(- (b^7 c d^4 + a^3 b^5 e^4 + a^3 e^4 \right. \right. \\
& (- (4 a c - b^2)^5)^{1/2} - 11 a b^5 c^2 d^4 - 48 a^3 b c^4 d^4 + a c^2 d^4 \\
& (- (4 a c - b^2)^5)^{1/2} - 8 a^4 b^3 c e^4 + 16 a^5 b c^2 e^4 - b^2 c d^4 (- (4 a c - \\
& b^2)^5)^{1/2} + 128 a^4 c^4 d^3 e - 128 a^5 c^3 d e^3 + 40 a^2 b^3 c^3 \\
& d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + \\
& 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 \\
& 4 b c^3 d^2 e^2 + 64 a^4 b^2 c^2 d e^3 - 6 a^2 c d^2 e^2 (- (4 a c - b^2)^5)^{1/2} \\
& \left. \right) \left(\frac{1}{4} \right) + 4 a b c d^3 e (- (4 a c - b^2)^5)^{1/2} \left. \right) / \left(512 (256 a^7 c^5 + a^3 b^8 c - \right. \\
& 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4) \left. \right) \left(\frac{1}{4} \right) * (262144 a^5 c^7 e - \\
& 49152 a^2 b^5 c^5 d + 196608 a^3 b^3 c^6 d - 4096 a^2 b^6 c^4 e + \\
& 49152 a^3 b^4 c^5 e - 196608 a^4 b^2 c^6 e + 4096 a b^7 c^4 d - 262144 a^4 \\
& b c^7 d) - x * (1024 b^7 c^4 d^2 - 11264 a b^5 c^5 d^2 - 49152 a^3 b c^7 d^2 \\
& + 16384 a^4 b c^6 e^2 + 40960 a^2 b^3 c^6 d^2 + 1024 a^2 b^5 c^4 e^2 - 819 \\
& 2 a^3 b^3 c^5 e^2 + 65536 a^4 c^7 d e - 2048 a b^6 c^4 d e + 20480 a^2 b^4 c^5 d e \\
& - 65536 a^3 b^2 c^6 d e) * \left(- (b^7 c d^4 + a^3 b^5 e^4 + a^3 e^4 (- (4 a c - b^2)^5)^{1/2} - \right. \\
& 11 a b^5 c^2 d^4 - 48 a^3 b c^4 d^4 + a c^2 d^4 (- (4 a c - b^2)^5)^{1/2} - 8 a^4 b^3 c e^4 + \\
& 16 a^5 b c^2 e^4 - b^2 c d^4 (- (4 a c - b^2)^5)^{1/2} + 128 a^4 c^4 d^3 e - 128 a^5 c^3 d e^3 + \\
& 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + 40 a^2 \\
& b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b c^3 d^2 e^2 + 64 a^4 b^2 c^2 d e^3 - \\
& 6 a^2 c d^2 e^2 (- (4 a c - b^2)^5)^{1/2} \left. \right) \left(\frac{1}{4} \right) + 4 a b c d^3 e (- (4 a c - b^2)^5)^{1/2} \left. \right) / \left(512 (256 a^7 c^5 + \right. \\
& 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4) \left. \right) \left(\frac{3}{4} \right) + 64 a c^7 d^5 \\
& - 16 b^2 c^6 d^5 + 64 a^3 b c^4 e^5 - 192 a^3 c^5 d e^4 + 16 b^3 c^5 d^4 e \\
& - 16 a^2 b^3 c^3 e^5 - 128 a^2 c^6 d^3 e^2 - 64 a b c^6 d^4 e + 16 a b^4 c^3 d e^4 + \\
& 32 a b^2 c^5 d^3 e^2 - 64 a b^3 c^4 d^2 e^3 + 256 a^2 b c^5 d^2 e^3 - 16 a^2 b^2 c^4 d e^4) - \\
& x * (8 c^7 d^6 - 8 a^3 c^4 e^6 + 8 a c^6 d^4 e^2 + 4 a^2 b^2 c^3 e^6 - 8 a^2 c^5 d^2 e^4 + \\
& 28 b^2 c^5 d^4 e^2 - 16 b^3 c^4 d^3 e^3 + 4 b^4 c^3 d^2 e^4 - 24 b c^6 d^5 e - 16 a b c^5 d^3 e^3 - \\
& 8 a b^3 c^3 d e^5 + 8 a^2 b c^4 d e^5 + 16 a b^2 c^4 d^2 e^4) * \left(- (b^7 c d^4 + a^3 b^5 e^4 + \right. \\
& a^3 e^4 (- (4 a c - b^2)^5)^{1/2} - 11 a b^5 c^2 d^4 - 48 a^3 b c^4 d^4 + a c^2 d^4 (- (4 a c - \\
& b^2)^5)^{1/2} - 8 a^4 b^3 c e^4 + 16 a^5 b c^2 e^4 - b^2 c d^4 (- (4 a c - b^2)^5)^{1/2} + \\
& 128 a^4 c^4 d^3 e - 128 a^5 c^3 d e^3 + 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - \\
& 8 a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b c^3 d^2 e^2 + \\
& 64 a^4 b^2 c^2 d e^3 - 6 a^2 c d^2 e^2 (- (4 a c - b^2)^5)^{1/2} \left. \right)
\end{aligned}$$

$$\begin{aligned}
& - (4ac - b^2)^5)^{1/2} + 4abcd^3e * (- (4ac - b^2)^5)^{1/2} / (512 * (256 \\
& * a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4)) \\
& ^{1/4} * i) / (((- (b^7cd^4 + a^3b^5e^4 + a^3e^4 * (- (4ac - b^2)^5)^{1/2} \\
& - 11ab^5c^2d^4 - 48a^3b^4cd^4 + a^2cd^4 * (- (4ac - b^2)^5)^{1/2} \\
& - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 * (- (4ac - b^2)^5)^{1/2} + \\
& 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3 \\
& * e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6 \\
& a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b \\
& ^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac - b^2)^5)^{1/2} + 4abcd^3e * (- (\\
& 4ac - b^2)^5)^{1/2} / (512 * (256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96 \\
& a^5b^4c^3 - 256a^6b^2c^4))^{1/4} * (((- (b^7cd^4 + a^3b^5e^4 + a^3e \\
& ^4 * (- (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - 48a^3b^4cd^4 + a^2cd \\
& ^4 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^ \\
& 4 * (- (4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2 \\
& * b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e^3 \\
& + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96 \\
& a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac - b^2)^ \\
& 5)^{1/2} + 4abcd^3e * (- (4ac - b^2)^5)^{1/2} / (512 * (256a^7c^5 + a^3 \\
& b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{1/4} * (262144 \\
& a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2b^6c^4e \\
& + 49152a^3b^4c^5e - 196608a^4b^2c^6e + 4096a^2b^7c^4d - 262144a \\
& ^4b^7c^4d) + x * (1024b^7c^4d^2 - 11264ab^5c^5d^2 - 49152a^3b^7c^7d \\
& ^2 + 16384a^4b^6c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 - 8 \\
& 192a^3b^3c^5e^2 + 65536a^4c^7d^2e - 2048ab^6c^4d^2e + 20480a^2b^ \\
& 4c^5d^2e - 65536a^3b^2c^6d^2e) * (- (b^7cd^4 + a^3b^5e^4 + a^3e^4 * (- \\
& (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - 48a^3b^4cd^4 + a^2cd^4 * (- \\
& (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 * (- (\\
& 4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3 \\
& c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e^3 + 40 \\
& a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b \\
& * c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac - b^2)^5)^{1 \\
& /2} + 4abcd^3e * (- (4ac - b^2)^5)^{1/2} / (512 * (256a^7c^5 + a^3b^8c \\
& - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{3/4} + 64a^2c^7d^ \\
& 5 - 16b^2c^6d^5 + 64a^3b^4c^4e^5 - 192a^3c^5d^4e^4 + 16b^3c^5d^4e \\
& e - 16a^2b^3c^3e^5 - 128a^2c^6d^3e^2 - 64ab^6c^6d^4e + 16ab^4 \\
& c^3d^4e^4 + 32ab^2c^5d^3e^2 - 64ab^3c^4d^2e^3 + 256a^2b^5c^5d^2 \\
& * e^3 - 16a^2b^2c^4d^4e^4) + x * (8c^7d^6 - 8a^3c^4e^6 + 8a^2c^6d^4e \\
& ^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - 16b^3c^ \\
& 4d^3e^3 + 4b^4c^3d^2e^4 - 24b^6c^6d^5e - 16ab^6c^5d^3e^3 - 8ab \\
& ^3c^3d^4e^5 + 8a^2b^6c^4d^4e^5 + 16ab^2c^4d^2e^4) * (- (b^7cd^4 + a^ \\
& 3b^5e^4 + a^3e^4 * (- (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - 48a^3b^ \\
& c^4d^4 + a^2cd^4 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2 \\
& c^2e^4 - b^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c \\
& ^3d^3e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - \\
& 8a^3b^4cd^3e^3 + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^
\end{aligned}$$

$$\begin{aligned}
& 2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2 \\
& *(-4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(2 \\
& 56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4) \\
&))^{(1/4)} + (((-b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3* \\
& e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a \\
& ^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^ \\
& 2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e*(-(4 \\
& *a*c - b^2)^5)^{(1/2)}/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a \\
& ^5*b^4*c^3 - 256*a^6*b^2*c^4)))^{(1/4)}*(((-b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4 \\
& *(-4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2* \\
& b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 \\
& + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a \\
& ^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^7*c^5 + a^3*b \\
& ^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^{(1/4)}*(262144*a \\
& ^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e \\
& + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^ \\
& 4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^ \\
& 2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 81 \\
& 92*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4 \\
& *c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c \\
& ^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40* \\
& a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b* \\
& c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^7*c^5 + a^3*b^8*c \\
& - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^{(3/4)} + 64*a*c^7*d^5 \\
& - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e \\
& - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c \\
& ^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2* \\
& e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^ \\
& 2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4 \\
& *d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^ \\
& 3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-b^7*c*d^4 + a^3 \\
& *b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c \\
& ^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^ \\
& 2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^ \\
& 3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8
\end{aligned}$$

$$\begin{aligned}
& a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b^3 c^3 d^2 e^2 + 64 a^4 b^2 c^2 d^3 e^3 - 6 a^2 c d^2 e^2 * \\
& (- (4 a c - b^2)^5)^{(1/2)} + 4 a b c d^3 e * (- (4 a c - b^2)^5)^{(1/2)} / (512 * (256 a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4)) \\
&)^{(1/4)}) * (- (b^7 c d^4 + a^3 b^5 e^4 + a^3 e^4 * (- (4 a c - b^2)^5)^{(1/2)} - 11 a b^5 c^2 d^4 - 48 a^3 b^3 c^4 d^4 - \\
& 1 a b^5 c^2 d^4 - 48 a^3 b^3 c^4 d^4 + a c^2 d^4 * (- (4 a c - b^2)^5)^{(1/2)} - 8 a^4 b^3 c e^4 + 16 a^5 b^3 c^2 e^4 - b^2 c d^4 * (- (4 a c - b^2)^5)^{(1/2)} + 12 \\
& 8 a^4 c^4 d^3 e - 128 a^5 c^3 d^3 e^3 + 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 \\
& b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b^3 c^3 d^2 e^2 + 64 a^4 b^2 c^2 d^3 e^3 - 6 a^2 c d^2 e^2 * (- (4 a c - b^2)^5)^{(1/2)} + 4 a b c d^3 e * (- (4 a \\
& c - b^2)^5)^{(1/2)} / (512 * (256 a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4))^{(1/4)} * 2i - \operatorname{atan}(\dots) \\
& (- (b^7 c d^4 + a^3 b^5 e^4 - a^3 e^4 * (- (4 a c - b^2)^5)^{(1/2)} - 11 a b^5 c^2 d^4 - 48 a^3 b^3 c^4 d^4 - \\
& a c^2 d^4 * (- (4 a c - b^2)^5)^{(1/2)} - 8 a^4 b^3 c e^4 + 16 a^5 b^3 c^2 e^4 + b^2 c d^4 * (- (4 a c - b^2)^5)^{(1/2)} + 128 a^4 c^4 d^3 e - 128 a^5 c^3 d^3 e^3 + \\
& 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e \\
& e + 96 a^4 b^3 c^3 d^2 e^2 + 64 a^4 b^2 c^2 d^3 e^3 + 6 a^2 c d^2 e^2 * (- (4 a c - b^2)^5)^{(1/2)} - 4 a b c d^3 e * (- (4 a c - b^2)^5)^{(1/2)} / (512 * (256 a^7 c^5 \\
& + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4))^{(1/4)} * (\\
& (- (b^7 c d^4 + a^3 b^5 e^4 - a^3 e^4 * (- (4 a c - b^2)^5)^{(1/2)} - 11 a b^5 c^2 d^4 - 48 a^3 b^3 c^4 d^4 - \\
& a c^2 d^4 * (- (4 a c - b^2)^5)^{(1/2)} - 8 a^4 b^3 c e^4 + 16 a^5 b^3 c^2 e^4 + b^2 c d^4 * (- (4 a c - b^2)^5)^{(1/2)} + 128 a^4 c^4 d^3 e - 128 a^5 c^3 d^3 e^3 + \\
& 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b^3 c^3 d^2 e^2 + \\
& 64 a^4 b^2 c^2 d^3 e^3 + 6 a^2 c d^2 e^2 * (- (4 a c - b^2)^5)^{(1/2)} - 4 a b c d^3 e * (- (4 a c - b^2)^5)^{(1/2)} / (512 * (256 a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + \\
& 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4))^{(1/4)} * (262144 a^5 c^7 e - 49152 a^2 b^5 c^5 d + 196608 a^3 b^3 c^6 d - 4096 a^2 b^6 c^4 e + 49152 a^3 b^4 c^5 e - 196608 a^4 b^2 c^6 e \\
& + 4096 a b^7 c^4 d - 262144 a^4 b^3 c^7 d) + x * (1024 b^7 c^4 d^2 - 11264 a b^5 c^5 d^2 - 49152 a^3 b^3 c^7 d^2 + 16384 a^4 b^3 c^6 e^2 + 40960 a^2 b^3 \\
& c^6 d^2 + 1024 a^2 b^5 c^4 e^2 - 8192 a^3 b^3 c^5 e^2 + 65536 a^4 c^7 d e - 2048 a b^6 c^4 d e + 20480 a^2 b^4 c^5 d e - 65536 a^3 b^2 c^6 d e)) * (- (b \\
& ^7 c d^4 + a^3 b^5 e^4 - a^3 e^4 * (- (4 a c - b^2)^5)^{(1/2)} - 11 a b^5 c^2 d^4 - 48 a^3 b^3 c^4 d^4 - a c^2 d^4 * (- (4 a c - b^2)^5)^{(1/2)} - 8 a^4 b^3 c e^4 \\
& + 16 a^5 b^3 c^2 e^4 + b^2 c d^4 * (- (4 a c - b^2)^5)^{(1/2)} + 128 a^4 c^4 d^3 e - 128 a^5 c^3 d^3 e^3 + 40 a^2 b^3 c^3 d^4 - 4 a b^6 c d^3 e - 48 a^3 b^3 c^2 d^2 e^2 - 8 a^3 b^4 c d e^3 + \\
& 40 a^2 b^4 c^2 d^3 e + 6 a^2 b^5 c d^2 e^2 - 128 a^3 b^2 c^3 d^3 e + 96 a^4 b^3 c^3 d^2 e^2 + 64 a^4 b^2 c^2 d^3 e^3 + 6 a^2 c d^2 e^2 * (- (4 a c - b^2)^5)^{(1/2)} - 4 a b c d^3 e * (- (4 a c - b^2)^5)^{(1/2)} / \\
& (512 * (256 a^7 c^5 + a^3 b^8 c - 16 a^4 b^6 c^2 + 96 a^5 b^4 c^3 - 256 a^6 b^2 c^4))^{(3/4)} + 64 a c^7 d^5 - 16 b^2 c^6 d^5 + 64 a^3 b^3 c^4 e^5 - \\
& 192 a^3 c^5 d e^4 + 16 b^3 c^5 d^4 e - 16 a^2 b^3 c^3 e^5 - 128 a^2 c^6 d^3
\end{aligned}$$

$$\begin{aligned}
& *e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a* \\
& b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7* \\
& d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e \\
& ^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6 \\
& *d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a* \\
& b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(\\
& 1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(\\
& 1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1 \\
& /2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6* \\
& c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e \\
& + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64* \\
& a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3* \\
& e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 \\
& + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i - (((-b^7*c*d^4 + a^3*b^5*e^ \\
& 4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 \\
& - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + \\
& b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 \\
& + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^ \\
& 4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^ \\
& 3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a* \\
& c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c \\
& ^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4) \\
& *(((- (b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5 \\
& *c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^ \\
& 3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c \\
& ^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^ \\
& 3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c* \\
& d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e \\
& ^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^ \\
& 2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^ \\
& 3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 1966 \\
& 08*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^ \\
& 2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11 \\
& 264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b \\
& ^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d* \\
& e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(- \\
& (b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2* \\
& d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e \\
& ^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^ \\
& 3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3 \\
& *c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e \\
& ^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + \\
& 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5) \\
& ^1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 2 \\
& 56*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5
\end{aligned}$$

$$\begin{aligned}
& - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4)*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*1i)/(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 -
\end{aligned}$$

$$\begin{aligned}
& 256*a^6*b^2*c^4))^{(3/4)} + 64*a*c^7*d^5 - 16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e - 16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)} + (((-b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)})*(((b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)}*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 1264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})
\end{aligned}$$

$$\begin{aligned}
&)^{(1/2)}) / (512 * (256 * a^7 * c^5 + a^3 * b^8 * c - 16 * a^4 * b^6 * c^2 + 96 * a^5 * b^4 * c^3 - \\
& 256 * a^6 * b^2 * c^4))^{(3/4)} + 64 * a * c^7 * d^5 - 16 * b^2 * c^6 * d^5 + 64 * a^3 * b * c^4 * e^5 \\
& - 192 * a^3 * c^5 * d * e^4 + 16 * b^3 * c^5 * d^4 * e - 16 * a^2 * b^3 * c^3 * e^5 - 128 * a^2 * c^6 * \\
& d^3 * e^2 - 64 * a * b * c^6 * d^4 * e + 16 * a * b^4 * c^3 * d * e^4 + 32 * a * b^2 * c^5 * d^3 * e^2 - 64 \\
& * a * b^3 * c^4 * d^2 * e^3 + 256 * a^2 * b * c^5 * d^2 * e^3 - 16 * a^2 * b^2 * c^4 * d * e^4) - x * (8 * c \\
& ^7 * d^6 - 8 * a^3 * c^4 * e^6 + 8 * a * c^6 * d^4 * e^2 + 4 * a^2 * b^2 * c^3 * e^6 - 8 * a^2 * c^5 * d^ \\
& 2 * e^4 + 28 * b^2 * c^5 * d^4 * e^2 - 16 * b^3 * c^4 * d^3 * e^3 + 4 * b^4 * c^3 * d^2 * e^4 - 24 * b * \\
& c^6 * d^5 * e - 16 * a * b * c^5 * d^3 * e^3 - 8 * a * b^3 * c^3 * d * e^5 + 8 * a^2 * b * c^4 * d * e^5 + 16 \\
& * a * b^2 * c^4 * d^2 * e^4)) * (- (b^7 * c * d^4 + a^3 * b^5 * e^4 - a^3 * e^4 * (- (4 * a * c - b^2)^5 \\
&)^{(1/2)} - 11 * a * b^5 * c^2 * d^4 - 48 * a^3 * b * c^4 * d^4 - a * c^2 * d^4 * (- (4 * a * c - b^2)^5 \\
&)^{(1/2)} - 8 * a^4 * b^3 * c * e^4 + 16 * a^5 * b * c^2 * e^4 + b^2 * c * d^4 * (- (4 * a * c - b^2)^5 \\
&)^{(1/2)} + 128 * a^4 * c^4 * d^3 * e - 128 * a^5 * c^3 * d * e^3 + 40 * a^2 * b^3 * c^3 * d^4 - 4 * a * b \\
& ^6 * c * d^3 * e - 48 * a^3 * b^3 * c^2 * d^2 * e^2 - 8 * a^3 * b^4 * c * d * e^3 + 40 * a^2 * b^4 * c^2 * d^ \\
& 3 * e + 6 * a^2 * b^5 * c * d^2 * e^2 - 128 * a^3 * b^2 * c^3 * d^3 * e + 96 * a^4 * b * c^3 * d^2 * e^2 + \\
& 64 * a^4 * b^2 * c^2 * d * e^3 + 6 * a^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * a * b * c * d \\
& ^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^7 * c^5 + a^3 * b^8 * c - 16 * a^4 * b^6 * c \\
& ^2 + 96 * a^5 * b^4 * c^3 - 256 * a^6 * b^2 * c^4))^{(1/4)}) * (- (b^7 * c * d^4 + a^3 * b^5 * e^4 \\
& - a^3 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 11 * a * b^5 * c^2 * d^4 - 48 * a^3 * b * c^4 * d^4 - \\
& a * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a^4 * b^3 * c * e^4 + 16 * a^5 * b * c^2 * e^4 + \\
& b^2 * c * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 128 * a^4 * c^4 * d^3 * e - 128 * a^5 * c^3 * d * e^3 \\
& + 40 * a^2 * b^3 * c^3 * d^4 - 4 * a * b^6 * c * d^3 * e - 48 * a^3 * b^3 * c^2 * d^2 * e^2 - 8 * a^3 * b^4 \\
& * c * d * e^3 + 40 * a^2 * b^4 * c^2 * d^3 * e + 6 * a^2 * b^5 * c * d^2 * e^2 - 128 * a^3 * b^2 * c^3 * d^3 \\
& * e + 96 * a^4 * b * c^3 * d^2 * e^2 + 64 * a^4 * b^2 * c^2 * d * e^3 + 6 * a^2 * c * d^2 * e^2 * (- (4 * a * c \\
& - b^2)^5)^{(1/2)} - 4 * a * b * c * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^7 * c^ \\
& 5 + a^3 * b^8 * c - 16 * a^4 * b^6 * c^2 + 96 * a^5 * b^4 * c^3 - 256 * a^6 * b^2 * c^4))^{(1/4)} * \\
& 2i - 2 * \operatorname{atan}(\left(\left(- (b^7 * c * d^4 + a^3 * b^5 * e^4 + a^3 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \right. \right. \\
& - 11 * a * b^5 * c^2 * d^4 - 48 * a^3 * b * c^4 * d^4 + a * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& - 8 * a^4 * b^3 * c * e^4 + 16 * a^5 * b * c^2 * e^4 - b^2 * c * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
& + 128 * a^4 * c^4 * d^3 * e - 128 * a^5 * c^3 * d * e^3 + 40 * a^2 * b^3 * c^3 * d^4 - 4 * a * b^6 * c * d^ \\
& 3 * e - 48 * a^3 * b^3 * c^2 * d^2 * e^2 - 8 * a^3 * b^4 * c * d * e^3 + 40 * a^2 * b^4 * c^2 * d^3 * e + 6 \\
& * a^2 * b^5 * c * d^2 * e^2 - 128 * a^3 * b^2 * c^3 * d^3 * e + 96 * a^4 * b * c^3 * d^2 * e^2 + 64 * a^4 * \\
& b^2 * c^2 * d * e^3 - 6 * a^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * a * b * c * d^3 * e * (- \\
& (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^7 * c^5 + a^3 * b^8 * c - 16 * a^4 * b^6 * c^2 + 96 \\
& * a^5 * b^4 * c^3 - 256 * a^6 * b^2 * c^4))^{(1/4)} * \left(\left(- (b^7 * c * d^4 + a^3 * b^5 * e^4 + a^3 * \\
& e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 11 * a * b^5 * c^2 * d^4 - 48 * a^3 * b * c^4 * d^4 + a * c^2 * \\
& d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a^4 * b^3 * c * e^4 + 16 * a^5 * b * c^2 * e^4 - b^2 * c * d \\
& ^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 128 * a^4 * c^4 * d^3 * e - 128 * a^5 * c^3 * d * e^3 + 40 * a^ \\
& 2 * b^3 * c^3 * d^4 - 4 * a * b^6 * c * d^3 * e - 48 * a^3 * b^3 * c^2 * d^2 * e^2 - 8 * a^3 * b^4 * c * d * e^ \\
& 3 + 40 * a^2 * b^4 * c^2 * d^3 * e + 6 * a^2 * b^5 * c * d^2 * e^2 - 128 * a^3 * b^2 * c^3 * d^3 * e + 96 \\
& * a^4 * b * c^3 * d^2 * e^2 + 64 * a^4 * b^2 * c^2 * d * e^3 - 6 * a^2 * c * d^2 * e^2 * (- (4 * a * c - b^2) \\
& ^5)^{(1/2)} + 4 * a * b * c * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)}) / (512 * (256 * a^7 * c^5 + a^3 \\
& * b^8 * c - 16 * a^4 * b^6 * c^2 + 96 * a^5 * b^4 * c^3 - 256 * a^6 * b^2 * c^4))^{(1/4)} * (262144 \\
& * a^5 * c^7 * e - 49152 * a^2 * b^5 * c^5 * d + 196608 * a^3 * b^3 * c^6 * d - 4096 * a^2 * b^6 * c^4 * \\
& e + 49152 * a^3 * b^4 * c^5 * e - 196608 * a^4 * b^2 * c^6 * e + 4096 * a * b^7 * c^4 * d - 262144 * \\
& a^4 * b * c^7 * d) * i + x * (1024 * b^7 * c^4 * d^2 - 11264 * a * b^5 * c^5 * d^2 - 49152 * a^3 * b * c
\end{aligned}$$

$$\begin{aligned}
& ^7d^2 + 16384a^4b^3c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 \\
& - 8192a^3b^3c^5e^2 + 65536a^4c^7d^2e - 2048a^2b^6c^4d^2e + 20480a^2 \\
& 2b^4c^5d^2e - 65536a^3b^2c^6d^2e)) * (- (b^7cd^4 + a^3b^5e^4 + a^3e^4 \\
& 4 * (- (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 + ac^2d^4 \\
& 4 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 \\
& * (- (4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 + 40a^2 * \\
& b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^2e^3 \\
& + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4 \\
& b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac - b^2)^5 \\
&)^{1/2} + 4ab^3cd^3e * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^7c^5 + a^3b \\
& ^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{3/4} * i - 64a \\
& c^7d^5 + 16b^2c^6d^5 - 64a^3b^3c^4e^5 + 192a^3c^5d^2e^4 - 16b^3c^5 \\
& d^4e + 16a^2b^3c^3e^5 + 128a^2c^6d^3e^2 + 64ab^3c^6d^4e - 16 \\
& a^2b^4c^3d^2e^4 - 32ab^2c^5d^3e^2 + 64ab^3c^4d^2e^3 - 256a^2b^3 \\
& c^5d^2e^3 + 16a^2b^2c^4d^2e^4) * i - x * (8c^7d^6 - 8a^3c^4e^6 + 8a \\
& c^6d^4e^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - \\
& 16b^3c^4d^3e^3 + 4b^4c^3d^2e^4 - 24b^3c^6d^5e - 16ab^3c^5d^3e^3 \\
& - 8a^2b^3c^3d^2e^5 + 8a^2b^3c^4d^2e^5 + 16ab^2c^4d^2e^4) * (- (b^7 * \\
& cd^4 + a^3b^5e^4 + a^3e^4 * (- (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - \\
& 48a^3b^3c^4d^4 + ac^2d^4 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + \\
& 16a^5b^2c^2e^4 - b^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - \\
& 128a^5c^3d^2e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2 \\
& e^2 - 8a^3b^4cd^2e^3 + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - \\
& 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2 \\
& cd^2e^2 * (- (4ac - b^2)^5)^{1/2} + 4ab^3cd^3e * (- (4ac - b^2)^5)^{1/2} \\
&) / (512 * (256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6 \\
& b^2c^4))^{1/4} - ((- (b^7cd^4 + a^3b^5e^4 + a^3e^4 * (- (4ac - b^2)^5)^{1/2} \\
& - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 + ac^2d^4 * (- (4ac - b^2)^5)^{1/2} \\
& - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 * (- (4ac - b^2)^5)^{1/2} \\
&)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 + 40a^2b^3c^3d^4 - 4ab^6 \\
& cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^2e^3 + 40a^2b^4c^2d^3 \\
& e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + \\
& 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac - b^2)^5)^{1/2} + 4ab^3cd^3 \\
& e * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^7c^5 + a^3b^8c - 16a^4b^6c^2 \\
& + 96a^5b^4c^3 - 256a^6b^2c^4))^{1/4} * (((- (b^7cd^4 + a^3b^5e^4 \\
& + a^3e^4 * (- (4ac - b^2)^5)^{1/2} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 \\
& + ac^2d^4 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - \\
& b^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 \\
& + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4 \\
& cd^2e^3 + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3 \\
& e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 * (- (4ac \\
& - b^2)^5)^{1/2} + 4ab^3cd^3e * (- (4ac - b^2)^5)^{1/2}) / (512 * (256a^7c^5 \\
& + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{1/4} \\
& * (262144a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2 * \\
& b^6c^4e + 49152a^3b^4c^5e - 196608a^4b^2c^6e + 4096a^2b^7c^4d -
\end{aligned}$$

$$\begin{aligned}
& 262144*a^4*b*c^7*d)*1i - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152 \\
& *a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5 \\
& *c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + \\
& 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 \\
& + a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + \\
& a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b \\
& ^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + \\
& 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4* \\
& c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3* \\
& e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^7*c^5 \\
& + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^{(3/4)}*1 \\
& i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - \\
& 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^ \\
& 4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 25 \\
& 6*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i + x*(8*c^7*d^6 - 8*a^3*c^4*e \\
& ^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d \\
& ^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c \\
& ^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4)) \\
& *(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c \\
& ^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3* \\
& c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4 \\
& *d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3* \\
& b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^ \\
& 2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 \\
& - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2) \\
& ^5)^{(1/2)))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 \\
& - 256*a^6*b^2*c^4)))^{(1/4)})/(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^ \\
& 4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b \\
& ^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d \\
& ^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(256*a^7*c^5 + a^3*b^8*c - 16* \\
& a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^{(1/4)}*(((-(b^7*c*d^4 + a^ \\
& 3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b* \\
& c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c \\
& ^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c \\
& ^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - \\
& 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^ \\
& 2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(2 \\
& 56*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4) \\
&))^{(1/4)}*(262144*a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4
\end{aligned}$$

$$\begin{aligned}
& 096*a^2*b^6*c^4*e + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b*c^7*d) * 1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 \\
& - 49152*a^3*b*c^7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024 \\
& *a^2*b^5*c^4*e^2 - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4 \\
& *d*e + 20480*a^2*b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e)) * (- (b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4 \\
& *d^4 + a*c^2*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2 \\
& *e^4 - b^2*c*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3 \\
& *d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8* \\
& a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3 \\
& *d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256 \\
& *a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)) \\
& ^{(3/4)} * 1i - 64*a*c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5* \\
& d*e^4 - 16*b^3*c^5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a* \\
& b*c^6*d^4*e - 16*a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2* \\
& e^3 - 256*a^2*b*c^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4) * 1i - x*(8*c^7*d^6 - 8*a \\
& ^3*c^4*e^6 + 8*a*c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b \\
& ^2*c^5*d^4*e^2 - 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - \\
& 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d \\
& ^2*e^4)) * (- (b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 11 \\
& *a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 8* \\
& a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 128 \\
& *a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - \\
& 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2* \\
& b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2 \\
& *d*e^3 - 6*a^2*c*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e * (- (4*a* \\
& c - b^2)^5)^{(1/2)}) / (512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5* \\
& b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)} * 1i + (((- (b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 \\
& - 48*a^3*b*c^4*d^4 + a*c^2*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e \\
& - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 4*a*b*c*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)}) / (512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\left(\left(6b^2c^4 \right) \right)^{1/4} \left(262144a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2b^6c^4e + 49152a^3b^4c^5e - 196608a^4b^2c^6e + 4096ab^7c^4d - 262144a^4b^6c^7d \right) \right) \right) i - x \left(1024b^7c^4d^2 - 11264ab^5c^5d^2 - 49152a^3b^6c^7d^2 + 16384a^4b^5c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 - 8192a^3b^3c^5e^2 + 65536a^4c^7d^2e - 2048ab^6c^4d^2e + 20480a^2b^4c^5d^2e - 65536a^3b^2c^6d^2e \right) \left(- \left(b^7cd^4 + a^3b^5e^4 + a^3e^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 + ac^2d^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 \left(- (4ac - b^2)^5 \right)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 \left(- (4ac - b^2)^5 \right)^{1/2} + 4ab^2cd^3e \left(- (4ac - b^2)^5 \right)^{1/2} \right) \right) / \left(512 \left(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4 \right) \right)^{3/4} i - 64aac^7d^5 + 16b^2c^6d^5 - 64a^3b^4c^4e^5 + 192a^3c^5d^4e - 16b^3c^5d^4e + 16a^2b^3c^3e^5 + 128a^2c^6d^3e^2 + 64ab^2c^6d^4e - 16ab^4c^3d^4e - 32ab^2c^5d^3e^2 + 64ab^3c^4d^2e^3 - 256a^2b^5d^2e^3 + 16a^2b^2c^4d^4e \right) i + x \left(8c^7d^6 - 8a^3c^4e^6 + 8aac^6d^4e^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - 16b^3c^4d^3e^3 + 4b^4c^3d^2e^4 - 24b^6c^5d^5e - 16ab^5c^5d^3e^3 - 8ab^3c^3d^4e^5 + 8a^2b^4cd^5e + 16ab^2c^4d^2e^4 \right) \left(- \left(b^7cd^4 + a^3b^5e^4 + a^3e^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 + ac^2d^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 \left(- (4ac - b^2)^5 \right)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 - 6a^2cd^2e^2 \left(- (4ac - b^2)^5 \right)^{1/2} + 4ab^2cd^3e \left(- (4ac - b^2)^5 \right)^{1/2} \right) \right) / \left(512 \left(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4 \right) \right)^{1/4} i \right) \left(- \left(b^7cd^4 + a^3b^5e^4 + a^3e^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 + ac^2d^4 \left(- (4ac - b^2)^5 \right)^{1/2} - 8a^4b^3c^2e^4 + 16a^5b^2c^2e^4 - b^2cd^4 \left(- (4ac - b^2)^5 \right)^{1/2} + 128a^4c^4d^3e - 128a^5c^3d^2e^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^2c^3d^2e^2 + 64a^4b^2c^2d^2e^3 + 6a^2cd^2e^2 \left(- (4ac - b^2)^5 \right)^{1/2} - 4ab^2cd^3e \left(- (4ac - b^2)^5 \right)^{1/2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96* \\
& a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)}*(((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e \\
& ^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d \\
& ^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2 \\
& *b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 \\
& + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96* \\
& a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3* \\
& b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)}*(262144* \\
& a^5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e \\
& + 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a \\
& ^4*b*c^7*d)*1i + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^ \\
& 7*d^2 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 \\
& - 8192*a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2 \\
& *b^4*c^5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4* \\
& (- (4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b \\
& ^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + \\
& 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^ \\
& 4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5) \\
& ^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(256*a^7*c^5 + a^3*b^ \\
& 8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(3/4)}*1i - 64*a* \\
& c^7*d^5 + 16*b^2*c^6*d^5 - 64*a^3*b*c^4*e^5 + 192*a^3*c^5*d*e^4 - 16*b^3*c^ \\
& 5*d^4*e + 16*a^2*b^3*c^3*e^5 + 128*a^2*c^6*d^3*e^2 + 64*a*b*c^6*d^4*e - 16* \\
& a*b^4*c^3*d*e^4 - 32*a*b^2*c^5*d^3*e^2 + 64*a*b^3*c^4*d^2*e^3 - 256*a^2*b*c \\
& ^5*d^2*e^3 + 16*a^2*b^2*c^4*d*e^4)*1i - x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a* \\
& c^6*d^4*e^2 + 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - \\
& 16*b^3*c^4*d^3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^ \\
& 3 - 8*a*b^3*c^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c \\
& *d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 11*a*b^5*c^2*d^4 - \\
& 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b^3*c*e^4 + 1 \\
& 6*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - \\
& 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d \\
& ^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 1 \\
& 28*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2* \\
& c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6 \\
& *b^2*c^4))^{(1/4)} - ((-(b^7*c*d^4 + a^3*b^5*e^4 - a^3*e^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 - a*c^2*d^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 + b^2*c*d^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b \\
& ^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^ \\
& 3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 +
\end{aligned}$$

$$\begin{aligned}
& 64a^4b^2c^2de^3 + 6a^2cd^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^3cd^3e(-4ac - b^2)^5)^{(1/2)}/(512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)} * (((-b^7cd^4 + a^3b^5e^4 - a^3e^4(-4ac - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^4c^4d^4 - ac^2d^4(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3ce^4 + 16a^5b^2e^4 + b^2cd^4(-4ac - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3de^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2de^3 + 6a^2cd^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^3cd^3e(-4ac - b^2)^5)^{(1/2)}/(512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)} * (262144a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2b^6c^4e + 49152a^3b^4c^5e - 196608a^4b^2c^6e + 4096ab^7c^4d - 262144a^4b^3c^7d) * 1i - x(1024b^7c^4d^2 - 11264ab^5c^5d^2 - 49152a^3b^3c^7d^2 + 16384a^4b^3c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 - 8192a^3b^3c^5e^2 + 65536a^4c^7de - 2048ab^6c^4de + 20480a^2b^4c^5de - 65536a^3b^2c^6de) * (-b^7cd^4 + a^3b^5e^4 - a^3e^4(-4ac - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^4c^4d^4 - ac^2d^4(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3ce^4 + 16a^5b^2e^4 + b^2cd^4(-4ac - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3de^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2de^3 + 6a^2cd^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^3cd^3e(-4ac - b^2)^5)^{(1/2)}/(512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(3/4)} * 1i - 64ac^7d^5 + 16b^2c^6d^5 - 64a^3b^3c^4e^5 + 192a^3c^5d^4e - 16b^3c^5d^4e + 16a^2b^3c^3e^5 + 128a^2c^6d^3e^2 + 64ab^3c^6d^4e - 16ab^4c^3de^4 - 32ab^2c^5d^3e^2 + 64ab^3c^4d^2e^3 - 256a^2b^3c^5d^2e^3 + 16a^2b^2c^4de^4) * 1i + x(8c^7d^6 - 8a^3c^4e^6 + 8ac^6d^4e^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - 16b^3c^4d^3e^3 + 4b^4c^3d^2e^4 - 24b^3c^6d^5e - 16ab^3c^5d^3e^3 - 8ab^3c^3de^5 + 8a^2b^3c^4de^5 + 16ab^2c^4d^2e^4) * (-b^7cd^4 + a^3b^5e^4 - a^3e^4(-4ac - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^4c^4d^4 - ac^2d^4(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3ce^4 + 16a^5b^2e^4 + b^2cd^4(-4ac - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3de^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^4c^2d^3e + 6a^2b^5cd^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2de^3 + 6a^2cd^2e^2(-4ac - b^2)^5)^{(1/2)} - 4ab^3cd^3e(-4ac - b^2)^5)^{(1/2)}/(512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)}/(((-b^7cd^4 + a^3b^5e^4 - a^3e^4(-4ac - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^4c^4d^4 - ac^2d^4(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3ce^4 + 16a^5b^2e^4 + b^2cd^4(-4ac - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3de^3 + 40a^2b^3c^3d^4 - 4ab^6cd^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4cd^3e + 40a^2b^
\end{aligned}$$

$$\begin{aligned}
& b^3c^3d^4 - 4ab^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^3d^3e^3 \\
& + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2d^3e^3 + 6a^2c^2d^2e^2(-4a^3c - b^2)^5 \\
&)^{(1/2)} - 4ab^6c^3d^3e^3(-4a^3c - b^2)^5)^{(1/2)} / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)} * (((-b^7c^3d^4 + a^3b^5e^4 - a^3e^4(-4a^3c - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 - a^3c^2d^4(-4a^3c - b^2)^5)^{(1/2)} - 8a^4b^3c^3e^4 + 16a^5b^3c^2e^4 + b^2c^3d^4(-4a^3c - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4ab^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^3d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2d^3e^3 + 6a^2c^2d^2e^2(-4a^3c - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4a^3c - b^2)^5)^{(1/2)})) / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)} * (262144a^5c^7e - 49152a^2b^5c^5d + 196608a^3b^3c^6d - 4096a^2b^6c^4e + 49152a^3b^4c^5e - 196608a^4b^2c^6e + 4096ab^7c^4d - 262144a^4b^3c^7d) * 1i - x(1024b^7c^4d^2 - 11264ab^5c^5d^2 - 49152a^3b^3c^7d^2 + 16384a^4b^3c^6e^2 + 40960a^2b^3c^6d^2 + 1024a^2b^5c^4e^2 - 8192a^3b^3c^5e^2 + 65536a^4c^7d^2e - 2048ab^6c^4d^2e + 20480a^2b^4c^5d^2e - 65536a^3b^2c^6d^2e) * (-b^7c^3d^4 + a^3b^5e^4 - a^3e^4(-4a^3c - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 - a^3c^2d^4(-4a^3c - b^2)^5)^{(1/2)} - 8a^4b^3c^3e^4 + 16a^5b^3c^2e^4 + b^2c^3d^4(-4a^3c - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4ab^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^3d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2d^3e^3 + 6a^2c^2d^2e^2(-4a^3c - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4a^3c - b^2)^5)^{(1/2)})) / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(3/4)} * 1i - 64a^3c^7d^5 + 16b^2c^6d^5 - 64a^3b^3c^4e^5 + 192a^3c^5d^4e - 16b^3c^5d^4e + 16a^2b^3c^3e^5 + 128a^2c^6d^3e^2 + 64ab^3c^6d^4e - 16ab^4c^3d^4e - 32ab^2c^5d^3e^2 + 64ab^3c^4d^2e^3 - 256a^2b^3c^5d^2e^3 + 16a^2b^2c^4d^4e^4) * 1i + x(8c^7d^6 - 8a^3c^4e^6 + 8a^3c^6d^4e^2 + 4a^2b^2c^3e^6 - 8a^2c^5d^2e^4 + 28b^2c^5d^4e^2 - 16b^3c^4d^3e^3 + 4b^4c^3d^2e^4 - 24b^3c^6d^5e - 16ab^3c^5d^3e^3 - 8ab^3c^3d^4e^5 + 8a^2b^3c^4d^4e^5 + 16ab^2c^4d^2e^4) * (-b^7c^3d^4 + a^3b^5e^4 - a^3e^4(-4a^3c - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 - a^3c^2d^4(-4a^3c - b^2)^5)^{(1/2)} - 8a^4b^3c^3e^4 + 16a^5b^3c^2e^4 + b^2c^3d^4(-4a^3c - b^2)^5)^{(1/2)} + 128a^4c^4d^3e - 128a^5c^3d^3e^3 + 40a^2b^3c^3d^4 - 4ab^6c^3d^3e - 48a^3b^3c^2d^2e^2 - 8a^3b^4c^3d^3e + 40a^2b^4c^2d^3e + 6a^2b^5c^2d^2e^2 - 128a^3b^2c^3d^3e + 96a^4b^3c^3d^2e^2 + 64a^4b^2c^2d^3e^3 + 6a^2c^2d^2e^2(-4a^3c - b^2)^5)^{(1/2)} - 4ab^6c^3d^3e^3(-4a^3c - b^2)^5)^{(1/2)})) / (512(256a^7c^5 + a^3b^8c - 16a^4b^6c^2 + 96a^5b^4c^3 - 256a^6b^2c^4))^{(1/4)} * 1i) * (-b^7c^3d^4 + a^3b^5e^4 - a^3e^4(-4a^3c - b^2)^5)^{(1/2)} - 11ab^5c^2d^4 - 48a^3b^3c^4d^4 - a^3c^2d^4(-4a^3c - b^2)^5)^{(1/2)} - 8a^4b^3c^3e^4 + 16a^5b^3c^2e^4 +
\end{aligned}$$

$$b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 + 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4))^{(1/4)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.48 \quad \int \frac{d+ex^4}{x(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=78

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} - \frac{d \log(a + bx^4 + cx^8)}{8a} + \frac{d \log(x)}{a}$$

[Out] d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd - 2ae) \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} - \frac{d \log(a + bx^4 + cx^8)}{8a} + \frac{d \log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)), x]

[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d._) + (e._)*(x_))/((a._) + (b._)*(x_) + (c._)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d._) + (e._)*(x_))^(m_)*((f._) + (g._)*(x_)))/((a._) + (b._)*(x_) + (c._)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^(m_)*((a_) + (c._)*(x_)^(n2_)) + (b._)*(x_)^(n_)]^(p_)*((d_) + (e._)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{d + ex^4}{x(a + bx^4 + cx^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{d + ex}{x(a + bx + cx^2)} dx, x, x^4 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{d}{ax} + \frac{-bd + ae - cdx}{a(a + bx + cx^2)} \right) dx, x, x^4 \right) \\
 &= \frac{d \log(x)}{a} + \frac{\text{Subst} \left(\int \frac{-bd + ae - cdx}{a + bx + cx^2} dx, x, x^4 \right)}{4a} \\
 &= \frac{d \log(x)}{a} - \frac{d \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^4 \right)}{8a} + \frac{(-bd + 2ae) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^4 \right)}{8a} \\
 &= \frac{d \log(x)}{a} - \frac{d \log(a + bx^4 + cx^8)}{8a} - \frac{(-bd + 2ae) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^4 \right)}{4a} \\
 &= \frac{(bd - 2ae) \tanh^{-1} \left(\frac{b + 2cx^4}{\sqrt{b^2 - 4ac}} \right)}{4a\sqrt{b^2 - 4ac}} + \frac{d \log(x)}{a} - \frac{d \log(a + bx^4 + cx^8)}{8a}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 1.03

$$\frac{d \log(x)}{a} - \frac{\text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 c d \log(x-\#1) - a e \log(x-\#1) + b d \log(x-\#1)}{2\#1^4 c + b} \&\right]}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)), x]

[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 &, (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &]/(4*a)

fricas [A] time = 2.40, size = 240, normalized size = 3.08

$$\left[\frac{(b^2 - 4ac)d \log(cx^8 + bx^4 + a) - 8(b^2 - 4ac)d \log(x) + \sqrt{b^2 - 4ac}(bd - 2ae) \log\left(\frac{2c^2x^8 + 2bcx^4 + b^2 - 2ac - (2cx^4 + b)\sqrt{b^2 - 4ac}}{cx^8 + bx^4 + a}\right)}{8(ab^2 - 4a^2c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a), x, algorithm="fricas")

[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) + sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), - 1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]

giac [A] time = 20.63, size = 78, normalized size = 1.00

$$-\frac{d \log(cx^8 + bx^4 + a)}{8a} + \frac{d \log(x^4)}{4a} - \frac{(bd - 2ae) \arctan\left(\frac{2cx^4 + b}{\sqrt{-b^2 + 4ac}}\right)}{4\sqrt{-b^2 + 4ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a), x, algorithm="giac")

[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)

maple [A] time = 0.01, size = 106, normalized size = 1.36

$$-\frac{bd \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{4\sqrt{4ac - b^2}a} + \frac{e \arctan\left(\frac{2cx^4 + b}{\sqrt{4ac - b^2}}\right)}{2\sqrt{4ac - b^2}} + \frac{d \ln(x)}{a} - \frac{d \ln(cx^8 + bx^4 + a)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x^4+d)/x/(c*x^8+b*x^4+a), x)$

[Out] $\frac{1}{a}d*\ln(x) - \frac{1}{8}d*\ln(c*x^8+b*x^4+a) / a + \frac{1}{2} / (4*a*c-b^2)^{(1/2)} * \arctan((2*c*x^4+b) / (4*a*c-b^2)^{(1/2)}) * e^{-1/4} / a / (4*a*c-b^2)^{(1/2)} * \arctan((2*c*x^4+b) / (4*a*c-b^2)^{(1/2)}) * b*d$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x^4+d)/x/(c*x^8+b*x^4+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 5.27, size = 8454, normalized size = 108.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^4)/(x*(a + b*x^4 + c*x^8)), x)$

[Out] $(d*\log(x))/a - (\log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d)) / (2*(16*a*b^2 - 64*a^2*c)) + (\text{atan}((128*a^5*x^4*((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b*c^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))))))))) / (2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)) / (2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e)) / (2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*c^5*d*e^2)) / (2*(16*a*b^2 - 64*a^2*c))) / (2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)) / (2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)) / (8*a*(4*a*c - b^2)^{(1/2)}) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)) / (16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)})) * (2*a*e - b*d)) / (8*a*(4*a*c - b^2)^{(1/2)}) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2) / (128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))) / (2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)) / (2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)) / (8*a*(4*a*c - b^2)^{(1/2)}) + ((4*b^2*d - 16*a*c*d)*(1$

$$\begin{aligned}
& 280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/((16*a*(16*a*b^2 - 64*a^2*c)*(4 \\
& *a*c - b^2)^{(1/2)}))*((4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a \\
& *e - b*d)*(((4*b^2*d - 16*a*c*d)*((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 460 \\
& 8*a*b^3*c^5)))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + \\
& 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5 \\
& *e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^{(1/2)}))/((8*a*(4*a*c - b^2)^{(1 \\
& /2)}))/((2*(16*a*b^2 - 64*a^2*c)) + (((((((2*a*e - b*d)*((4*b^2*d - 16*a*c \\
& *d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^ \\
& 5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^{(1/2)) + ((4*b \\
& ^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a \\
& *b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)}))*((2*a*e - b*d))/(8*a*(4*a*c - b^2)^{(1 \\
& /2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^ \\
& 2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))*((2*a*e - b*d))/(8*a*(4*a*c \\
& - b^2)^{(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2* \\
& a*e - b*d)^3)/(1024*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(3/2)}))*((2*a*e \\
& - b*d))/(8*a*(4*a*c - b^2)^{(1/2)) - (((((4*b^2*d - 16*a*c*d)*(((2*a*e - b* \\
& d)*((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 6 \\
& 4*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c \\
& - b^2)^{(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a \\
& *e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)}))*((4*b^2*d - 16* \\
& a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)* \\
& ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^ \\
& 2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - \\
& 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4* \\
& a*c - b^2)^{(1/2)}))/((2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - \\
& 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 46 \\
& 08*a*b^3*c^5)))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + \\
& 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c \\
& ^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^ \\
& 2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^{(1/2)}))*((2*a*e - b*d))/(8* \\
& a*(4*a*c - b^2)^{(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c \\
& ^5)*(2*a*e - b*d)^4)/(8192*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2))*((5*b \\
& ^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e) \\
&)/(32*a^5*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)) - (((((4*b^2*d \\
& - 16*a*c*d)*((((2*a*e - b*d)*((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608 \\
& *a*b^3*c^5)))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3 \\
& 456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^{(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b \\
& ^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c \\
& - b^2)^{(1/2)}))*((2*a*e - b*d))/(8*a*(4*a*c - b^2)^{(1/2)) + ((4*b^2*d - 16*a* \\
& c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 - \\
& 64*a^2*c)*(4*a*c - b^2)))*((2*a*e - b*d))/(8*a*(4*a*c - b^2)^{(1/2)) + ((4*b^ \\
& 2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^3)/(1024*a^3* \\
& (16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(3/2)})))/((2*(16*a*b^2 - 64*a^2*c)) - ((\\
& 4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((4*b^2*d - 1 \\
& 6*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b
\end{aligned}$$

$$\begin{aligned}
& ^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (8a(4ac - b^2)^{1/2}) + \\
& ((4b^2d - 16ac^2d)(1280b^5c^4 - 4608a^2b^3c^5)(2ae - bd)) / (16a^2(16ab^2 - 64a^2c)(4ac - b^2)^{1/2}) * (4b^2d - 16ac^2d) / (2(16ab^2 - 64a^2c)) + \\
& ((2ae - bd) * ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (2(16ab^2 - 64a^2c)) + 224b^3c^4e^2 - 864ab^2c^5e^2 - 432b^2c^5d^2e)) / (8a(4ac - b^2)^{1/2}) \\
&)) / (2(16ab^2 - 64a^2c)) + ((2ae - bd) * ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (2(16ab^2 - 64a^2c)) + 224b^3c^4e^2 - 864ab^2c^5e^2 - 432b^2c^5d^2e)) / (2(16ab^2 - 64a^2c)) + 72a^2c^5e^3 + 16b^2c^4e^3 + 108b^2c^5d^2e^2)) / (8a(4ac - b^2)^{1/2}) \\
&)) / (2(16ab^2 - 64a^2c)) + ((2ae - bd) * (11b^2c^4e^4 + 9c^5d^2e^3 - ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (2(16ab^2 - 64a^2c)) + 224b^3c^4e^2 - 864ab^2c^5e^2 - 432b^2c^5d^2e)) / (2(16ab^2 - 64a^2c)) + 72a^2c^5e^3 + 16b^2c^4e^3 + 108b^2c^5d^2e^2)) / (8a(4ac - b^2)^{1/2}) \\
&)) + ((2ae - bd) * ((4b^2d - 16ac^2d) * (((2ae - bd) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (8a(4ac - b^2)^{1/2}) + ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5) * (2ae - bd)) / (16a^2(16ab^2 - 64a^2c)(4ac - b^2)^{1/2})) * (2ae - bd)) / (8a(4ac - b^2)^{1/2}) + ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5) * (2ae - bd)^2) / (128a^2(16ab^2 - 64a^2c)(4ac - b^2))) / (2(16ab^2 - 64a^2c)) + ((2ae - bd) * (((2ae - bd) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (8a(4ac - b^2)^{1/2}) + ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5) * (2ae - bd)) / (16a^2(16ab^2 - 64a^2c)(4ac - b^2)^{1/2})) * (4b^2d - 16ac^2d)) / (2(16ab^2 - 64a^2c)) + ((2ae - bd) * (((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * (1280b^5c^4 - 4608a^2b^3c^5)) / (2(16ab^2 - 64a^2c)) + 576b^3c^5d - 1024b^4c^4e + 3456a^2b^2c^5e)) / (2(16ab^2 - 64a^2c)) + 224b^3c^4e^2 - 864ab^2c^5e^2 - 432b^2c^5d^2e)) / (8a(4ac - b^2)^{1/2})) / (8a(4ac - b^2)^{1/2}) - ((1280b^5c^4 - 4608a^2b^3c^5) * (2ae - bd)^5) / (32768a^5(4ac - b^2)^{5/2})) * (144a^3c^3d - 40b^6d + 8a^2b^5e - 488a^2b^2c^2d + 272a^2b^4c^2d - 40a^2b^3c^2e + 40a^3b^2c^2e)) / (256a^5c^4(4ac - b^2)^{1/2}(a^2e^2 - 20b^2d^2 + 81a^2c^2d^2 - a^2bd^2e)) * (4ac - b^2)^{5/2} / (16a^4c^4e^4 + b^4c^4d^4 + 24a^2b^2c^4d^2e^2 - 8a^2b^3c^4d^3e - 32a^3b^2c^4d^2e^3) + (4(4ac - b^2)^{5/2}(5b^5d - a^3c^2e - a^2b^4e - 24a^2b^3c^2d + 23a^2b^2c^2d + 3a^2b^2c^2e) * (c^4d^2e^4 + ((4b^2d - 16ac^2d) * (a^2c^4e^4 + ((4b^2d - 16ac^2d) * ((4b^2d - 16ac^2d) * (256b^4c^4d - 256a^2b^3c^4e + (128a^2b^4c^4(4b^2d - 16ac^2d)) / (16ab^2 - 64a^2c)
\end{aligned}$$

$$\begin{aligned}
&)) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + 96 * a * b^2 * c^4 * e^2 - 256 * b^3 * c^4 * d * e)) / (2 * (16 * \\
& a * b^2 - 64 * a^2 * c)) + 96 * b^2 * c^4 * d * e^2 - 16 * a * b * c^4 * e^3)) / (2 * (16 * a * b^2 - 64 * \\
& a^2 * c)) - 16 * b * c^4 * d * e^3)) / (2 * (16 * a * b^2 - 64 * a^2 * c)) - (((4 * b^2 * d - 16 * a * c \\
& * d) * ((2 * a * e - b * d) * ((2 * a * e - b * d) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (128 \\
& * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (16 * a * b^2 - 64 * a^2 * c))) / (8 * a * (4 * a * c - b^2) \\
& ^{(1/2)}) + (16 * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)) / ((16 * a * b^2 - 64 * a \\
& ^2 * c) * (4 * a * c - b^2)^{(1/2)})) / (8 * a * (4 * a * c - b^2)^{(1/2)}) + (2 * b^4 * c^4 * (4 * b^2 * \\
& d - 16 * a * c * d) * (2 * a * e - b * d)^2) / (a * (16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2)))) / (2 \\
& * (16 * a * b^2 - 64 * a^2 * c)) + ((2 * a * e - b * d) * (((4 * b^2 * d - 16 * a * c * d) * ((2 * a * e - \\
& b * d) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) \\
&)) / (16 * a * b^2 - 64 * a^2 * c))) / (8 * a * (4 * a * c - b^2)^{(1/2)}) + (16 * b^4 * c^4 * (4 * b^2 * d \\
& - 16 * a * c * d) * (2 * a * e - b * d)) / ((16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2)^{(1/2)})) / (2 \\
& * (16 * a * b^2 - 64 * a^2 * c)) + ((2 * a * e - b * d) * (((4 * b^2 * d - 16 * a * c * d) * (256 * b^4 * c^ \\
& 4 * d - 256 * a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (16 * a * b^2 - 64 \\
& * a^2 * c)))) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + 96 * a * b^2 * c^4 * e^2 - 256 * b^3 * c^4 * d * e)) / \\
& (8 * a * (4 * a * c - b^2)^{(1/2)})) / (8 * a * (4 * a * c - b^2)^{(1/2)}) * (4 * b^2 * d - 16 * a * c * d) \\
&)) / (2 * (16 * a * b^2 - 64 * a^2 * c)) - (((4 * b^2 * d - 16 * a * c * d) * ((4 * b^2 * d - 16 * a * c * d) \\
&) * ((2 * a * e - b * d) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * \\
& d - 16 * a * c * d)) / (16 * a * b^2 - 64 * a^2 * c))) / (8 * a * (4 * a * c - b^2)^{(1/2)}) + (16 * b^4 * \\
& c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)) / ((16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2) \\
&)^{(1/2)})) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + ((2 * a * e - b * d) * (((4 * b^2 * d - 16 * a * c * d) \\
&) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (\\
& 16 * a * b^2 - 64 * a^2 * c)))) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + 96 * a * b^2 * c^4 * e^2 - 256 * b \\
& ^3 * c^4 * d * e)) / (8 * a * (4 * a * c - b^2)^{(1/2)})) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + ((2 * a * \\
& e - b * d) * (((4 * b^2 * d - 16 * a * c * d) * ((4 * b^2 * d - 16 * a * c * d) * (256 * b^4 * c^4 * d - 256 \\
& * a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (16 * a * b^2 - 64 * a^2 * c))) \\
& / (2 * (16 * a * b^2 - 64 * a^2 * c)) + 96 * a * b^2 * c^4 * e^2 - 256 * b^3 * c^4 * d * e)) / (2 * (16 * a * \\
& b^2 - 64 * a^2 * c)) + 96 * b^2 * c^4 * d * e^2 - 16 * a * b * c^4 * e^3)) / (8 * a * (4 * a * c - b^2)^(\\
& 1/2)) * (2 * a * e - b * d) / (8 * a * (4 * a * c - b^2)^{(1/2)}) + ((2 * a * e - b * d) * ((2 * a * e - \\
& b * d) * ((2 * a * e - b * d) * ((2 * a * e - b * d) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (1 \\
& 28 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (16 * a * b^2 - 64 * a^2 * c))) / (8 * a * (4 * a * c - b^ \\
& 2)^{(1/2)}) + (16 * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)) / ((16 * a * b^2 - 64 \\
& * a^2 * c) * (4 * a * c - b^2)^{(1/2)})) / (8 * a * (4 * a * c - b^2)^{(1/2)}) + (2 * b^4 * c^4 * (4 * b^ \\
& 2 * d - 16 * a * c * d) * (2 * a * e - b * d)^2) / (a * (16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2)))) / \\
& (8 * a * (4 * a * c - b^2)^{(1/2)}) + (b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)^3) / \\
& (4 * a^2 * (16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2)^{(3/2)})) / (8 * a * (4 * a * c - b^2)^{(1/2) \\
&)) + (b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)^4) / (32 * a^3 * (16 * a * b^2 - 64 * \\
& a^2 * c) * (4 * a * c - b^2)^2)) / (c^4 * (a^2 * e^2 - 20 * b^2 * d^2 + 81 * a * c * d^2 - a * b * d * e \\
&) * (16 * a^4 * c^4 * e^4 + b^4 * c^4 * d^4 + 24 * a^2 * b^2 * c^4 * d^2 * e^2 - 8 * a * b^3 * c^4 * d^3 * \\
& e - 32 * a^3 * b * c^4 * d * e^3)) + ((4 * a * c - b^2)^2 * (((4 * b^2 * d - 16 * a * c * d) * ((4 * b^2 \\
& * d - 16 * a * c * d) * ((4 * b^2 * d - 16 * a * c * d) * ((2 * a * e - b * d) * (256 * b^4 * c^4 * d - 256 * \\
& a * b^3 * c^4 * e + (128 * a * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d)) / (16 * a * b^2 - 64 * a^2 * c)))) / \\
& (8 * a * (4 * a * c - b^2)^{(1/2)}) + (16 * b^4 * c^4 * (4 * b^2 * d - 16 * a * c * d) * (2 * a * e - b * d)) \\
& / ((16 * a * b^2 - 64 * a^2 * c) * (4 * a * c - b^2)^{(1/2)})) / (2 * (16 * a * b^2 - 64 * a^2 * c)) + \\
& ((2 * a * e - b * d) * (((4 * b^2 * d - 16 * a * c * d) * (256 * b^4 * c^4 * d - 256 * a * b^3 * c^4 * e + (1
\end{aligned}$$

$$\begin{aligned}
& 28*a*b^4*c^4*(4*b^2*d - 16*a*c*d)/(16*a*b^2 - 64*a^2*c))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e)/(8*a*(4*a*c - b^2)^{(1/2)}) \\
&)/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e)/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(8*a*(4*a*c - b^2)^{(1/2)})))/(2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^{(1/2))} + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)})))/(8*a*(4*a*c - b^2)^{(1/2))} + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))))/(8*a*(4*a*c - b^2)^{(1/2))} + (b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^3)/(4*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(3/2)})))/(2*(16*a*b^2 - 64*a^2*c)) - (((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^{(1/2))} + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)})))/(8*a*(4*a*c - b^2)^{(1/2))} + (2*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d)^2)/(a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((2*a*e - b*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(8*a*(4*a*c - b^2)^{(1/2))} + (16*b^4*c^4*(4*b^2*d - 16*a*c*d)*(2*a*e - b*d))/((16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^{(1/2)})))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e)/(8*a*(4*a*c - b^2)^{(1/2)})))/(8*a*(4*a*c - b^2)^{(1/2))}*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^{(1/2))} + ((2*a*e - b*d)*(a*c^4*e^4 + ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(256*b^4*c^4*d - 256*a*b^3*c^4*e + (128*a*b^4*c^4*(4*b^2*d - 16*a*c*d))/(16*a*b^2 - 64*a^2*c)))/(2*(16*a*b^2 - 64*a^2*c)) + 96*a*b^2*c^4*e^2 - 256*b^3*c^4*d*e)/(2*(16*a*b^2 - 64*a^2*c)) + 96*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3))/(2*(16*a*b^2 - 64*a^2*c)) - 16*b*c^4*d*e^3))/(8*a*(4*a*c - b^2)^{(1/2))} + (b^4*c^4*(2*a*e - b*d)^5)/(128*a^4*(4*a*c - b^2)^{(5/2))}*(144*a^3*c^3*d - 40*b^6*d + 8*a*b^5*e - 488*a^2*b^2*c^2*d + 272*a*b^4*c*d - 40*a^2*b^3*c*e + 40*a^3*b*c^2*e))/(2*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)*(16*a^4*c^4*e^4 + b^4*c^4*d^4 + 24*a^2*b^2*c^4*d^2*e^2 - 8*a*b^3*c^4*d^3*e - 32*a^3*b*c^4*d*e^3))*(2*a*e - b*d))/(4*a*(4*a*c - b^2)^{(1/2))}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

$$3.49 \quad \int \frac{d+ex^4}{x^2(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=392

$$\frac{\sqrt[4]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

[Out] $-d/a/x-1/4*c^{(1/4)*\arctan(2^{(1/4)*c^{(1/4)*x}/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2))*2^{(1/4)}/a/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)}+1/4*c^{(1/4)*\operatorname{arctanh}(2^{(1/4)*c^{(1/4)*x}/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2))*2^{(1/4)}/a/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)}-1/4*c^{(1/4)*\arctan(2^{(1/4)*c^{(1/4)*x}/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2))*2^{(1/4)}/a/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)}+1/4*c^{(1/4)*\operatorname{arctanh}(2^{(1/4)*c^{(1/4)*x}/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2))*2^{(1/4)}/a/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)}$

Rubi [A] time = 0.68, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1504, 1510, 298, 205, 208}

$$\frac{\sqrt[4]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{-\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{\sqrt{b^2-4ac}-b}} + \frac{\sqrt[4]{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2^{2^{3/4}} a \sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)), x]

[Out] $-(d/(a*x)) - (c^{(1/4)*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c]})*\text{ArcTan}[(2^{(1/4)*c^{(1/4)*x}/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}})/(2*2^{(3/4)*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}}) - (c^{(1/4)*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c]})*\text{ArcTan}[(2^{(1/4)*c^{(1/4)*x}/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}})/(2*2^{(3/4)*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}}) + (c^{(1/4)*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c]})*\text{ArcTanh}[(2^{(1/4)*c^{(1/4)*x}/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}})/(2*2^{(3/4)*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}}) + (c^{(1/4)*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c]})*\text{ArcTanh}[(2^{(1/4)*c^{(1/4)*x}/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}})/(2*2^{(3/4)*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}})$

Rule 205

$\text{Int}[\frac{((a_) + (b_)*(x_)^2)^{-1}}{a, x}] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 208

$\text{Int}[\frac{((a_) + (b_)*(x_)^2)^{-1}}{a, x}] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

Rule 298

$\text{Int}[\frac{(x_)^2}{((a_) + (b_)*(x_)^4)}, x_Symbol] :> \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[1/(r + s*x^2), x], x] - \text{Dist}[s/(2*b), \text{Int}[1/(r - s*x^2), x], x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ !\text{GtQ}[a/b, 0]$

Rule 1504

$\text{Int}[\frac{((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^{(n_)})*((a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)})^{(p_)}}{x_Symbol}] :> \text{Simp}[\frac{d*(f*x)^{(m+1)}*(a + b*x^n + c*x^{(2*n)})^{(p+1)}}{a*f*(m+1)}, x] + \text{Dist}[1/(a*f^n*(m+1)), \text{Int}[(f*x)^{(m+n)}*(a + b*x^n + c*x^{(2*n)})^p * \text{Simp}[a*e*(m+1) - b*d*(m+n*(p+1)+1) - c*d*(m+2*n*(p+1)+1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[p]$

Rule 1510

$\text{Int}[\frac{(((f_)*(x_))^{(m_)}*((d_) + (e_)*(x_)^{(n_)})/((a_) + (b_)*(x_)^{(n_)} + (c_)*(x_)^{(n2_)}))}{x_Symbol}] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[(f*x)^m/(b/2 + q/2 + c*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^4}{x^2(a + bx^4 + cx^8)} dx &= -\frac{d}{ax} - \frac{\int \frac{x^2(bd - ae + cd x^4)}{a + bx^4 + cx^8} dx}{a} \\
&= -\frac{d}{ax} - \frac{\left(c \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{x^2}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx}{2a} - \frac{\left(c \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{x^2}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^4} dx}{2a} \\
&= -\frac{d}{ax} + \frac{\left(\sqrt{c} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\sqrt{-b - \sqrt{b^2 - 4ac}} - \sqrt{2} \sqrt{c} x^2} dx}{2\sqrt{2} a} - \frac{\left(\sqrt{c} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\sqrt{-b + \sqrt{b^2 - 4ac}} + \sqrt{2} \sqrt{c} x^2} dx}{2\sqrt{2} a} \\
&= -\frac{d}{ax} - \frac{\sqrt[4]{c} \left(d - \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b - \sqrt{b^2 - 4ac}}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt[4]{c} \left(d + \frac{bd - 2ae}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b + \sqrt{b^2 - 4ac}}}\right)}{2 \cdot 2^{3/4} a \sqrt[4]{-b + \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 85, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^8 c + \#1^4 b + a \&, \frac{\#1^4 c d \log(x - \#1) - a e \log(x - \#1) + b d \log(x - \#1)}{2 \#1^5 c + \#1 b} \& \right]}{4a} - \frac{d}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)), x]

[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) &]/(4*a)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.01, size = 72, normalized size = 0.18

$$\frac{\left(\text{RootOf}\left(-Z^8c + Z^4b + a\right)^6 cd + (-ae + bd) \text{RootOf}\left(-Z^8c + Z^4b + a\right)^2\right) \ln\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right)\right) + 4a\left(2 \text{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + \text{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)

[Out] -1/a*d/x-1/4/a*sum((_R^6*c*d+(-a*e+b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(-_R+x),_R=RootOf(-Z^8*c+Z^4*b+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 9.46, size = 39028, normalized size = 99.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)

[Out] atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*

$$\begin{aligned}
& b^5 e^4 + a^4 e^4 (-4ac - b^2)^5)^{1/2} + b^4 d^4 (-4ac - b^2)^5)^{1/2} \\
& + 80a^4 b^3 c^4 d^4 - 8a^5 b^3 c^3 e^4 + 16a^6 b^3 c^2 e^4 - 4a^3 b^6 d^3 e^3 \\
& - 128a^5 c^4 d^3 e + 128a^6 c^3 d^3 e^3 + 61a^2 b^5 c^2 d^4 - 120a^3 b^3 c^3 d^4 \\
& + a^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} + 6a^2 b^7 d^2 e^2 - 13a^3 b^7 c^3 d^4 \\
& - 4a^3 b^8 d^3 e + 6a^2 b^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 240a^4 b^3 c^2 d^2 e^2 \\
& - 3a^3 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} - 4a^3 b^3 d^3 e (-4ac - b^2)^5)^{1/2} \\
& + 48a^2 b^6 c^3 d^3 e + 40a^4 b^4 c^3 d^3 e - 200a^3 b^4 c^2 d^3 e - 66a^3 b^5 c^2 d^2 e^2 \\
& + 320a^4 b^2 c^3 d^3 e - 288a^5 b^3 c^3 d^2 e^2 - 128a^5 b^2 c^2 d^2 e^3 \\
& - 6a^3 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 8a^2 b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} \\
& + 8a^2 b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} / (512(a^5 b^8 + 256a^9 c^4 - 16a^6 b^6 c + 96a^7 b^4 c^2 \\
& - 256a^8 b^2 c^3))^{1/4} * (32768a^16 c^8 d^2 - 32768a^17 c^7 e^2 + 1024a^12 b^8 c^4 d^2 \\
& - 12288a^13 b^6 c^5 d^2 + 51200a^14 b^4 c^6 d^2 - 81920a^15 b^2 c^7 d^2 + 1024a^14 b^6 c^4 e^2 \\
& - 10240a^15 b^4 c^5 e^2 + 32768a^16 b^2 c^6 e^2 + 98304a^16 b^3 c^7 d^2 e - 2048a^13 b^7 c^4 d^2 e \\
& + 22528a^14 b^5 c^5 d^2 e - 81920a^15 b^3 c^6 d^2 e) - 4096a^15 c^8 d^3 + 4096a^16 b^3 c^6 e^3 \\
& + 12288a^16 c^7 d^2 e^2 - 256a^11 b^8 c^4 d^3 + 2816a^12 b^6 c^5 d^3 - 10496a^13 b^4 c^6 d^3 \\
& + 14336a^14 b^2 c^7 d^3 + 256a^14 b^5 c^4 e^3 - 2048a^15 b^3 c^5 e^3 - 24576a^15 b^3 c^7 d^2 e \\
& + 768a^12 b^7 c^4 d^2 e - 7680a^13 b^5 c^5 d^2 e - 768a^13 b^6 c^4 d^2 e^2 + 24576a^14 b^3 c^6 d^2 e \\
& + 6912a^14 b^4 c^5 d^2 e^2 - 18432a^15 b^2 c^6 d^2 e^2) + x(4a^11 b^3 c^8 d^6 + 4a^14 b^3 c^5 e^6 \\
& - 16a^12 c^8 d^5 e - 16a^14 c^6 d^5 e^5 - 32a^13 c^7 d^3 e^3 + 4a^11 b^3 c^6 d^4 e^2 \\
& - 32a^12 b^2 c^6 d^3 e^3 + 4a^12 b^3 c^5 d^2 e^4 - 8a^11 b^2 c^7 d^5 e + 44a^12 b^3 c^7 d^4 e^2 \\
& + 44a^13 b^3 c^6 d^2 e^4 - 8a^13 b^2 c^5 d^2 e^5) * (-b^9 d^4 + a^4 b^5 e^4 + a^4 e^4 (-4ac - b^2)^5)^{1/2} \\
& + b^4 d^4 (-4ac - b^2)^5)^{1/2} + 80a^4 b^3 c^4 d^4 - 8a^5 b^3 c^3 e^4 + 16a^6 b^3 c^2 e^4 \\
& - 4a^3 b^6 d^3 e^3 - 128a^5 c^4 d^3 e + 128a^6 c^3 d^3 e^3 + 61a^2 b^5 c^2 d^4 - 120a^3 b^3 c^3 d^4 \\
& + a^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} + 6a^2 b^7 d^2 e^2 - 13a^3 b^7 c^3 d^4 - 4a^3 b^8 d^3 e \\
& + 6a^2 b^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 240a^4 b^3 c^2 d^2 e^2 - 3a^3 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} \\
& - 4a^3 b^3 d^3 e (-4ac - b^2)^5)^{1/2} + 48a^2 b^6 c^3 d^3 e + 40a^4 b^4 c^3 d^3 e \\
& - 200a^3 b^4 c^2 d^3 e - 66a^3 b^5 c^2 d^2 e^2 + 320a^4 b^2 c^3 d^3 e - 288a^5 b^3 c^3 d^2 e^2 \\
& - 128a^5 b^2 c^2 d^2 e^3 - 6a^3 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 8a^2 b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} \\
& + 8a^2 b^3 c^3 d^3 e (-4ac - b^2)^5)^{1/2} / (512(a^5 b^8 + 256a^9 c^4 - 16a^6 b^6 c + 96a^7 b^4 c^2 - 256a^8 b^2 c^3))^{1/4} \\
& * i + ((-b^9 d^4 + a^4 b^5 e^4 + a^4 e^4 (-4ac - b^2)^5)^{1/2} + b^4 d^4 (-4ac - b^2)^5)^{1/2} \\
& + 80a^4 b^3 c^4 d^4 - 8a^5 b^3 c^3 e^4 + 16a^6 b^3 c^2 e^4 - 4a^3 b^6 d^3 e^3 - 128a^5 c^4 d^3 e \\
& + 128a^6 c^3 d^3 e^3 + 61a^2 b^5 c^2 d^4 - 120a^3 b^3 c^3 d^4 + a^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} \\
& + 6a^2 b^7 d^2 e^2 - 13a^3 b^7 c^3 d^4 - 4a^3 b^8 d^3 e + 6a^2 b^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} \\
& + 240a^4 b^3 c^2 d^2 e^2 - 3a^3 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} - 4a^3 b^3 d^3 e (-4ac - b^2)^5)^{1/2} \\
& - 4a^3 b^3 d^3 e (-4ac - b^2)^5)^{1/2} + 48a^2 b^6 c^3 d^3 e + 40a^4 b^4 c^3 d^3 e - 200a^3 b^4 c^2 d^3 e \\
& - 66a^3 b^5 c^2 d^2 e^2 + 320a^4 b^2 c^3 d^3 e - 288a^5 b^3 c^3 d^2 e^2
\end{aligned}$$

$$\begin{aligned}
& *c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)}*(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*1i)/((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*1i)
\end{aligned}$$

$$\begin{aligned}
& (- (4ac - b^2)^5)^{1/2} - 4ab^3d^3e(- (4ac - b^2)^5)^{1/2} - 4a^3b \\
& * d^3e^3(- (4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e^3 - \\
& 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288 \\
& a^5b^3cd^2e^2 - 128a^5b^2c^2d^3e - 6a^3cd^2e^2(- (4ac - b^2 \\
&)^5)^{1/2} + 8a^2b^3cd^3e(- (4ac - b^2)^5)^{1/2} / (512(a^5b^8 + 256a \\
& a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{3/4} * (x * (- (b^ \\
& 9d^4 + a^4b^5e^4 + a^4e^4(- (4ac - b^2)^5)^{1/2} + b^4d^4(- (4ac - \\
& b^2)^5)^{1/2} + 80a^4b^3cd^4 - 8a^5b^3c^2e^4 + 16a^6b^3c^2e^4 - 4 \\
& a^3b^6d^3e - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 \\
& - 120a^3b^3c^3d^4 + a^2c^2d^4(- (4ac - b^2)^5)^{1/2} + 6a^2b^7d^ \\
& 2e^2 - 13ab^7cd^4 - 4ab^8d^3e + 6a^2b^2d^2e^2(- (4ac - b^2)^ \\
& 5)^{1/2} + 240a^4b^3c^2d^2e^2 - 3ab^2cd^4(- (4ac - b^2)^5)^{1/2} \\
& - 4ab^3d^3e(- (4ac - b^2)^5)^{1/2} - 4a^3b^3d^3e(- (4ac - b^2)^5 \\
&)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e^3 - 200a^3b^4c^2d^3e - \\
& 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3cd^2e^2 - 128 \\
& a^5b^2c^2d^3e - 6a^3cd^2e^2(- (4ac - b^2)^5)^{1/2} + 8a^2b^3cd \\
& ^3e(- (4ac - b^2)^5)^{1/2} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + \\
& 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} * (32768a^16c^8d^2 - 32768a^17 \\
& c^7e^2 + 1024a^12b^8c^4d^2 - 12288a^13b^6c^5d^2 + 51200a^14b^4c \\
& c^6d^2 - 81920a^15b^2c^7d^2 + 1024a^14b^6c^4e^2 - 10240a^15b^4c \\
& ^5e^2 + 32768a^16b^2c^6e^2 + 98304a^16b^3c^7d^2e - 2048a^13b^7c^4 \\
& d^2e + 22528a^14b^5c^5d^2e - 81920a^15b^3c^6d^2e) - 4096a^15c^8d^3 \\
& + 4096a^16b^3c^6e^3 + 12288a^16c^7d^2e^2 - 256a^11b^8c^4d^3 + 2816a \\
& a^12b^6c^5d^3 - 10496a^13b^4c^6d^3 + 14336a^14b^2c^7d^3 + 256a^ \\
& 14b^5c^4e^3 - 2048a^15b^3c^5e^3 - 24576a^15b^3c^7d^2e + 768a^12 \\
& b^7c^4d^2e - 7680a^13b^5c^5d^2e - 768a^13b^6c^4d^2e^2 + 24576a^ \\
& 14b^3c^6d^2e + 6912a^14b^4c^5d^2e^2 - 18432a^15b^2c^6d^2e^2) + x * \\
& (4a^11b^3c^8d^6 + 4a^14b^3c^5e^6 - 16a^12c^8d^5e - 16a^14c^6d^2e^ \\
& 5 - 32a^13c^7d^3e^3 + 4a^11b^3c^6d^4e^2 - 32a^12b^2c^6d^3e^3 \\
& + 4a^12b^3c^5d^2e^4 - 8a^11b^2c^7d^5e + 44a^12b^3c^7d^4e^2 + 4 \\
& 4a^13b^3c^6d^2e^4 - 8a^13b^2c^5d^2e^5) * (- (b^9d^4 + a^4b^5e^4 + a^ \\
& 4e^4(- (4ac - b^2)^5)^{1/2} + b^4d^4(- (4ac - b^2)^5)^{1/2} + 80a^4 \\
& b^3cd^4 - 8a^5b^3c^2e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e - 128a^5 \\
& c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + \\
& a^2c^2d^4(- (4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - \\
& 4ab^8d^3e + 6a^2b^2d^2e^2(- (4ac - b^2)^5)^{1/2} + 240a^4b^3c \\
& ^2d^2e^2 - 3ab^2cd^4(- (4ac - b^2)^5)^{1/2} - 4ab^3d^3e(- (4ac \\
& - b^2)^5)^{1/2} - 4a^3b^3d^3e(- (4ac - b^2)^5)^{1/2} + 48a^2b^6cd \\
& ^3e + 40a^4b^4cd^3e^3 - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + \\
& 320a^4b^2c^3d^3e - 288a^5b^3cd^2e^2 - 128a^5b^2c^2d^3e - 6a \\
& ^3cd^2e^2(- (4ac - b^2)^5)^{1/2} + 8a^2b^3cd^3e(- (4ac - b^2)^5)^ \\
& (1/2) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^ \\
& 8b^2c^3))^{1/4} - ((- (b^9d^4 + a^4b^5e^4 + a^4e^4(- (4ac - b^2)^5) \\
&)^{1/2} + b^4d^4(- (4ac - b^2)^5)^{1/2} + 80a^4b^3cd^4 - 8a^5b^3c \\
& e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e - 128a^5c^4d^3e + 128a^6c^3
\end{aligned}$$

$$\begin{aligned}
& d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^{1/2} + 240a^4b^3c^2d^2e^2 - 3ab^2cd^4(-4ac - b^2)^{1/2} - 4ab^3d^3e(-4ac - b^2)^{1/2} - 4a^3bd^3e(-4ac - b^2)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e^3 - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3d^2e^2 - 128a^5b^2c^2d^3e - 6a^3cd^2e^2(-4ac - b^2)^{1/2} + 8a^2b^3cd^3e(-4ac - b^2)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{3/4} * (4096a^15c^8d^3 + x(-b^9d^4 + a^4b^5e^4 + a^4e^4(-4ac - b^2)^{1/2}) + b^4d^4(-4ac - b^2)^{1/2} + 80a^4b^3cd^4 - 8a^5b^3c^2e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^{1/2} + 240a^4b^3c^2d^2e^2 - 3ab^2cd^4(-4ac - b^2)^{1/2} - 4ab^3d^3e(-4ac - b^2)^{1/2} - 4a^3bd^3e(-4ac - b^2)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e^3 - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3d^2e^2 - 128a^5b^2c^2d^3e - 6a^3cd^2e^2(-4ac - b^2)^{1/2} + 8a^2b^3cd^3e(-4ac - b^2)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} * (32768a^16c^8d^2 - 32768a^17c^7e^2 + 1024a^12b^8c^4d^2 - 12288a^13b^6c^5d^2 + 51200a^14b^4c^6d^2 - 81920a^15b^2c^7d^2 + 1024a^14b^6c^4e^2 - 10240a^15b^4c^5e^2 + 32768a^16b^2c^6e^2 + 98304a^16b^3c^7d^2e - 2048a^13b^7c^4d^2e + 22528a^14b^5c^5d^2e - 81920a^15b^3c^6d^2e) - 4096a^16b^3c^6e^3 - 12288a^16c^7d^2e^2 + 256a^11b^8c^4d^3 - 2816a^12b^6c^5d^3 + 10496a^13b^4c^6d^3 - 14336a^14b^2c^7d^3 - 256a^14b^5c^4e^3 + 2048a^15b^3c^5e^3 + 24576a^15b^3c^7d^2e - 768a^12b^7c^4d^2e + 7680a^13b^5c^5d^2e + 768a^13b^6c^4d^2e^2 - 24576a^14b^3c^6d^2e - 6912a^14b^4c^5d^2e^2 + 18432a^15b^2c^6d^2e^2) + x(4a^11b^3c^8d^6 + 4a^14b^3c^5e^6 - 16a^12c^8d^5e - 16a^14c^6d^5e^5 - 32a^13c^7d^3e^3 + 4a^11b^3c^6d^4e^2 - 32a^12b^2c^6d^3e^3 + 4a^12b^3c^5d^2e^4 - 8a^11b^2c^7d^5e + 44a^12b^3c^7d^4e^2 + 44a^13b^3c^6d^2e^4 - 8a^13b^2c^5d^2e^5) * (-b^9d^4 + a^4b^5e^4 + a^4e^4(-4ac - b^2)^{1/2} + b^4d^4(-4ac - b^2)^{1/2} + 80a^4b^3cd^4 - 8a^5b^3c^2e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^{1/2} + 240a^4b^3c^2d^2e^2 - 3ab^2cd^4(-4ac - b^2)^{1/2} - 4ab^3d^3e(-4ac - b^2)^{1/2} - 4a^3bd^3e(-4ac - b^2)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e^3 - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3d^2e^2 - 128a^5b^2c^2d^3e - 6a^3cd^2e^2(-4ac - b^2)^{1/2} + 8a^2b^3cd^3e(-4ac - b^2)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256
\end{aligned}$$

$$\begin{aligned}
& *a^8*b^2*c^3))^{(1/4)} + 2*a^{14}*c^5*e^7 + 2*a^{11}*c^8*d^6*e + 6*a^{12}*c^7*d^4* \\
& e^3 + 6*a^{13}*c^6*d^2*e^5 + 6*a^{11}*b^2*c^6*d^4*e^3 - 2*a^{11}*b^3*c^5*d^3*e^4 \\
& + 6*a^{12}*b^2*c^5*d^2*e^5 - 6*a^{13}*b*c^5*d*e^6 - 6*a^{11}*b*c^7*d^5*e^2 - 12*a \\
& ^{12}*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 \\
& + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d* \\
& e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2 \\
& *d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3* \\
& b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 \\
& - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 28 \\
& 8*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256 \\
& *a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*2i + at \\
& an((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4* \\
& (-4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^ \\
& 2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^ \\
& 5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6* \\
& a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c \\
& ^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2 \\
& *e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8 \\
& *a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a \\
& ^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)}*(x*(-(b^9*d^4 + a^4*b^ \\
& 5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 \\
& - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3* \\
& c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b \\
& ^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240 \\
& *a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3 \\
& *e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a \\
& ^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c* \\
& d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d \\
& *e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c \\
& - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^ \\
& 2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102 \\
& 4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192 \\
& 0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768 \\
& *a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^ \\
& 14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*b* \\
& c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d \\
& ^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3
\end{aligned}$$

$$\begin{aligned}
& - 2048a^{15}b^3c^5e^3 - 24576a^{15}b^3c^7d^2e + 768a^{12}b^7c^4d^2e \\
& - 7680a^{13}b^5c^5d^2e - 768a^{13}b^6c^4d^2e + 24576a^{14}b^3c^6d^2 \\
& * e + 6912a^{14}b^4c^5d^2e - 18432a^{15}b^2c^6d^2e + x*(4a^{11}b^3c^8 \\
& d^6 + 4a^{14}b^3c^5e^6 - 16a^{12}c^8d^5e - 16a^{14}c^6d^5e - 32a^{13}c^7 \\
& d^3e^3 + 4a^{11}b^3c^6d^4e^2 - 32a^{12}b^2c^6d^3e^3 + 4a^{12}b^3c^5 \\
& d^2e^4 - 8a^{11}b^2c^7d^5e + 44a^{12}b^3c^7d^4e^2 + 44a^{13}b^3c^6d^2 \\
& e^4 - 8a^{13}b^2c^5d^5e^5)*(-(b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - \\
& b^2)^5)^{1/2} - b^4d^4*(-(4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5 \\
& b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3 \\
& d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{1/2} \\
& + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{1/2} \\
& + 240a^4b^3c^2d^2e^2 + 3ab^2c^4d^4*(-(4ac - b^2)^5)^{1/2} + 4ab^3d^3e*(-(4ac - b^2)^5)^{1/2} \\
& + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{1/2} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e \\
& + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3 \\
& d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{1/2} \\
& - 8a^2b^3c^3d^3e*(-(4ac - b^2)^5)^{1/2})/(512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} \\
& * 1i + ((-b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{1/2} - b^4d^4*(-(4ac - b^2)^5)^{1/2} \\
& + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2 \\
& b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2 \\
& (-4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2c^4d^4*(-(4ac - b^2)^5)^{1/2} + 4ab^3d^3e*(-(4ac - b^2)^5)^{1/2} \\
& + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{1/2} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 \\
& + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{1/2} - 8a^2b^3c^3d^3e \\
& (-4ac - b^2)^5)^{1/2})/(512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{3/4} * (4096a^{15}c^8d^3 \\
& + x*(-b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{1/2} - b^4d^4*(-(4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 \\
& + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4 \\
& (-4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 \\
& + 3ab^2c^4d^4*(-(4ac - b^2)^5)^{1/2} + 4ab^3d^3e*(-(4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{1/2} \\
& + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 \\
& - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{1/2} - 8a^2b^3c^3d^3e*(-(4ac - b^2)^5)^{1/2})/(512*(a^5b^8 + 256a^9c^4 - 16 \\
& a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} * (32768a^{16}c^8d^2 - 32768a^{17}c^7e^2 + 1024a^{12}b^8c^4d^2 - 12288a^{13}b^6c^5d^2 \\
& + 51200a^{14}b^4c^6d^2 - 81920a^{15}b^2c^7d^2 + 1024a^{14}b^6c^4e^2 - 10240a^{15}b^4c^5e^2 + 32768a^{16}b^2c^6e^2 + 98304a^{16}b^3c^7d^2e \\
& - 2048a
\end{aligned}$$

$$\begin{aligned}
& ^{13}b^7c^4d^3e + 22528a^{14}b^5c^5d^3e - 81920a^{15}b^3c^6d^3e) - 4096a^{16}b^3c^6e^3 - 12288a^{16}c^7d^3e^2 + 256a^{11}b^8c^4d^3 - 2816a^{12}b^6c^5d^3 + 10496a^{13}b^4c^6d^3 - 14336a^{14}b^2c^7d^3 - 256a^{14}b^5c^4e^3 + 2048a^{15}b^3c^5e^3 + 24576a^{15}b^3c^7d^2e - 768a^{12}b^7c^4d^2e + 7680a^{13}b^5c^5d^2e + 768a^{13}b^6c^4d^2e - 24576a^{14}b^3c^6d^2e - 6912a^{14}b^4c^5d^2e + 18432a^{15}b^2c^6d^2e) + x(4a^{11}b^3c^8d^6 + 4a^{14}b^3c^5e^6 - 16a^{12}c^8d^5e - 16a^{14}c^6d^5e - 32a^{13}c^7d^3e^3 + 4a^{11}b^3c^6d^4e^2 - 32a^{12}b^2c^6d^3e^3 + 4a^{12}b^3c^5d^2e^4 - 8a^{11}b^2c^7d^5e + 44a^{12}b^3c^7d^4e^2 + 44a^{13}b^4c^6d^2e^4 - 8a^{13}b^2c^5d^4e^5)) * (- (b^9d^4 + a^4b^5e^4 - a^4e^4 * (- (4ac - b^2)^5)^{1/2} - b^4d^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 4ab^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3cd^2e^2 - 128a^5b^2c^2d^3e^3 + 6a^3cd^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e * (- (4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} * i) / (((- (b^9d^4 + a^4b^5e^4 - a^4e^4 * (- (4ac - b^2)^5)^{1/2} - b^4d^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 4ab^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3cd^2e^2 - 128a^5b^2c^2d^3e^3 + 6a^3cd^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e * (- (4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{3/4} * (x * (- (b^9d^4 + a^4b^5e^4 - a^4e^4 * (- (4ac - b^2)^5)^{1/2} - b^4d^4 * (- (4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4 * (- (4ac - b^2)^5)^{1/2} + 4ab^3d^3e * (- (4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4cd^3e - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3cd^2e^2 - 128a^5b^2c^2d^3e^3 + 6a^3cd^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e * (- (4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{1/4} * (32768a^{16}c^8d^2 - 32768a^{17}c
\end{aligned}$$

$$\begin{aligned}
& ^7e^2 + 1024a^{12}b^8c^4d^2 - 12288a^{13}b^6c^5d^2 + 51200a^{14}b^4c^6d^2 - 81920a^{15}b^2c^7d^2 + 1024a^{14}b^6c^4e^2 - 10240a^{15}b^4c^5e^2 \\
& + 32768a^{16}b^2c^6e^2 + 98304a^{16}b^3c^7d^2e - 2048a^{13}b^7c^4d^2e + 22528a^{14}b^5c^5d^2e - 81920a^{15}b^3c^6d^2e) - 4096a^{15}c^8d^3 + \\
& 4096a^{16}b^3c^6e^3 + 12288a^{16}c^7d^2e^2 - 256a^{11}b^8c^4d^3 + 2816a^{12}b^6c^5d^3 - 10496a^{13}b^4c^6d^3 + 14336a^{14}b^2c^7d^3 + 256a^{14} \\
& *b^5c^4e^3 - 2048a^{15}b^3c^5e^3 - 24576a^{15}b^3c^7d^2e + 768a^{12}b^7c^4d^2e - 7680a^{13}b^5c^5d^2e - 768a^{13}b^6c^4d^2e^2 + 24576a^{14} \\
& *b^3c^6d^2e + 6912a^{14}b^4c^5d^2e^2 - 18432a^{15}b^2c^6d^2e^2) + x*(4 \\
& *a^{11}b^3c^8d^6 + 4a^{14}b^3c^5e^6 - 16a^{12}c^8d^5e - 16a^{14}c^6d^5e^5 - 32a^{13}c^7d^3e^3 + 4a^{11}b^3c^6d^4e^2 - 32a^{12}b^2c^6d^3e^3 + \\
& 4a^{12}b^3c^5d^2e^4 - 8a^{11}b^2c^7d^5e + 44a^{12}b^3c^7d^4e^2 + 44a^{13}b^3c^6d^2e^4 - 8a^{13}b^2c^5d^2e^5))*(-(b^9d^4 + a^4b^5e^4 - a^4e^4 \\
& *(-4ac - b^2)^5)^{(1/2)} - b^4d^4*(-(4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e \\
& + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^3b^7c^3d^4 - 4 \\
& *a^2b^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3a^2b^2c^3d^4*(-(4ac - b^2)^5)^{(1/2)} + 4a^2b^3d^3e*(-(4ac \\
& - b^2)^5)^{(1/2)} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^3d^2e^2 + 32 \\
& 0a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)}) \\
& /((512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3)))^{(1/4)} - (((-b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{(1/2)} - b^4d^4 \\
& *(-4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 \\
& - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^3b^7c^3d^4 - 4a^2b^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} \\
& + 240a^4b^3c^2d^2e^2 + 3a^2b^2c^3d^4*(-(4ac - b^2)^5)^{(1/2)} + 4a^2b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} \\
& + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^3d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 \\
& + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)})/(512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(3/4)} \\
& *(4096a^{15}c^8d^3 + x*(-(b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{(1/2)} - b^4d^4*(-(4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 \\
& + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} \\
& + 6a^2b^7d^2e^2 - 13a^3b^7c^3d^4 - 4a^2b^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3a^2b^2c^3d^4*(-(4ac - b^2)^5)^{(1/2)} \\
& + 4a^2b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^3d^2e^2 \\
& + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^3d^3e*(-(4ac - b^2)^5)^{(1/2)})
\end{aligned}$$

$$\begin{aligned}
 &5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) \\
 &- 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2) + x \\
 &*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} + 2*a^14*c^5*e^7 + 2*a^11*c^8*d^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2*a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^11*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)}*2i - 2*atan(((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4
 \end{aligned}$$

$$\begin{aligned}
& 5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6* \\
& a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c \\
& ^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2 \\
& *e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8 \\
& *a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a \\
& ^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(3/4)}*(x*(-(b^9*d^4 + a^4*b^ \\
& 5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 \\
& - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c \\
& ^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b \\
& ^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240 \\
& *a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3 \\
& *e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a \\
& ^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c* \\
& d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d \\
& *e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c \\
& - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^ \\
& 2 - 256*a^8*b^2*c^3)))^{(1/4)}*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 102 \\
& 4*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 8192 \\
& 0*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768 \\
& *a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^ \\
& 14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^15*c^8*d^3 + 4096*a^16 \\
& *b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^ \\
& 5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4* \\
& e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2 \\
& *e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6* \\
& d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2)*1i - x*(4*a^11* \\
& b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a \\
& ^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12 \\
& *b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b \\
& *c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^ \\
& 4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3* \\
& e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2* \\
& d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8 \\
& *d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e \\
& ^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 4 \\
& 0*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4* \\
& b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2 \\
& *e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(\\
& 512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^
\end{aligned}$$

$$\begin{aligned}
& 3)))^{(1/4)} + (((-b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16 \\
& *a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + \\
& 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2 * \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^ \\
& 3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200* \\
& a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5* \\
& b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c \\
& ^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(3/4)}*(4096*a^15*c^ \\
& 8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4 \\
& *d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6 \\
& *b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a \\
& ^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2 * \\
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3 * \\
& ^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3* \\
& b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^ \\
& 3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^5*b^8 + 256*a^9*c^4 - \\
& 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(1/4)}*(32768*a^16*c^8*d \\
& ^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + \\
& 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 1 \\
& 0240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 204 \\
& 8*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - \\
& 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 - 2816*a^ \\
& 12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14 \\
& *b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e - 768*a^12*b^ \\
& 7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14 \\
& *b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i - x \\
& *(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e \\
& ^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 \\
& + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + \\
& 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a \\
& ^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4 \\
& *b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5 \\
& *c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + \\
& a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 \\
& - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3* \\
& c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a \\
& *c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c* \\
& d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 +
\end{aligned}$$

$$\begin{aligned}
& 320a^4b^2c^3d^3e - 288a^5b^2c^3d^2e^2 - 128a^5b^2c^2d^2e^3 - 6a^3c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^2b^2c^3d^3e(-4ac - b^2)^5)^{(1/2)} \\
& / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} / (((-b^9d^4 + a^4b^5e^4 + a^4e^4(-4ac - b^2)^5)^{(1/2)} + b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^2c^4d^4 - 8a^5b^3c^2e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^2e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^2d^4 - 4a^2b^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 - 3a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} - 4a^2b^3d^3e(-4ac - b^2)^5)^{(1/2)} - 4a^3b^2d^2e^3(-4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^2d^3e + 40a^4b^4c^2d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^2c^3d^2e^2 - 128a^5b^2c^2d^2e^3 - 6a^3c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^2b^2c^3d^3e(-4ac - b^2)^5)^{(1/2)} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(3/4)} * (4096a^15c^8d^3 + x(-b^9d^4 + a^4b^5e^4 + a^4e^4(-4ac - b^2)^5)^{(1/2)} + b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^2c^4d^4 - 8a^5b^3c^2e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^2e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^2d^4 - 4a^2b^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 - 3a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} - 4a^2b^3d^3e(-4ac - b^2)^5)^{(1/2)} - 4a^3b^2d^2e^3(-4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^2d^3e + 40a^4b^4c^2d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^2c^3d^2e^2 - 128a^5b^2c^2d^2e^3 - 6a^3c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^2b^2c^3d^3e(-4ac - b^2)^5)^{(1/2)} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} * (32768a^16c^8d^2 - 32768a^17c^7e^2 + 1024a^12b^8c^4d^2 - 12288a^13b^6c^5d^2 + 51200a^14b^4c^6d^2 - 81920a^15b^2c^7d^2 + 1024a^14b^6c^4e^2 - 10240a^15b^4c^5e^2 + 32768a^16b^2c^6e^2 + 98304a^16b^2c^7d^2e - 2048a^13b^7c^4d^2e + 22528a^14b^5c^5d^2e - 81920a^15b^3c^6d^2e) * i - 4096a^16b^2c^6e^3 - 12288a^16c^7d^2e^2 + 256a^11b^8c^4d^3 - 2816a^12b^6c^5d^3 + 10496a^13b^4c^6d^3 - 14336a^14b^2c^7d^3 - 256a^14b^5c^4e^3 + 2048a^15b^3c^5e^3 + 24576a^15b^2c^7d^2e - 768a^12b^7c^4d^2e + 7680a^13b^5c^5d^2e + 768a^13b^6c^4d^2e^2 - 24576a^14b^3c^6d^2e - 6912a^14b^4c^5d^2e^2 + 18432a^15b^2c^6d^2e^2) * i - x(4a^11b^2c^8d^6 + 4a^14b^2c^5e^6 - 16a^12c^8d^5e - 16a^14c^6d^2e^5 - 32a^13c^7d^3e^3 + 4a^11b^3c^6d^4e^2 - 32a^12b^2c^6d^3e^3 + 4a^12b^3c^5d^2e^4 - 8a^11b^2c^7d^5e + 44a^12b^2c^7d^4e^2 + 44a^13b^2c^6d^2e^4 - 8a^13b^2c^5d^2e^5)) * (-b^9d^4 + a^4b^5e^4 + a^4e^4(-4ac - b^2)^5)^{(1/2)} + b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^2c^4d^4 - 8a^5b^3c^2e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^2e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 + a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^2d^4 - 4a^2b^8d^3e + 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240
\end{aligned}$$

$$\begin{aligned}
& a^4 b^3 c^2 d^2 e^2 - 3 a^3 b^2 c^2 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 4 a^3 b^3 d^3 \\
& * e (-4 a^2 c - b^2)^5)^{(1/2)} - 4 a^3 b^3 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)} + 48 a^2 \\
& b^6 c^2 d^3 e + 40 a^4 b^4 c^2 d^3 e^3 - 200 a^3 b^4 c^2 d^3 e - 66 a^3 b^5 c^2 \\
& d^2 e^2 + 320 a^4 b^2 c^3 d^3 e - 288 a^5 b^3 c^3 d^2 e^2 - 128 a^5 b^2 c^2 d^2 \\
& e^3 - 6 a^3 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 8 a^2 b^3 c^2 d^3 e (-4 a^2 c \\
& - b^2)^5)^{(1/2)} / (512 (a^5 b^8 + 256 a^9 c^4 - 16 a^6 b^6 c + 96 a^7 b^4 c^2 \\
& - 256 a^8 b^2 c^3))^{(1/4)} * i - ((- (b^9 d^4 + a^4 b^5 e^4 + a^4 e^4 (-4 a^2 c \\
& - b^2)^5)^{(1/2)} + b^4 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} + 80 a^4 b^3 c^4 d^4 - \\
& 8 a^5 b^3 c^2 e^4 + 16 a^6 b^3 c^2 e^4 - 4 a^3 b^6 d^3 e^3 - 128 a^5 c^4 d^3 e + \\
& 128 a^6 c^3 d^3 e^3 + 61 a^2 b^5 c^2 d^4 - 120 a^3 b^3 c^3 d^4 + a^2 c^2 d^4 \\
& * (-4 a^2 c - b^2)^5)^{(1/2)} + 6 a^2 b^7 d^2 e^2 - 13 a^3 b^7 c^2 d^4 - 4 a^3 b^8 d^3 \\
& e + 6 a^2 b^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} + 240 a^4 b^3 c^2 d^2 e^2 \\
& - 3 a^3 b^2 c^2 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} - 4 a^3 b^3 d^3 e (-4 a^2 c - b^2)^5)^{(1/2)} \\
& - 4 a^3 b^3 d^3 e^3 (-4 a^2 c - b^2)^5)^{(1/2)} + 48 a^2 b^6 c^2 d^3 e + 40 a^4 \\
& b^4 c^2 d^3 e^3 - 200 a^3 b^4 c^2 d^3 e - 66 a^3 b^5 c^2 d^2 e^2 + 320 a^4 b^2 \\
& c^3 d^3 e - 288 a^5 b^3 c^3 d^2 e^2 - 128 a^5 b^2 c^2 d^2 e^3 - 6 a^3 c^2 d^2 e^2 \\
& * (-4 a^2 c - b^2)^5)^{(1/2)} + 8 a^2 b^3 c^2 d^3 e (-4 a^2 c - b^2)^5)^{(1/2)} / (512 \\
& * (a^5 b^8 + 256 a^9 c^4 - 16 a^6 b^6 c + 96 a^7 b^4 c^2 - 256 a^8 b^2 c^3)) \\
&)^{(3/4)} * (x (- (b^9 d^4 + a^4 b^5 e^4 + a^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} + b^4 \\
& d^4 (-4 a^2 c - b^2)^5)^{(1/2)} + 80 a^4 b^3 c^4 d^4 - 8 a^5 b^3 c^2 e^4 + 16 a^6 \\
& b^3 c^2 e^4 - 4 a^3 b^6 d^3 e^3 - 128 a^5 c^4 d^3 e + 128 a^6 c^3 d^3 e^3 + 61 a^2 \\
& b^5 c^2 d^4 - 120 a^3 b^3 c^3 d^4 + a^2 c^2 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} \\
&) + 6 a^2 b^7 d^2 e^2 - 13 a^3 b^7 c^2 d^4 - 4 a^3 b^8 d^3 e + 6 a^2 b^2 d^2 e^2 * \\
& (-4 a^2 c - b^2)^5)^{(1/2)} + 240 a^4 b^3 c^2 d^2 e^2 - 3 a^3 b^2 c^2 d^4 (-4 a^2 c \\
& - b^2)^5)^{(1/2)} - 4 a^3 b^3 d^3 e (-4 a^2 c - b^2)^5)^{(1/2)} - 4 a^3 b^3 d^3 e^3 * \\
& (-4 a^2 c - b^2)^5)^{(1/2)} + 48 a^2 b^6 c^2 d^3 e + 40 a^4 b^4 c^2 d^3 e - 200 a^3 \\
& b^4 c^2 d^3 e - 66 a^3 b^5 c^2 d^2 e^2 + 320 a^4 b^2 c^3 d^3 e - 288 a^5 b^3 c^3 \\
& d^2 e^2 - 128 a^5 b^2 c^2 d^2 e^3 - 6 a^3 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{(1/2)} \\
& + 8 a^2 b^3 c^2 d^3 e (-4 a^2 c - b^2)^5)^{(1/2)} / (512 (a^5 b^8 + 256 a^9 c^4 \\
& - 16 a^6 b^6 c + 96 a^7 b^4 c^2 - 256 a^8 b^2 c^3))^{(1/4)} * (32768 a^16 c^8 d^2 \\
& - 32768 a^17 c^7 e^2 + 1024 a^12 b^8 c^4 d^2 - 12288 a^13 b^6 c^5 d^2 + \\
& 51200 a^14 b^4 c^6 d^2 - 81920 a^15 b^2 c^7 d^2 + 1024 a^14 b^6 c^4 e^2 - \\
& 10240 a^15 b^4 c^5 e^2 + 32768 a^16 b^2 c^6 e^2 + 98304 a^16 b^3 c^7 d e - 20 \\
& 48 a^13 b^7 c^4 d e + 22528 a^14 b^5 c^5 d e - 81920 a^15 b^3 c^6 d e) * i - \\
& 4096 a^15 c^8 d^3 + 4096 a^16 b^3 c^6 e^3 + 12288 a^16 c^7 d e^2 - 256 a^11 b^8 \\
& c^4 d^3 + 2816 a^12 b^6 c^5 d^3 - 10496 a^13 b^4 c^6 d^3 + 14336 a^14 b^2 c^7 d^3 \\
& + 256 a^14 b^5 c^4 e^3 - 2048 a^15 b^3 c^5 e^3 - 24576 a^15 b^3 c^7 d^2 e + 768 a^12 \\
& b^7 c^4 d^2 e - 7680 a^13 b^5 c^5 d^2 e - 768 a^13 b^6 c^4 d^2 e^2 + 24576 a^14 \\
& b^3 c^6 d^2 e + 6912 a^14 b^4 c^5 d^2 e^2 - 18432 a^15 b^2 c^6 d^2 e^2) * i - x (4 a^11 \\
& b^3 c^8 d^6 + 4 a^14 b^3 c^5 e^6 - 16 a^12 c^8 d^5 e - 16 a^14 c^6 d^5 e - 32 a^13 c^7 \\
& d^3 e^3 + 4 a^11 b^3 c^6 d^4 e^2 - 32 a^12 b^2 c^6 d^3 e^3 + 4 a^12 b^3 c^5 d^2 e^4 - 8 a^11 \\
& b^2 c^7 d^5 e + 44 a^12 b^3 c^7 d^4 e^2 + 44 a^13 b^3 c^6 d^2 e^4 - 8 a^13 b^2 c^5 d^5 e^5) * \\
& (- (b^9 d^4 + a^4 b^5 e^4 + a^4 e^4 (-4 a^2 c - b^2)^5)^{(1/2)} + b^4 d^4 (-4 a^2 c - b^2)^5)^{(1/2)} \\
& + 80 a^4 b^3 c^4 d^4 - 8 a^5 b^3 c^2 e^4 + 16 a^6 b^3 c^2 e^4 - 4 a^
\end{aligned}$$

$$\begin{aligned}
& 3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - \\
& 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2* \\
& e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5) \\
& ^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 6 \\
& 6*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a \\
& ^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3 \\
& *e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 9 \\
& 6*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(1/4)}*i + 2*a^14*c^5*e^7 + 2*a^11*c^8*d \\
& ^6*e + 6*a^12*c^7*d^4*e^3 + 6*a^13*c^6*d^2*e^5 + 6*a^11*b^2*c^6*d^4*e^3 - 2 \\
& *a^11*b^3*c^5*d^3*e^4 + 6*a^12*b^2*c^5*d^2*e^5 - 6*a^13*b*c^5*d*e^6 - 6*a^1 \\
& 1*b*c^7*d^5*e^2 - 12*a^12*b*c^6*d^3*e^4))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} + b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^ \\
& 4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4* \\
& d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2* \\
& c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a \\
& *b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d \\
& ^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^3*d^3*e*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e \\
& + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320* \\
& a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c \\
& *d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^ \\
& 2*c^3)))^{(1/4)} - 2*atan((((-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2) \\
&)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^ \\
& 3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6* \\
& c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a \\
& ^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2 \\
& *c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c* \\
& d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3* \\
& e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^5*b^8 \\
& + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^{(3/4)}*(\\
& x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e \\
& ^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c \\
& ^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2 \\
& *b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5) \\
&)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a*c - \\
& b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2* \\
& d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^
\end{aligned}$$

$$\begin{aligned}
& 2 - 128a^5b^2c^2d^3e^3 + 6a^3c^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3cd^3e^3(-4ac - b^2)^5)^{(1/2)} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} * (32768a^{16}c^8d^2 - 32768a^{17}c^7e^2 + 1024a^{12}b^8c^4d^2 - 12288a^{13}b^6c^5d^2 + 51200a^{14}b^4c^6d^2 - 81920a^{15}b^2c^7d^2 + 1024a^{14}b^6c^4e^2 - 10240a^{15}b^4c^5e^2 + 32768a^{16}b^2c^6e^2 + 98304a^{16}b^3c^7de - 2048a^{13}b^7c^4de + 22528a^{14}b^5c^5de - 81920a^{15}b^3c^6de) * i - 4096a^{15}c^8d^3 + 4096a^{16}b^3c^6e^3 + 12288a^{16}c^7de^2 - 256a^{11}b^8c^4d^3 + 2816a^{12}b^6c^5d^3 - 10496a^{13}b^4c^6d^3 + 14336a^{14}b^2c^7d^3 + 256a^{14}b^5c^4e^3 - 2048a^{15}b^3c^5e^3 - 24576a^{15}b^3c^7d^2e + 768a^{12}b^7c^4d^2e - 7680a^{13}b^5c^5d^2e - 768a^{13}b^6c^4d^2e^2 + 24576a^{14}b^3c^6d^2e + 6912a^{14}b^4c^5d^2e^2 - 18432a^{15}b^2c^6de^2) * i - x(4a^{11}b^3c^8d^6 + 4a^{14}b^3c^5e^6 - 16a^{12}c^8d^5e - 16a^{14}c^6d^5e - 32a^{13}c^7d^3e^3 + 4a^{11}b^3c^6d^4e^2 - 32a^{12}b^2c^6d^3e^3 + 4a^{12}b^3c^5d^2e^4 - 8a^{11}b^2c^7d^5e + 44a^{12}b^3c^7d^4e^2 + 44a^{13}b^3c^6d^2e^4 - 8a^{13}b^2c^5d^2e^5) * (-b^9d^4 + a^4b^5e^4 - a^4e^4(-4ac - b^2)^5)^{(1/2)} - b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^3d^4 - 4a^2b^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3a^2b^2c^3d^4(-4ac - b^2)^5)^{(1/2)} + 4a^2b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)} + ((-b^9d^4 + a^4b^5e^4 - a^4e^4(-4ac - b^2)^5)^{(1/2)} - b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^3d^4 - 4a^2b^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3a^2b^2c^3d^4(-4ac - b^2)^5)^{(1/2)} + 4a^2b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} + 48a^2b^6c^3d^3e + 40a^4b^4c^3d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3c^3d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(3/4)} * (4096a^{15}c^8d^3 + x(-b^9d^4 + a^4b^5e^4 - a^4e^4(-4ac - b^2)^5)^{(1/2)} - b^4d^4(-4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^4e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13a^2b^7c^3d^4 - 4a^2b^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3
\end{aligned}$$

$$\begin{aligned}
& a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e \\
& - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} \\
& *(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 \\
& - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e \\
& - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e)*1i - 4096*a^16*b*c^6*e^3 - 12288*a^16*c^7*d*e^2 + 256*a^11*b^8*c^4*d^3 \\
& - 2816*a^12*b^6*c^5*d^3 + 10496*a^13*b^4*c^6*d^3 - 14336*a^14*b^2*c^7*d^3 - 256*a^14*b^5*c^4*e^3 + 2048*a^15*b^3*c^5*e^3 + 24576*a^15*b*c^7*d^2*e \\
& - 768*a^12*b^7*c^4*d^2*e + 7680*a^13*b^5*c^5*d^2*e + 768*a^13*b^6*c^4*d*e^2 - 24576*a^14*b^3*c^6*d^2*e - 6912*a^14*b^4*c^5*d*e^2 + 18432*a^15*b^2*c^6*d*e^2)*1i \\
& - x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 \\
& + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5)*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 \\
& - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 \\
& + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 \\
& + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} \\
& /(((b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e \\
& + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 - a^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e - 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 240*a^4*b^3*c^2*d^2*e^2 \\
& + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e \\
& - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3))^{(3/4)} \\
& *(4096*a^15*c^8*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 - a^4*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - b^4*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3 \\
& - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3
\end{aligned}$$

$$\begin{aligned}
& 3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^8d^4(-4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4(-4ac - b^2)^5)^{1/2} + 4ab^3d^3e(-4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4c^2d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3cd^2e^2(-4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e(-4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^1/4 * (32768a^16c^8d^2 - 32768a^17c^7e^2 + 1024a^12b^8c^4d^2 - 12288a^13b^6c^5d^2 + 51200a^14b^4c^6d^2 - 81920a^15b^2c^7d^2 + 1024a^14b^6c^4e^2 - 10240a^15b^4c^5e^2 + 32768a^16b^2c^6e^2 + 98304a^16b^3c^7d^2e - 2048a^13b^7c^4d^2e + 22528a^14b^5c^5d^2e - 81920a^15b^3c^6d^2e) * 1i - 4096a^16b^3c^6e^3 - 12288a^16c^7d^2e^2 + 256a^11b^8c^4d^3 - 2816a^12b^6c^5d^3 + 10496a^13b^4c^6d^3 - 14336a^14b^2c^7d^3 - 256a^14b^5c^4e^3 + 2048a^15b^3c^5e^3 + 24576a^15b^3c^7d^2e - 768a^12b^7c^4d^2e + 7680a^13b^5c^5d^2e + 768a^13b^6c^4d^2e^2 - 24576a^14b^3c^6d^2e - 6912a^14b^4c^5d^2e^2 + 18432a^15b^2c^6d^2e^2) * 1i - x*(4a^11b^3c^8d^6 + 4a^14b^3c^5e^6 - 16a^12c^8d^5e - 16a^14c^6d^5e^5 - 32a^13c^7d^3e^3 + 4a^11b^3c^6d^4e^2 - 32a^12b^2c^6d^3e^3 + 4a^12b^3c^5d^2e^4 - 8a^11b^2c^7d^5e + 44a^12b^3c^7d^4e^2 + 44a^13b^3c^6d^2e^4 - 8a^13b^2c^5d^2e^5) * (-b^9d^4 + a^4b^5e^4 - a^4e^4(-4ac - b^2)^5)^{1/2} - b^4d^4(-4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4(-4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4(-4ac - b^2)^5)^{1/2} + 4ab^3d^3e(-4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4c^2d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3cd^2e^2(-4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e(-4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^1/4 * 1i - ((-b^9d^4 + a^4b^5e^4 - a^4e^4(-4ac - b^2)^5)^{1/2} - b^4d^4(-4ac - b^2)^5)^{1/2} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^3c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^3e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4(-4ac - b^2)^5)^{1/2} + 6a^2b^7d^2e^2 - 13ab^7cd^4 - 4ab^8d^3e - 6a^2b^2d^2e^2(-4ac - b^2)^5)^{1/2} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4(-4ac - b^2)^5)^{1/2} + 4ab^3d^3e(-4ac - b^2)^5)^{1/2} + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{1/2} + 48a^2b^6cd^3e + 40a^4b^4c^2d^3e - 200a^3b^4c^2d^3e - 66a^3b^5c^2d^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^3c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3cd^2e^2(-4ac - b^2)^5)^{1/2} - 8a^2b^3cd^3e(-4ac - b^2)^5)^{1/2}) / (512(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^1/4 * 1i
\end{aligned}$$

$$\begin{aligned}
& 2 - 256a^8b^2c^3))^{(3/4)}(x*(-(b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{(1/2)} - b^4d^4*(-(4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4*(-(4ac - b^2)^5)^{(1/2)} + 4ab^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 48a^2b^6cd^3e + 40a^4b^4cd^3e - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^2c^2d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3cd^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8a^2b^3cd^3e*(-(4ac - b^2)^5)^{(1/2)))/(512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)}*(32768a^16c^8d^2 - 32768a^17c^7e^2 + 1024a^12b^8c^4d^2 - 12288a^13b^6c^5d^2 + 51200a^14b^4c^6d^2 - 81920a^15b^2c^7d^2 + 1024a^14b^6c^4e^2 - 10240a^15b^4c^5e^2 + 32768a^16b^2c^6e^2 + 98304a^16b^3c^7d^2e - 2048a^13b^7c^4d^2e + 22528a^14b^5c^5d^2e - 81920a^15b^3c^6d^2e)*1i - 4096a^15c^8d^3 + 4096a^16b^3c^6e^3 + 12288a^16c^7d^2e^2 - 256a^11b^8c^4d^3 + 2816a^12b^6c^5d^3 - 10496a^13b^4c^6d^3 + 14336a^14b^2c^7d^3 + 256a^14b^5c^4e^3 - 2048a^15b^3c^5e^3 - 24576a^15b^3c^7d^2e + 768a^12b^7c^4d^2e - 7680a^13b^5c^5d^2e - 768a^13b^6c^4d^2e^2 + 24576a^14b^3c^6d^2e + 6912a^14b^4c^5d^2e^2 - 18432a^15b^2c^6d^2e^2)*1i - x*(4a^11b^3c^8d^6 + 4a^14b^3c^5e^6 - 16a^12c^8d^5e - 16a^14c^6d^5e - 32a^13c^7d^3e^3 + 4a^11b^3c^6d^4e^2 - 32a^12b^2c^6d^3e^3 + 4a^12b^3c^5d^2e^4 - 8a^11b^2c^7d^5e + 44a^12b^3c^7d^4e^2 + 44a^13b^3c^6d^2e^4 - 8a^13b^2c^5d^2e^5))*(-(b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{(1/2)} - b^4d^4*(-(4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)} + 240a^4b^3c^2d^2e^2 + 3ab^2cd^4*(-(4ac - b^2)^5)^{(1/2)} + 4ab^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 4a^3b^3d^3e*(-(4ac - b^2)^5)^{(1/2)} + 48a^2b^6cd^3e + 40a^4b^4cd^3e - 200a^3b^4c^2d^3e - 66a^3b^5cd^2e^2 + 320a^4b^2c^3d^3e - 288a^5b^2c^3d^2e^2 - 128a^5b^2c^2d^2e^3 + 6a^3cd^2e^2*(-(4ac - b^2)^5)^{(1/2)} - 8a^2b^3cd^3e*(-(4ac - b^2)^5)^{(1/2)))/(512*(a^5b^8 + 256a^9c^4 - 16a^6b^6c + 96a^7b^4c^2 - 256a^8b^2c^3))^{(1/4)}*1i + 2a^14c^5e^7 + 2a^11c^8d^6e + 6a^12c^7d^4e^3 + 6a^13c^6d^2e^5 + 6a^11b^2c^6d^4e^3 - 2a^11b^3c^5d^3e^4 + 6a^12b^2c^5d^2e^5 - 6a^13b^3c^5d^2e^6 - 6a^11b^3c^7d^5e^2 - 12a^12b^3c^6d^3e^4))*(-(b^9d^4 + a^4b^5e^4 - a^4e^4*(-(4ac - b^2)^5)^{(1/2)} - b^4d^4*(-(4ac - b^2)^5)^{(1/2)} + 80a^4b^3c^4d^4 - 8a^5b^3c^3e^4 + 16a^6b^2c^2e^4 - 4a^3b^6d^3e^3 - 128a^5c^4d^3e + 128a^6c^3d^2e^3 + 61a^2b^5c^2d^4 - 120a^3b^3c^3d^4 - a^2c^2d^4*(-(4ac - b^2)^5)^{(1/2)} + 6a^2b^7d^2e^2 - 13ab^7c^4d^4 - 4ab^8d^3e - 6a^2b^2d^2e^2*(-(4ac - b^2)^5)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
&) + 240*a^4*b^3*c^2*d^2*e^2 + 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a* \\
& b^3*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3 \\
& *b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^ \\
& 2*c^2*d*e^3 + 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^2*b*c*d^3*e*(- \\
& (4*a*c - b^2)^5)^{(1/2)})/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7 \\
& *b^4*c^2 - 256*a^8*b^2*c^3))^{(1/4)} - d/(a*x)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.50 \quad \int \frac{d+ex^4}{x^3(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=199

$$\frac{\sqrt{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{d}{2ax^2}$$

[Out] $-1/2*d/a/x^2-1/4*\arctan(x^2*2^{(1/2)}*c^{(1/2)/(b-(-4*a*c+b^2)^{(1/2))}^{(1/2)})^c$
 $^{(1/2)}*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})/a*2^{(1/2)/(b-(-4*a*c+b^2)^{(1/2))}^{(1/2)}$
 $^{(1/2)}-1/4*\arctan(x^2*2^{(1/2)}*c^{(1/2)/(b+(-4*a*c+b^2)^{(1/2))}^{(1/2)})^c$
 $^{(1/2)}*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})/a*2^{(1/2)/(b+(-4*a*c+b^2)^{(1/2))}^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1490, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b-\sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{\sqrt{b^2-4ac}+b}} \right)}{2\sqrt{2} a \sqrt{\sqrt{b^2-4ac}+b}} - \frac{d}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)), x]

[Out] $-d/(2*a*x^2) - (\text{Sqrt}[c]*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x^2)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x^2)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne

$Q[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 1281

$\text{Int}[(f_)*(x_)]^{(m_)}*((d_)+(e_)*(x_)^2)*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(d*(f*x)^{(m+1)}*(a+b*x^2+c*x^4)^{(p+1)})/(a*f*(m+1)), x] + \text{Dist}[1/(a*f^2*(m+1)), \text{Int}[(f*x)^{(m+2)}*(a+b*x^2+c*x^4)^p*\text{Simp}[a*e*(m+1)-b*d*(m+2*p+3)-c*d*(m+4*p+5)*x^2, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2*p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1490

$\text{Int}[(x_)]^{(m_)}*((a_)+(c_)*(x_)^{(n2_)}+(b_)*(x_)^{(n_)})^{(p_)}*((d_)+(e_)*(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow \text{With}[\{k = \text{GCD}[m+1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m+1)/k-1)}*(d+e*x^{(n/k)})^q*(a+b*x^{(n/k)}+c*x^{((2*n)/k)})^p, x], x, x^k], x] /; k \neq 1] /; \text{FreeQ}[\{a, b, c, d, e, p, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rubi steps

$$\begin{aligned} \int \frac{d+ex^4}{x^3(a+bx^4+cx^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{d+ex^2}{x^2(a+bx^2+cx^4)} dx, x, x^2 \right) \\ &= \frac{d}{2ax^2} - \frac{\text{Subst} \left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4} dx, x, x^2 \right)}{2a} \\ &= \frac{d}{2ax^2} - \frac{\left(c \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} + \frac{1}{2} \sqrt{b^2-4ac} + cx^2} dx, x, x^2 \right)}{4a} - \frac{\left(c \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \right) \text{Subst} \left(\int \frac{1}{\frac{b}{2} - \frac{1}{2} \sqrt{b^2-4ac} + cx^2} dx, x, x^2 \right)}{4a} \\ &= \frac{d}{2ax^2} - \frac{\sqrt{c} \left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b - \sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b - \sqrt{b^2-4ac}}} - \frac{\sqrt{c} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt{2} \sqrt{c} x^2}{\sqrt{b + \sqrt{b^2-4ac}}} \right)}{2\sqrt{2} a \sqrt{b + \sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.45

$$-\frac{\text{RootSum} \left[\#1^8 c + \#1^4 b + a \ \&\& \ \frac{\#1^4 cd \log(x-\#1) - ae \log(x-\#1) + bd \log(x-\#1)}{2\#1^6 c + \#1^2 b} \ \&\& \right]}{4a} - \frac{d}{2ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]

[Out] $-1/2*d/(a*x^2) - \text{RootSum}[a + b*\#1^4 + c*\#1^8 \& , (b*d*\text{Log}[x - \#1] - a*e*\text{Log}[x - \#1] + c*d*\text{Log}[x - \#1]*\#1^4)/(b*\#1^2 + 2*c*\#1^6) \&]/(4*a)$

fricas [B] time = 2.63, size = 2772, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] $1/4*(\text{sqrt}(1/2)*a*x^2*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\text{log}((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 + 1/2*\text{sqrt}(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - \text{sqrt}(1/2)*a*x^2*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\text{log}((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*\text{sqrt}(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + \text{sqrt}(1/2)*a*x^2*\text{sqrt}(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*\text{sqrt}(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*\text{log}((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 + 1/2*\text{sqrt}(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13$

$$\begin{aligned}
& *a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - \\
& 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c) \\
& *e)*\sqrt{-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4* \\
& (a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)} \\
&))*\sqrt{-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3 \\
& *b^2 - 4*a^4*c)*\sqrt{-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2) \\
&)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 \\
& - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - \sqrt{1/2}*a*x^2*\sqrt{-(a^2*b*e^2 + (b \\
& ^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*\sqrt{-(4* \\
& a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c) \\
&)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - \\
& 4*a^4*c))*\log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - \\
& a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*\sqrt{1/2}*((b^5 - 5*a*b^3*c \\
& + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 \\
& - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c \\
& + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*\sqrt{-(4*a^3*b*d*e^3 - a^4*e^4 - \\
& (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 \\
& - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*\sqrt{-(a^2*b*e^2 + (b^3 - 3*a*b*c) \\
&)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*\sqrt{-(4*a^3*b*d*e^3 - \\
& a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(\\
& 3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2 \\
& *d)/(a*x^2)
\end{aligned}$$

giac [B] time = 22.52, size = 3006, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/8*((\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 8*\sqrt{2}*\sqrt{b*c + \\
& \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c) \\
& *b^3*c^2 - 2*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 + \\
& 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + \sqrt{2}*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c}}*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - \\
& 4*a*c}}*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a \\
& *c^3)*d*x^4*\text{abs}(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c}) \\
& *\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{ \\
& b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c \\
& + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (\sqrt{2})*\sqrt{ \\
& b*c + \sqrt{b^2 - 4*a*c}}*c)*b^5 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c) \\
& *a*b^3*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 2*b^5*c + 16*s \\
& \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{ \\
& b^2 - 4*a*c}}*c)*a*b^2*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*b^3*c
\end{aligned}$$

$$\begin{aligned}
&^2 + 16*a*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 32* \\
&a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (\sqrt{2})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4 - 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c - 2*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c \\
&- 2*a*b^4*c + 16*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*c^2 + 8*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + \sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*\sqrt{2}*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2*c^2)*abs(a)*e + (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2)*d - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^3 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^3*b*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c + \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*e)*arctan(2*\sqrt{1/2}*x^2/\sqrt{(a*b + \sqrt{a^2*b^2 - 4*a^3*c}))/a*c))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) - 1/8*((\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 + 2*b^4*c^2 + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a) - (2*a*b^3*c^3 - 8*a^2*b*c^4 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^5 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^4*c + 2*b^5*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (\sqrt{2})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4 - 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b^2*c - 2*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^3*c + 2*a*b^4*c + 16*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^3*c^2 + 8*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*b*c^2 + \sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*\sqrt{2}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(a)*e - (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - \sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a*b^4 + 4*\sqrt{2}*\sqrt{b^2 - 4*a*c})*\sqrt{b*c - \sqrt{b^2 - 4*a*c}}*c)*a^
\end{aligned}$$

$2*b^2*c + 2*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^3$
 $*c - \sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a*b^2*c^2 -$
 $2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - \sqrt{2}*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^3 + 4*\sqrt{2}*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^3*b*c + 2*\sqrt{2}*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b^2*c - \sqrt{2}*\sqrt{2}*\sqrt{b^2 - 4*a*c}*\sqrt{b*c - \sqrt{b^2 - 4*a*c}*c})*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*e)*\arctan(2*\sqrt{1/2}*x^2/\sqrt{(a*b - \sqrt{a^2*b^2 - 4*a^3*c})/(a*c)}))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*\text{abs}(a)*\text{abs}(c)) - 1/2*d/(a*x^2)$

maple [B] time = 0.02, size = 365, normalized size = 1.83

$$\frac{\sqrt{2} bcd \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right) + \sqrt{2} bcd \operatorname{arctan}\left(\frac{\sqrt{2} c x^2}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) - \sqrt{2} ce \operatorname{arctanh}\left(\frac{\sqrt{2} c x^2}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)}{4\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c} a + 4\sqrt{-4ac + b^2} \sqrt{(b + \sqrt{-4ac + b^2})c} a - 2\sqrt{-4ac + b^2} \sqrt{(-b + \sqrt{-4ac + b^2})c} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^4+d)/x^3/(c*x^8+b*x^4+a), x)`

[Out] $1/4/a*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*d-1/2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*e+1/4/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*b*d-1/4/a*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*d-1/2*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*e+1/4/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*c*x^2)*b*d-1/2/a*d/x^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a), x, algorithm="maxima")`

[Out] `-integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2)`

mupad [B] time = 7.62, size = 15013, normalized size = 75.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)), x)$

[Out] $-\text{atan}\left(\frac{\left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} \cdot \left(\frac{\left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} \cdot \left(\frac{\left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} \cdot (4096a^{12}b^6c^4 - 32768a^{13}b^4c^5 + 65536a^{14}b^2c^6) + x^2(9216a^{11}b^5c^5d - 1024a^{10}b^7c^4d - 24576a^{12}b^3c^6d + 1024a^{11}b^6c^4e - 8192a^{12}b^4c^5e + 16384a^{13}b^2c^6e + 16384a^{13}b^3c^7d) \cdot \left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} + 4096a^{12}bc^7d^2 - 4096a^{13}bc^6e^2 + 512a^{10}b^5c^5d^2 - 3072a^{11}b^3c^6d^2 + 1024a^{12}b^3c^5e^2 - 1024a^{11}b^4c^5de + 4096a^{12}b^2c^6de) + x^2(512a^{11}c^8d^3 - 768a^{12}bc^6e^3 - 512a^{12}c^7de^2 - 64a^8b^6c^5d^3 + 448a^9b^4c^6d^3 - 896a^{10}b^2c^7d^3 + 192a^{11}b^3c^5e^3 + 768a^{11}b^3c^7d^2e + 192a^9b^5c^5d^2e - 960a^{10}b^3c^6d^2e - 320a^{10}b^4c^5de^2 + 1408a^{11}b^2c^6de^2) \cdot \left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} + 64a^{10}c^8d^4 + 64a^{12}c^6e^4 + 16a^8b^4c^6d^4 - 64a^9b^2c^7d^4 - 128a^{11}c^7d^2e^2 + 128a^{10}b^2c^6d^2e^2 + 128a^{10}bc^7d^3e - 128a^{11}bc^6de^3 - 64a^9b^3c^6d^3e) + x^2(8a^{11}c^6e^5 - 8a^9c^8d^4e - 4a^8b^3c^6d^3e^2 + 12a^9b^2c^6d^2e^3 - 16a^{10}bc^6de^4 + 4a^8b^2c^7d^4e) \cdot \left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2(-4ac - b^2)^3\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} \cdot 1i - \left(\frac{\left(-\left(b^5d^2 + a^2b^3e^2 + a^2e^2(-4ac - b^2)^3\right)^{1/2} + b^2d^2(-4ac - b^2)^3\right)^{1/2} + 12a^2bc^2d^2 - 2ab^4de - 7ab^3cd^2 - a\right)^{1/2} - 4a^3bce^2 - 16a^3c^2de + 12a^2b^2cde - 2abd^2e(-4ac - b^2)^3\right)^{1/2}}{32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)}\right)^{1/2} \cdot 1i$

$$\begin{aligned}
& *c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b \\
& ^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - \\
& 8*a^4*b^2*c))^{(1/2)}*(((b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3 \\
&)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e \\
& - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3 \\
& c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3 \\
& *b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(((b^5*d^2 + a^2*b^3*e^2 + a^2*e \\
& ^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c \\
& ^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4 \\
& *a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2 \\
&)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(4096*a^12*b^6 \\
& *c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d \\
& - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a \\
& ^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^ \\
& 2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(\\
& 1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - \\
& b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d \\
& *e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/ \\
& 2)} + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 307 \\
& 2*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a \\
& ^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^ \\
& 7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + \\
& 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960* \\
& a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(\\
& b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d \\
& *e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4* \\
& b^2*c))^{(1/2)} + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 6 \\
& 4*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a \\
& ^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^1 \\
& 1*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^ \\
& 3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + \\
& a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^ \\
& 2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c \\
& - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*i)/(((b^5 \\
& *d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d \\
& *e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4* \\
& b^2*c))^{(1/2)}*(((b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
&) + b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a \\
& *b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2* \\
& d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 +
\end{aligned}$$

$$\begin{aligned}
& (16a^5c^2 - 8a^4b^2c))^{(1/2)} * (((- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} * (4096a^{12}b^6c^4 - 32768a^{13}b^4c^5 + 65536a^{14}b^2c^6) + x^2(9216a^{11}b^5c^5d - 1024a^{10}b^7c^4d - 24576a^{12}b^3c^6d + 1024a^{11}b^6c^4e - 8192a^{12}b^4c^5e + 16384a^{13}b^2c^6e + 16384a^{13}b^3c^7d)) * (- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} + 4096a^{12}b^3c^7d^2 - 4096a^{13}b^3c^6e^2 + 512a^{10}b^5c^5d^2 - 3072a^{11}b^3c^6d^2 + 1024a^{12}b^3c^5e^2 - 1024a^{11}b^4c^5de + 4096a^{12}b^2c^6de + x^2(512a^{11}c^8d^3 - 768a^{12}b^3c^6e^3 - 512a^{12}c^7de^2 - 64a^8b^6c^5d^3 + 448a^9b^4c^6d^3 - 896a^{10}b^2c^7d^3 + 192a^{11}b^3c^5e^3 + 768a^{11}b^3c^7d^2e + 192a^9b^5c^5d^2e - 960a^{10}b^3c^6d^2e - 320a^{10}b^4c^5de^2 + 1408a^{11}b^2c^6de^2)) * (- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} + 64a^{10}c^8d^4 + 64a^{12}c^6e^4 + 16a^8b^4c^6d^4 - 64a^9b^2c^7d^4 - 128a^{11}c^7d^2e^2 + 128a^{10}b^2c^6d^2e^2 + 128a^{10}b^2c^7d^3e - 128a^{11}b^3c^6d^3e - 64a^9b^3c^6d^3e) + x^2(8a^{11}c^6e^5 - 8a^9c^8d^4e - 4a^8b^3c^6d^3e^2 + 12a^9b^2c^6d^2e^3 - 16a^{10}b^3c^6de^4 + 4a^8b^2c^7d^4e)) * (- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} + (((- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} * (((- (b^5d^2 + a^2b^3e^2 + a^2e^2 * (- (4ac - b^2)^3)^{(1/2)} + b^2d^2 * (- (4ac - b^2)^3)^{(1/2)} + 12a^2b^2c^2d^2 - 2ab^4de - 7ab^3cd^2 - acd^2 * (- (4ac - b^2)^3)^{(1/2)} - 4a^3b^2c^2e^2 - 16a^3c^2de + 12a^2b^2cd^2 - 2abd^2 * (- (4ac - b^2)^3)^{(1/2)})) / (32(a^3b^4 + 16a^5c^2 - 8a^4b^2c)))^{(1/2)} * (4096a^{12}b^6c^4 - 32768a^{13}b^4c^5 + 65536a^{14}b^2c^6) - x^2(9216a^{11}b^5c^5d - 1024a^{10}b^7
\end{aligned}$$

$$\begin{aligned}
& *c^4*d - 24576*a^{12}*b^3*c^6*d + 1024*a^{11}*b^6*c^4*e - 8192*a^{12}*b^4*c^5*e + \\
& 16384*a^{13}*b^2*c^6*e + 16384*a^{13}*b*c^7*d)) * (-(b^5*d^2 + a^2*b^3*e^2 + a^2 \\
& *e^2 * (-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b \\
& *c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e * (-(4*a*c - b \\
& ^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 4096*a^{12} \\
& *b*c^7*d^2 - 4096*a^{13}*b*c^6*e^2 + 512*a^{10}*b^5*c^5*d^2 - 3072*a^{11}*b^3*c^6* \\
& d^2 + 1024*a^{12}*b^3*c^5*e^2 - 1024*a^{11}*b^4*c^5*d*e + 4096*a^{12}*b^2*c^6*d*e \\
&) - x^2*(512*a^{11}*c^8*d^3 - 768*a^{12}*b*c^6*e^3 - 512*a^{12}*c^7*d*e^2 - 64*a^ \\
& 8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^{10}*b^2*c^7*d^3 + 192*a^{11}*b^3*c \\
& ^5*e^3 + 768*a^{11}*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^{10}*b^3*c^6*d^ \\
& 2*e - 320*a^{10}*b^4*c^5*d*e^2 + 1408*a^{11}*b^2*c^6*d*e^2) * (-(b^5*d^2 + a^2*b \\
& ^3*e^2 + a^2*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} \\
&) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 * (-(4*a*c - b^2 \\
&)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e * \\
& (-(4*a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} \\
& + 64*a^{10}*c^8*d^4 + 64*a^{12}*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d \\
& ^4 - 128*a^{11}*c^7*d^2*e^2 + 128*a^{10}*b^2*c^6*d^2*e^2 + 128*a^{10}*b*c^7*d^3*e \\
& - 128*a^{11}*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^{11}*c^6*e^5 - 8*a \\
& ^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^{10}*b*c \\
& ^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e) * (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2 * (-(4*a* \\
& c - b^2)^3)^{(1/2)} + b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2 \\
& *a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e \\
& ^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e * (-(4*a*c - b^2)^3)^{(1/2)} \\
&) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}) * (-(b^5*d^2 + a^2*b^3*e \\
& ^2 + a^2*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} + b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} + \\
& 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2 * (-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e * (-(4 \\
& *a*c - b^2)^3)^{(1/2)}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * 2i - \\
& \operatorname{atan}(\frac{(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} - b^2*d \\
& ^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^ \\
& 2 + a*c*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12* \\
& a^2*b^2*c*d*e + 2*a*b*d*e * (-(4*a*c - b^2)^3)^{(1/2)})}{(32*(a^3*b^4 + 16*a^5*c \\
& ^2 - 8*a^4*b^2*c))^{(1/2)} * ((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2 * (-(4*a*c - b \\
& ^2)^3)^{(1/2)} - b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^ \\
& 4*d*e - 7*a*b^3*c*d^2 + a*c*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - \\
& 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e * (-(4*a*c - b^2)^3)^{(1/2)})}{(32 \\
& *(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * ((-(b^5*d^2 + a^2*b^3*e^2 - \\
& a^2*e^2 * (-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} + 12*a^ \\
& 2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2 * (-(4*a*c - b^2)^3)^{(1/2)} \\
&) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e * (-(4*a*c \\
& - b^2)^3)^{(1/2)})}{(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}} * (4096*a^1 \\
& 2*b^6*c^4 - 32768*a^{13}*b^4*c^5 + 65536*a^{14}*b^2*c^6) + x^2*(9216*a^{11}*b^5*c \\
& ^5*d - 1024*a^{10}*b^7*c^4*d - 24576*a^{12}*b^3*c^6*d + 1024*a^{11}*b^6*c^4*e - 8 \\
& 192*a^{12}*b^4*c^5*e + 16384*a^{13}*b^2*c^6*e + 16384*a^{13}*b*c^7*d)) * (-(b^5*d^2
\end{aligned}$$

$$\begin{aligned}
& + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} + 4096a^{12} b^3 c^7 d^2 - 4096a^{13} b^3 c^6 e^2 + 512a^{10} b^5 c^5 d^2 - 3072a^{11} b^3 c^6 d^2 + 1024a^{12} b^3 c^5 e^2 - 1024a^{11} b^4 c^5 d^2 e + 4096a^{12} b^2 c^6 d^2 e \\
& + x^2 (512a^{11} c^8 d^3 - 768a^{12} b^3 c^6 e^3 - 512a^{12} c^7 d^2 e^2 - 64a^8 b^6 c^5 d^3 + 448a^9 b^4 c^6 d^3 - 896a^{10} b^2 c^7 d^3 + 192a^{11} b^3 c^5 e^3 + 768a^{11} b^3 c^7 d^2 e + 192a^9 b^5 c^5 d^2 e - 960a^{10} b^3 c^6 d^2 e - 320a^{10} b^4 c^5 d^2 e^2 + 1408a^{11} b^2 c^6 d^2 e^2) \\
&) * (-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} + 64a^{10} c^8 d^4 + 64a^{12} c^6 e^4 + 16a^8 b^4 c^6 d^4 - 64a^9 b^2 c^7 d^4 - 128a^{11} c^7 d^2 e^2 + 128a^{10} b^2 c^6 d^2 e^2 + 128a^{10} b^3 c^7 d^3 e - 128a^{11} b^3 c^6 d^3 e^3 - 64a^9 b^3 c^6 d^3 e + x^2 (8a^{11} c^6 e^5 - 8a^9 c^8 d^4 e - 4a^8 b^3 c^6 d^3 e^2 + 12a^9 b^2 c^6 d^2 e^3 - 16a^{10} b^3 c^6 d^2 e^4 + 4a^8 b^2 c^7 d^4 e) \\
&) * (-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} * i - ((-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} * (((-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} * (((-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} * (4096a^{12} b^6 c^4 - 32768a^{13} b^4 c^5 + 65536a^{14} b^2 c^6) - x^2 (9216a^{11} b^5 c^5 d - 1024a^{10} b^7 c^4 d - 24576a^{12} b^3 c^6 d + 1024a^{11} b^6 c^4 e - 8192a^{12} b^4 c^5 e + 16384a^{13} b^2 c^6 e + 16384a^{13} b^3 c^7 d) \\
&) * (-b^5 d^2 + a^2 b^3 e^2 - a^2 e^2 (-4ac - b^2)^3)^{(1/2)} - b^2 d^2 (-4ac - b^2)^3)^{(1/2)} + 12a^2 b^3 c^2 d^2 - 2a^2 b^4 d^2 e - 7a^2 b^3 c^2 d^2 + a^2 c^2 d^2 (-4ac - b^2)^3)^{(1/2)} - 4a^3 b^3 c^2 e^2 - 16a^3 c^2 d^2 e + 12a^2 b^2 c^2 d^2 e + 2a^2 b^3 d^2 e (-4ac - b^2)^3)^{(1/2)} / (32(a^3 b^4 + 16a^5 c^2 - 8a^4 b^2 c)) \\
&)^{(1/2)} + 4096a^{12} b^3 c^7 d^2 - 4096a^{13} b^3 c^6 e^2 + 512a^{10} b^5 c^5 d^2 - 3072a^{11} b^3 c^6 d^2 + 1024a^{12} b^3 c^5 e^2 - 1024a^{11} b^4 c^5 d^2 e + 4096a^{12}
\end{aligned}$$

$$\begin{aligned}
& 12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7 \\
& *d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + \\
& 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a \\
& ^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b \\
& ^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2* \\
& (-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d* \\
& e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64 \\
& *a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 + 128*a^ \\
& 10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^11 \\
& *c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 \\
& - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a \\
& ^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2 \\
& *b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - \\
& b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*1i)/(((-(b \\
& ^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a* \\
& b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d \\
& *e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + \\
& 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a* \\
& b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d \\
& *e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + \\
& 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4 \\
& *a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 \\
& - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b* \\
& c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1 \\
& /2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)}*(4096*a^12*b^6*c^4 - \\
& 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*c^5*d - 1024* \\
& a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a^12*b^4 \\
& *c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e \\
& ^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + \\
& 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4 \\
& *a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 40 \\
& 96*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 3072*a^11* \\
& b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a^12*b^2 \\
& *c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^7*d*e^2 \\
& - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 + 192*a^ \\
& 11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^10*b^ \\
& 3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(b^5*d^2 \\
& + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a
\end{aligned}$$

$$\begin{aligned}
& *c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2* \\
& a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) \\
&)^{(1/2)} + 64*a^{10}*c^8*d^4 + 64*a^{12}*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b \\
& ^2*c^7*d^4 - 128*a^{11}*c^7*d^2*e^2 + 128*a^{10}*b^2*c^6*d^2*e^2 + 128*a^{10}*b*c \\
& ^7*d^3*e - 128*a^{11}*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*(8*a^{11}*c^6*e \\
& ^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16* \\
& a^{10}*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2 \\
& *d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a \\
& ^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^ \\
& 3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + ((-(b^5*d^2 + \\
& a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3) \\
& ^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c \\
& - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b \\
& *d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * \\
& (((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2 \\
& *(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 \\
& + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a \\
& ^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^ \\
& 2 - 8*a^4*b^2*c))^{(1/2)} * (((-(b^5*d^2 + a^2*b^3*e^2 - a^2*e^2*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b*c^2*d^2 - 2*a*b^4 \\
& *d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - 4*a^3*b*c*e^2 - 1 \\
& 6*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^3)^{(1/2)})/(32* \\
& (a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} * (4096*a^{12}*b^6*c^4 - 32768*a^{1 \\
& 3}*b^4*c^5 + 65536*a^{14}*b^2*c^6) - x^2*(9216*a^{11}*b^5*c^5*d - 1024*a^{10}*b^7* \\
& c^4*d - 24576*a^{12}*b^3*c^6*d + 1024*a^{11}*b^6*c^4*e - 8192*a^{12}*b^4*c^5*e + \\
& 16384*a^{13}*b^2*c^6*e + 16384*a^{13}*b*c^7*d))*(-(b^5*d^2 + a^2*b^3*e^2 - a^2* \\
& e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} + 12*a^2*b* \\
& c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2)^3)^{(1/2)} - \\
& 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(-(4*a*c - b^ \\
& 2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + 4096*a^{12}*b \\
& *c^7*d^2 - 4096*a^{13}*b*c^6*e^2 + 512*a^{10}*b^5*c^5*d^2 - 3072*a^{11}*b^3*c^6*d \\
& ^2 + 1024*a^{12}*b^3*c^5*e^2 - 1024*a^{11}*b^4*c^5*d*e + 4096*a^{12}*b^2*c^6*d*e) \\
& - x^2*(512*a^{11}*c^8*d^3 - 768*a^{12}*b*c^6*e^3 - 512*a^{12}*c^7*d*e^2 - 64*a^8 \\
& *b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^{10}*b^2*c^7*d^3 + 192*a^{11}*b^3*c^ \\
& 5*e^3 + 768*a^{11}*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*a^{10}*b^3*c^6*d^2 \\
& *e - 320*a^{10}*b^4*c^5*d*e^2 + 1408*a^{11}*b^2*c^6*d*e^2))*(-(b^5*d^2 + a^2*b^ \\
& 3*e^2 - a^2*e^2*(-(4*a*c - b^2)^3)^{(1/2)} - b^2*d^2*(-(4*a*c - b^2)^3)^{(1/2)} \\
& + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2*(-(4*a*c - b^2) \\
& ^3)^{(1/2)} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e*(\\
& -(4*a*c - b^2)^3)^{(1/2)})/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{(1/2)} + \\
& 64*a^{10}*c^8*d^4 + 64*a^{12}*c^6*e^4 + 16*a^8*b^4*c^6*d^4 - 64*a^9*b^2*c^7*d^ \\
& 4 - 128*a^{11}*c^7*d^2*e^2 + 128*a^{10}*b^2*c^6*d^2*e^2 + 128*a^{10}*b*c^7*d^3*e \\
& - 128*a^{11}*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) - x^2*(8*a^{11}*c^6*e^5 - 8*a^ \\
& 9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*d^2*e^3 - 16*a^{10}*b*c^
\end{aligned}$$

$$6*d*e^4 + 4*a^8*b^2*c^7*d^4*e)) * (- (b^5*d^2 + a^2*b^3*e^2 - a^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - b^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{1/2})) * (- (b^5*d^2 + a^2*b^3*e^2 - a^2*e^2 * (- (4*a*c - b^2)^3)^{1/2} - b^2*d^2 * (- (4*a*c - b^2)^3)^{1/2} + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 + a*c*d^2 * (- (4*a*c - b^2)^3)^{1/2} - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e + 2*a*b*d*e * (- (4*a*c - b^2)^3)^{1/2}) / (32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))^{1/2}) * 2i - d / (2*a*x^2)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a), x)

[Out] Timed out

$$3.51 \quad \int \frac{d+ex^4}{x^4(a+bx^4+cx^8)} dx$$

Optimal. Leaf size=394

$$\frac{c^{3/4} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac} - b \right)^{3/4}} + \frac{c^{3/4} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(\sqrt{b^2-4ac} - b \right)^{3/4}} + \frac{c^{3/4} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac} - b \right)^{3/4}}$$

[Out] $-1/3*d/a/x^3+1/4*c^{(3/4)}*arctan(2^{(1/4)}*c^{(1/4)}*x/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/a/(-b-(-4*a*c+b^2)^{(1/2)})^{(3/4)}+1/4*c^{(3/4)}*arctanh(2^{(1/4)}*c^{(1/4)}*x/(-b-(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(d+(2*a*e-b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/a/(-b-(-4*a*c+b^2)^{(1/2)})^{(3/4)}+1/4*c^{(3/4)}*arctan(2^{(1/4)}*c^{(1/4)}*x/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/a/(-b+(-4*a*c+b^2)^{(1/2)})^{(3/4)}+1/4*c^{(3/4)}*arctanh(2^{(1/4)}*c^{(1/4)}*x/(-b+(-4*a*c+b^2)^{(1/2)})^{(1/4)})*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^{(1/2)})*2^{(3/4)}/a/(-b+(-4*a*c+b^2)^{(1/2)})^{(3/4)}$

Rubi [A] time = 0.63, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1504, 1422, 212, 208, 205}

$$\frac{c^{3/4} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac} - b \right)^{3/4}} + \frac{c^{3/4} \left(\frac{bd-2ae}{\sqrt{b^2-4ac}} + d \right) \tan^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(\sqrt{b^2-4ac} - b \right)^{3/4}} + \frac{c^{3/4} \left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}} \right)}{2\sqrt[4]{2} a \left(-\sqrt{b^2-4ac} - b \right)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)), x]

[Out] $-d/(3*a*x^3) + (c^{(3/4)}*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) + (c^{(3/4)}*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) + (c^{(3/4)}*(d - (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b - \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*a*(-b - \text{Sqrt}[b^2 - 4*a*c])^{(3/4)}) + (c^{(3/4)}*(d + (b*d - 2*a*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(2^{(1/4)}*c^{(1/4)}*x)/(-b + \text{Sqrt}[b^2 - 4*a*c])^{(1/4)}])/(2*2^{(1/4)}*a*(-b + \text{Sqrt}[b^2 - 4*a*c])^{(3/4)})$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1422

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a*c] || !IGtQ[n/2, 0])

Rule 1504

Int[((f_.)*(x_)^(m_))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m+1)*(a + b*x^n + c*x^(2*n))^(p+1))/(a*f*(m+1)), x] + Dist[1/(a*f^n*(m+1)), Int[(f*x)^(m+n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m+1) - b*d*(m+n*(p+1)+1] - c*d*(m+2*n*(p+1)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{d + ex^4}{x^4(a + bx^4 + cx^8)} dx &= -\frac{d}{3ax^3} - \frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8} dx}{3a} \\
&= -\frac{d}{3ax^3} - \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} - \frac{\left(c\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^4} dx}{2a} \\
&= -\frac{d}{3ax^3} + \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} - \sqrt{2}\sqrt{c}x^2} dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}} + \sqrt{2}\sqrt{c}x^2} dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} \\
&= -\frac{d}{3ax^3} + \frac{c^{3/4}\left(d - \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(d + \frac{bd-2ae}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.22

$$\frac{3\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4cd \log(x-\#1) - ae \log(x-\#1) + bd \log(x-\#1)}{2\#1^7c + \#1^3b}\right] \& + \frac{4d}{x^3}}{12a}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]

[Out] -1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &])/a

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")

[Out] Timed out

maple [C] time = 0.01, size = 68, normalized size = 0.17

$$\frac{\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right)^4 cd + ae - bd\right) \ln\left(-\text{RootOf}\left(-Z^8c + Z^4b + a\right) + x\right)}{4a\left(2\text{RootOf}\left(-Z^8c + Z^4b + a\right)^7 c + \text{RootOf}\left(-Z^8c + Z^4b + a\right)^3 b\right)} - \frac{d}{3ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)

[Out] -1/3/a*d/x^3+1/4/a*sum((-R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(-R+x),_R=RootOf(-Z^8*c+Z^4*b+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 10.22, size = 65350, normalized size = 165.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)

[Out] atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^

$$\begin{aligned}
& 4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(3/4)} - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^
\end{aligned}$$

$$\begin{aligned}
& 5 + 192a^{12}c^9d^4e - 16a^9b^5c^7d^5 + 96a^{10}b^3c^8d^5 + 16a^{13} \\
& *b^2c^6e^5 + 128a^{13}c^8d^2e^3 - 64a^{10}b^5c^6d^3e^2 + 288a^{11}b^3 \\
& c^7d^3e^2 + 96a^{11}b^4c^6d^2e^3 - 416a^{12}b^2c^7d^2e^3 + 256a^{13} \\
& b^3c^7d^3e^2 + 16a^9b^6c^6d^4e - 48a^{10}b^4c^7d^4e - 112a^{11}b^2 \\
& c^8d^4e - 128a^{12}b^3c^8d^3e^2 - 64a^{12}b^3c^6d^4e + x(8a^{13}c \\
& ^7e^6 - 8a^{10}c^{10}d^6 + 4a^9b^2c^9d^6 - 8a^{11}c^9d^4e^2 + 8a^{12} \\
& c^8d^2e^4 + 4a^9b^4c^7d^4e^2 + 16a^{10}b^2c^8d^4e^2 - 16a^{10}b^3 \\
& c^7d^3e^3 + 28a^{11}b^2c^7d^2e^4 + 8a^{10}b^3c^9d^5e - 24a^{12}b^3c^7 \\
& *d^4e^5 - 8a^9b^3c^8d^5e - 16a^{11}b^3c^8d^3e^3) * (- (b^{11}d^4 + a^4b^7 \\
& e^4 + b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^3c^5d^4 - 11a^5b^5c \\
& *e^4 - 48a^7b^3c^3e^4 - a^5c^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^4 \\
& e^3 + 128a^6c^5d^3e - 128a^7c^4d^4e^3 + 86a^2b^7c^2d^4 - 231a^3b^5 \\
& c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} + \\
& a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 \\
& - 15a^9b^3c^4d^4 - 4a^9b^10d^3e + 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} \\
& + 6a^2b^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 \\
& - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} - \\
& 5a^9b^4c^4d^4 * (- (4ac - b^2)^5)^{1/2} - 4a^9b^5d^3e * (- (4ac - b^2)^5)^{1/2} \\
& + 56a^2b^8c^3d^3e + 48a^4b^6c^3d^3e - 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} \\
& - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e \\
& - 200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e + 16a^2b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} \\
& - 12a^3b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} \\
& + 8a^4b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} \\
& * i - ((- (b^{11}d^4 + a^4b^7e^4 + b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 - a^5c^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^4e^3 + 128a^6c^5d^3e - 128a^7c^4d^4e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} + a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a^9b^3c^4d^4 - 4a^9b^10d^3e + 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 5a^9b^4c^4d^4 * (- (4ac - b^2)^5)^{1/2} - 4a^9b^5d^3e * (- (4ac - b^2)^5)^{1/2} + 56a^2b^8c^3d^3e + 48a^4b^6c^3d^3e - 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e + 16a^2b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} - 12a^3b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 8a^4b^3c^3d^3e * (- (4ac - b^2)^5)^{1/2} / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} * ((- (b^{11}d^4 + a^4b^7e^4 + b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 - a^5c^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^4e^3 + 128a^6c^5d^3e - 128a^7c^4d^4e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - 231a^3b^5c^3d^4 + 2
\end{aligned}$$

$$\begin{aligned}
& 80a^4b^3c^4d^4 - a^3c^3d^4(-4ac - b^2)^5)^{(1/2)} + a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c \\
& *d^4 - 4ab^{10}d^3e + 6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^4d^2e^2(-4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5b^3 \\
& c^3d^2e^2 + 6a^4c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 5ab^4c^3d^4(-4ac - b^2)^5)^{(1/2)} - 4ab^5d^3e(-4ac - b^2)^5)^{(1/2)} + 56a^2b^8 \\
& c^3d^3e + 48a^4b^6c^3d^3e - 4a^3b^3d^3e(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 6 \\
& 40a^5b^2c^4d^3e - 200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e + 16a^2b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 12a^3b^3c^2 \\
& d^3e(-4ac - b^2)^5)^{(1/2)} - 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} / (512(a^7b^8 + 256a^1 \\
& 1c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{(1/4)} * (262144a^{17}c^8d + 4096a^{13}b^8c^4d - 53248a^{14}b^6c^5d + 245760a^{15}b^4c^6 \\
& d - 458752a^{16}b^2c^7d - 4096a^{14}b^7c^4e + 49152a^{15}b^5c^5e - 196608a^{16}b^3c^6e + 262144a^{17}b^3c^7e) - x(81920a^{15}b^3c^8d^2 - 49 \\
& 152a^{16}b^3c^7e^2 + 1024a^{11}b^9c^4d^2 - 13312a^{12}b^7c^5d^2 + 62464a^{13}b^5c^6d^2 - 122880a^{14}b^3c^7d^2 + 1024a^{13}b^7c^4e^2 - 11264 \\
& a^{14}b^5c^5e^2 + 40960a^{15}b^3c^6e^2 - 65536a^{16}c^8d^2e - 2048a^{12}b^8c^4d^2e + 24576a^{13}b^6c^5d^2e - 102400a^{14}b^4c^6d^2e + 163840a^{15} \\
& b^2c^7d^2e) * (-b^{11}d^4 + a^4b^7e^4 + b^6d^4(-4ac - b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 - a^5c^4e^4(- \\
& (4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3 \\
& c^3d^4(-4ac - b^2)^5)^{(1/2)} + a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c^3d^4 - 4ab^{10}d^3e + \\
& 6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^4d^2e^2(-4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2 \\
& d^2e^2(-4ac - b^2)^5)^{(1/2)} - 5ab^4c^3d^4(-4ac - b^2)^5)^{(1/2)} - 4ab^5d^3e(-4ac - b^2)^5)^{(1/2)} + 56a^2b^8c^3d^3e + 48a^4b^6 \\
& c^3d^3e - 4a^3b^3d^3e(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - \\
& 200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e + 16a^2b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 12a^3b^3c^2d^3e(-4ac - b^2)^5)^{(1/2)} \\
& - 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^4b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10} \\
& b^2c^3))^{(3/4)} - 64a^{14}c^7e^5 - 128a^{11}b^3c^9d^5 + 192a^{12}c^9d^4e - 16a^9b^5c^7d^5 + 96a^{10}b^3c^8d^5 + 16 \\
& a^{13}b^2c^6e^5 + 128a^{13}c^8d^2e^3 - 64a^{10}b^5c^6d^3e^2 + 288a^{11}b^3c^7d^3e^2 + 96a^{11}b^4c^6d^2e^3 - 416a^{12}b^2c^7d^2e^3 + 2 \\
& 56a^{13}b^3c^7d^3e^4 + 16a^9b^6c^6d^4e - 48a^{10}b^4c^7d^4e - 112a^{11}b^2c^8d^4e - 128a^{12}b^3c^6d^4e - 64a^{12}b^3c^6d^4e) - x(8a^{13} \\
& c^7e^6 - 8a^{10}c^{10}d^6 + 4a^9b^2c^9d^6 - 8a^{11}c^9d^4e^2 + 8a^{12}c^8d^2e^4 + 4a^9b^4c^7d^4e^2 + 16a^{10}b^2c^8d^4e^2 - 16a^{10} \\
& b^3c^7d^3e^3 + 28a^{11}b^2c^7d^2e^4 + 8a^{10}b^3c^9d^5e - 24a^{12}
\end{aligned}$$

$$\begin{aligned}
& b^7 c^7 d^5 e^5 - 8 a^9 b^3 c^8 d^5 e - 16 a^{11} b^3 c^8 d^3 e^3) * (- (b^{11} d^4 + a^4 b^7 e^4 + b^6 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^3 e^4 - 48 a^7 b^3 c^3 e^4 - a^5 c^3 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a^3 b^8 d^3 e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^3 e^3 + 86 a^2 b^7 c^2 d^4 - 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 - a^3 c^3 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + a^4 b^2 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 - 15 a b^9 c d^4 - 4 a b^{10} d^3 e + 6 a^2 b^2 c^2 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 6 a^2 b^4 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} + 366 a^4 b^5 c^2 d^2 e^2 - 720 a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} - 5 a b^4 c d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a b^5 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} + 56 a^2 b^8 c d^3 e + 48 a^4 b^6 c d^3 e - 4 a^3 b^3 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c d^2 e^2 + 680 a^4 b^4 c^3 d^3 e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^3 e + 480 a^6 b^3 c^4 d^2 e^2 + 320 a^6 b^2 c^3 d^3 e + 16 a^2 b^3 c d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 12 a^3 b^3 c^2 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 18 a^3 b^2 c^2 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} + 8 a^4 b^3 c d^3 e * (- (4 a^2 c - b^2)^5)^{1/2}) / (512 * (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * i) / (((- (b^{11} d^4 + a^4 b^7 e^4 + b^6 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^3 e^4 - 48 a^7 b^3 c^3 e^4 - a^5 c^3 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a^3 b^8 d^3 e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^3 e^3 + 86 a^2 b^7 c^2 d^4 - 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 - a^3 c^3 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + a^4 b^2 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 - 15 a b^9 c d^4 - 4 a b^{10} d^3 e + 6 a^2 b^2 c^2 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 6 a^2 b^4 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} + 366 a^4 b^5 c^2 d^2 e^2 - 720 a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} - 5 a b^4 c d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a b^5 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} + 56 a^2 b^8 c d^3 e + 48 a^4 b^6 c d^3 e - 4 a^3 b^3 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c d^2 e^2 + 680 a^4 b^4 c^3 d^3 e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^3 e + 480 a^6 b^3 c^4 d^2 e^2 + 320 a^6 b^2 c^3 d^3 e + 16 a^2 b^3 c d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 12 a^3 b^3 c^2 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 18 a^3 b^2 c^2 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} + 8 a^4 b^3 c d^3 e * (- (4 a^2 c - b^2)^5)^{1/2}) / (512 * (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * (((- (b^{11} d^4 + a^4 b^7 e^4 + b^6 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^3 e^4 - 48 a^7 b^3 c^3 e^4 - a^5 c^3 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a^3 b^8 d^3 e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^3 e^3 + 86 a^2 b^7 c^2 d^4 - 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 - a^3 c^3 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + a^4 b^2 e^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 - 15 a b^9 c d^4 - 4 a b^{10} d^3 e + 6 a^2 b^2 c^2 d^4 * (- (4 a^2 c - b^2)^5)^{1/2} + 6 a^2 b^4 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} + 366 a^4 b^5 c^2 d^2 e^2 - 720 a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 * (- (4 a^2 c - b^2)^5)^{1/2} - 5 a b^4 c d^4 * (- (4 a^2 c - b^2)^5)^{1/2} - 4 a b^5 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} + 56 a^2 b^8 c d^3 e + 48 a^4 b^6 c d^3 e - 4 a^3 b^3 d^3 e * (- (4 a^2 c - b^2)^5)^{1/2} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c d^2 e^2 + 680 a^4 b^4 c^3 d^3 e
\end{aligned}$$

$$\begin{aligned}
& e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + \\
& 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} / (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)} * (262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e) * (-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} / (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(3/4)} - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3) * (-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5
\end{aligned}$$

$$\begin{aligned}
& *c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)} + (((-b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*(((- (b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
&)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d
\end{aligned}$$

$$\begin{aligned}
&^2 - 49152*a^{16}*b*c^7*e^2 + 1024*a^{11}*b^9*c^4*d^2 - 13312*a^{12}*b^7*c^5*d^2 \\
&+ 62464*a^{13}*b^5*c^6*d^2 - 122880*a^{14}*b^3*c^7*d^2 + 1024*a^{13}*b^7*c^4*e^2 \\
&- 11264*a^{14}*b^5*c^5*e^2 + 40960*a^{15}*b^3*c^6*e^2 - 65536*a^{16}*c^8*d*e - 20 \\
&48*a^{12}*b^8*c^4*d*e + 24576*a^{13}*b^6*c^5*d*e - 102400*a^{14}*b^4*c^6*d*e + 16 \\
&3840*a^{15}*b^2*c^7*d*e)) * (-(b^{11}*d^4 + a^4*b^7*e^4 + b^6*d^4 * (-(4*a*c - b^2) \\
&^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c \\
&*e^4 * (-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a \\
&^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d \\
&^4 - a^3*c^3*d^4 * (-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4 * (-(4*a*c - b^2)^5)^{(1/2)} \\
&+ 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4 * (-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2 * (-(4 \\
&*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + \\
&6*a^4*c^2*d^2*e^2 * (-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4 * (-(4*a*c - b^2)^ \\
&5)^{(1/2)} - 4*a*b^5*d^3*e * (-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48 \\
&*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3 * (-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c \\
&^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d \\
&^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^ \\
&3 + 16*a^2*b^3*c*d^3*e * (-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e * (-(4*a \\
&*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2 * (-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b \\
&*c*d*e^3 * (-(4*a*c - b^2)^5)^{(1/2)}) / (512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b \\
&^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3)))^{(3/4)} - 64*a^{14}*c^7*e^5 - 128*a \\
&^{11}*b*c^9*d^5 + 192*a^{12}*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^{10}*b^3*c^8*d \\
&^5 + 16*a^{13}*b^2*c^6*e^5 + 128*a^{13}*c^8*d^2*e^3 - 64*a^{10}*b^5*c^6*d^3*e^2 + \\
&288*a^{11}*b^3*c^7*d^3*e^2 + 96*a^{11}*b^4*c^6*d^2*e^3 - 416*a^{12}*b^2*c^7*d^2* \\
&e^3 + 256*a^{13}*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^{10}*b^4*c^7*d^4*e - \\
&112*a^{11}*b^2*c^8*d^4*e - 128*a^{12}*b*c^8*d^3*e^2 - 64*a^{12}*b^3*c^6*d*e^4) - \\
&x*(8*a^{13}*c^7*e^6 - 8*a^{10}*c^{10}*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^{11}*c^9*d^4*e \\
&^2 + 8*a^{12}*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^{10}*b^2*c^8*d^4*e^2 - \\
&16*a^{10}*b^3*c^7*d^3*e^3 + 28*a^{11}*b^2*c^7*d^2*e^4 + 8*a^{10}*b*c^9*d^5*e - 2 \\
&4*a^{12}*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^{11}*b*c^8*d^3*e^3)) * (-(b^{11}* \\
&d^4 + a^4*b^7*e^4 + b^6*d^4 * (-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - \\
&11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4 * (-(4*a*c - b^2)^5)^{(1/2)} - \\
&4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^ \\
&4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4 * (-(4*a*c - b^2) \\
&^5)^{(1/2)} + a^4*b^2*e^4 * (-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a \\
&^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4 * (-(4*a \\
&*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2 * (-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b \\
&^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2 * (-(4*a*c - b^2 \\
&)^5)^{(1/2)} - 5*a*b^4*c*d^4 * (-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e * (-(4*a* \\
&c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e \\
&^3 * (-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 \\
&+ 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 4 \\
&80*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e * (-(4*a*c \\
&- b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e * (-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c \\
&d^2*e^2 * (-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3 * (-(4*a*c - b^2)^5)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2)) / (512 * (a^7 * b^8 + 256 * a^{11} * c^4 - 16 * a^8 * b^6 * c + 96 * a^9 * b^4 * c^2 - 256 * a^{10} \\
& * b^2 * c^3))^{(1/4)}) * (- (b^{11} * d^4 + a^4 * b^7 * e^4 + b^6 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} \\
& - 112 * a^5 * b * c^5 * d^4 - 11 * a^5 * b^5 * c * e^4 - 48 * a^7 * b * c^3 * e^4 - a^5 * c * e^4 \\
& * (- (4 * a * c - b^2)^5)^{1/2} - 4 * a^3 * b^8 * d * e^3 + 128 * a^6 * c^5 * d^3 * e - 128 * a^7 * c \\
& ^4 * d * e^3 + 86 * a^2 * b^7 * c^2 * d^4 - 231 * a^3 * b^5 * c^3 * d^4 + 280 * a^4 * b^3 * c^4 * d^4 - \\
& a^3 * c^3 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} + a^4 * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{1/2} \\
&) + 40 * a^6 * b^3 * c^2 * e^4 + 6 * a^2 * b^9 * d^2 * e^2 - 15 * a * b^9 * c * d^4 - 4 * a * b^{10} * d^3 * \\
& e + 6 * a^2 * b^2 * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} + 6 * a^2 * b^4 * d^2 * e^2 * (- (4 * a * c \\
& - b^2)^5)^{1/2} + 366 * a^4 * b^5 * c^2 * d^2 * e^2 - 720 * a^5 * b^3 * c^3 * d^2 * e^2 + 6 * a^4 \\
& * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} - 5 * a * b^4 * c * d^4 * (- (4 * a * c - b^2)^5)^{1/2} \\
& - 4 * a * b^5 * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} + 56 * a^2 * b^8 * c * d^3 * e + 48 * a^4 \\
& * b^6 * c * d * e^3 - 4 * a^3 * b^3 * d * e^3 * (- (4 * a * c - b^2)^5)^{1/2} - 292 * a^3 * b^6 * c^2 * d \\
& ^3 * e - 78 * a^3 * b^7 * c * d^2 * e^2 + 680 * a^4 * b^4 * c^3 * d^3 * e - 640 * a^5 * b^2 * c^4 * d^3 * e \\
& - 200 * a^5 * b^4 * c^2 * d * e^3 + 480 * a^6 * b * c^4 * d^2 * e^2 + 320 * a^6 * b^2 * c^3 * d * e^3 + \\
& 16 * a^2 * b^3 * c * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} - 12 * a^3 * b * c^2 * d^3 * e * (- (4 * a * c - \\
& b^2)^5)^{1/2} - 18 * a^3 * b^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} + 8 * a^4 * b * c * \\
& d * e^3 * (- (4 * a * c - b^2)^5)^{1/2} / (512 * (a^7 * b^8 + 256 * a^{11} * c^4 - 16 * a^8 * b^6 * c \\
& + 96 * a^9 * b^4 * c^2 - 256 * a^{10} * b^2 * c^3))^{(1/4)} * 2i + \operatorname{atan}(\frac{(- (b^{11} * d^4 + a^4 * b^7 * e^4 - b^6 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - 112 * a^5 * b * c^5 * d^4 - 11 * a^5 * b^5 * c * e^4 - 48 * a^7 * b * c^3 * e^4 + a^5 * c * e^4 * (- (4 * a * c - b^2)^5)^{1/2} - 4 * a^3 * b^8 * d * e^3 + 128 * a^6 * c^5 * d^3 * e - 128 * a^7 * c^4 * d * e^3 + 86 * a^2 * b^7 * c^2 * d^4 - 231 * a^3 * b^5 * c^3 * d^4 + 280 * a^4 * b^3 * c^4 * d^4 + a^3 * c^3 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - a^4 * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{1/2} + 40 * a^6 * b^3 * c^2 * e^4 + 6 * a^2 * b^9 * d^2 * e^2 - 15 * a * b^9 * c * d^4 - 4 * a * b^{10} * d^3 * e - 6 * a^2 * b^2 * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - 6 * a^2 * b^4 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} + 366 * a^4 * b^5 * c^2 * d^2 * e^2 - 720 * a^5 * b^3 * c^3 * d^2 * e^2 - 6 * a^4 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} + 5 * a * b^4 * c * d^4 * (- (4 * a * c - b^2)^5)^{1/2} + 4 * a * b^5 * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} + 56 * a^2 * b^8 * c * d^3 * e + 48 * a^4 * b^6 * c * d * e^3 + 4 * a^3 * b^3 * d * e^3 * (- (4 * a * c - b^2)^5)^{1/2} - 292 * a^3 * b^6 * c^2 * d^3 * e - 78 * a^3 * b^7 * c * d^2 * e^2 + 680 * a^4 * b^4 * c^3 * d^3 * e - 640 * a^5 * b^2 * c^4 * d^3 * e - 200 * a^5 * b^4 * c^2 * d * e^3 + 480 * a^6 * b * c^4 * d^2 * e^2 + 320 * a^6 * b^2 * c^3 * d * e^3 - 16 * a^2 * b^3 * c * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} + 12 * a^3 * b * c^2 * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} + 18 * a^3 * b^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} - 8 * a^4 * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{1/2} / (512 * (a^7 * b^8 + 256 * a^{11} * c^4 - 16 * a^8 * b^6 * c + 96 * a^9 * b^4 * c^2 - 256 * a^{10} * b^2 * c^3))^{(1/4)} * (((- (b^{11} * d^4 + a^4 * b^7 * e^4 - b^6 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - 112 * a^5 * b * c^5 * d^4 - 11 * a^5 * b^5 * c * e^4 - 48 * a^7 * b * c^3 * e^4 + a^5 * c * e^4 * (- (4 * a * c - b^2)^5)^{1/2} - 4 * a^3 * b^8 * d * e^3 + 128 * a^6 * c^5 * d^3 * e - 128 * a^7 * c^4 * d * e^3 + 86 * a^2 * b^7 * c^2 * d^4 - 231 * a^3 * b^5 * c^3 * d^4 + 280 * a^4 * b^3 * c^4 * d^4 + a^3 * c^3 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - a^4 * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{1/2} + 40 * a^6 * b^3 * c^2 * e^4 + 6 * a^2 * b^9 * d^2 * e^2 - 15 * a * b^9 * c * d^4 - 4 * a * b^{10} * d^3 * e - 6 * a^2 * b^2 * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{1/2} - 6 * a^2 * b^4 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} + 366 * a^4 * b^5 * c^2 * d^2 * e^2 - 720 * a^5 * b^3 * c^3 * d^2 * e^2 - 6 * a^4 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{1/2} + 5 * a * b^4 * c * d^4 * (- (4 * a * c - b^2)^5)^{1/2} + 4 * a * b^5 * d^3 * e * (- (4 * a * c - b^2)^5)^{1/2} + 56 * a^2 * b^8 * c * d^3 * e + 48 * a^4 * b^6 * c * d * e^3 + 4 * a^3 * b^3 * d * e^3 * (- (4 * a * c - b^2)^5)^{1/2} - 292 * a^3 * b^6 * c^2 * d^3 * e - 78
\end{aligned}$$

$$\begin{aligned}
& a^3 b^7 c^2 d^2 e^2 + 680 a^4 b^4 c^3 d^3 e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^2 e^3 + 480 a^6 b^3 c^4 d^2 e^2 + 320 a^6 b^2 c^3 d^2 e^3 - 16 a^2 b^3 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{1/2} + 12 a^3 b^3 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{1/2} \\
& + 18 a^3 b^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{1/2} - 8 a^4 b^3 c^2 d^2 e^3 (-4 a^2 c - b^2)^5)^{1/2} / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{1/4} * (262144 a^{17} c^8 d + 4096 a^{13} b^8 c^4 \\
& * d - 53248 a^{14} b^6 c^5 d + 245760 a^{15} b^4 c^6 d - 458752 a^{16} b^2 c^7 d - 4096 a^{14} b^7 c^4 e + 49152 a^{15} b^5 c^5 e - 196608 a^{16} b^3 c^6 e + 26214 \\
& 4 a^{17} b^3 c^7 e) + x * (81920 a^{15} b^3 c^8 d^2 - 49152 a^{16} b^2 c^7 e^2 + 1024 a^{11} b^9 c^4 d^2 - 13312 a^{12} b^7 c^5 d^2 + 62464 a^{13} b^5 c^6 d^2 - 122880 a^{14} b^3 c^7 d^2 \\
& + 1024 a^{13} b^7 c^4 e^2 - 11264 a^{14} b^5 c^5 e^2 + 40960 a^{15} b^3 c^6 e^2 - 65536 a^{16} c^8 d e - 2048 a^{12} b^8 c^4 d e + 24576 a^{13} b^6 c^5 d e - 102400 a^{14} b^4 c^6 d e + 163840 a^{15} b^2 c^7 d e) * (-b^{11} d^4 \\
& + a^4 b^7 e^4 - b^6 d^4 (-4 a^2 c - b^2)^5)^{1/2} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^4 e^4 - 48 a^7 b^3 c^3 e^4 + a^5 c^4 e^4 (-4 a^2 c - b^2)^5)^{1/2} - 4 a^3 b^8 d^2 e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^2 e^3 + 86 a^2 b^7 c^2 d^4 - \\
& 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 + a^3 c^3 d^4 (-4 a^2 c - b^2)^5)^{1/2} - a^4 b^2 e^4 (-4 a^2 c - b^2)^5)^{1/2} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 - 15 a^2 b^9 c^2 d^4 - 4 a^2 b^10 d^3 e - 6 a^2 b^2 c^2 d^4 (-4 a^2 c - \\
& b^2)^5)^{1/2} - 6 a^2 b^4 d^2 e^2 (-4 a^2 c - b^2)^5)^{1/2} + 366 a^4 b^5 c^2 d^2 e^2 - 720 a^5 b^3 c^3 d^2 e^2 - 6 a^4 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{1/2} + 5 a^2 b^4 c^2 d^4 (-4 a^2 c - b^2)^5)^{1/2} + 4 a^2 b^5 d^3 e^3 (-4 a^2 c - \\
& b^2)^5)^{1/2} + 56 a^2 b^8 c^2 d^3 e + 48 a^4 b^6 c^2 d^3 e^3 + 4 a^3 b^3 d^2 e^3 (-4 a^2 c - b^2)^5)^{1/2} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c^2 d^2 e^2 + 68 \\
& 0 a^4 b^4 c^3 d^3 e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^2 e^3 + 480 a^6 b^3 c^4 d^2 e^2 + 320 a^6 b^2 c^3 d^2 e^3 - 16 a^2 b^3 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{1/2} + 12 a^3 b^3 c^2 d^3 e^3 (-4 a^2 c - b^2)^5)^{1/2} + 18 a^3 b^2 c^2 d^2 e^2 (-4 a^2 c - b^2)^5)^{1/2} - 8 a^4 b^3 c^2 d^2 e^3 (-4 a^2 c - b^2)^5)^{1/2} / \\
& (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{3/4} - 64 a^{14} c^7 e^5 - 128 a^{11} b^3 c^9 d^5 + 192 a^{12} c^9 d^4 e - 16 a^9 b^5 c^7 d^5 + 96 a^{10} b^3 c^8 d^5 + 16 a^{13} b^2 c^6 e^5 + 128 a^{13} c^8 d^2 e^3 - 64 a^{10} b^5 c^6 d^3 e^2 + 288 a^{11} b^3 c^7 d^3 e^2 + 96 a^{11} b^4 c^6 d^2 e^3 - 416 a^{12} b^2 c^7 d^2 e^3 + 256 a^{13} b^3 c^7 d^2 e^4 + 16 a^9 b^6 c^6 d^4 e - 48 a^{10} b^4 c^7 d^4 e - 112 a^{11} b^2 c^8 d^4 e - 128 a^{12} b^3 c^8 d^3 e^2 - 64 a^{12} b^3 c^6 d^4 e^4) + x * (8 a^{13} c^7 e^6 - 8 a^{10} c^{10} d^6 + 4 a^9 b^2 c^9 d^6 - 8 a^{11} c^9 d^4 e^2 + 8 a^{12} c^8 d^2 e^4 + 4 a^9 b^4 c^7 d^4 e^2 + 16 a^{10} b^2 c^8 d^4 e^2 - 16 a^{10} b^3 c^7 d^3 e^3 + 28 a^{11} b^2 c^7 d^2 e^4 + 8 a^{10} b^3 c^9 d^5 e - 24 a^{12} b^3 c^7 d^5 e - 8 a^9 b^3 c^8 d^5 e - 16 a^{11} b^3 c^8 d^3 e^3) * (-b^{11} d^4 + a^4 b^7 e^4 - b^6 d^4 (-4 a^2 c - b^2)^5)^{1/2} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^4 e^4 - 48 a^7 b^3 c^3 e^4 + a^5 c^4 e^4 (-4 a^2 c - b^2)^5)^{1/2} - 4 a^3 b^8 d^2 e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^2 e^3 + 86 a^2 b^7 c^2 d^4 - 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 + a^3 c^3 d^4 (-4 a^2 c - b^2)^5)^{1/2} - a^4 b^2 e^4 (-4 a^2 c - b^2)^5)^{1/2} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 - 15 a^2 b^9 c^2 d^4 - 4 a^2 b^10 d^3 e - 6 a^2 b^2 c^2 d^4 (-4 a^2 c - b^2)^5)^{1/2} - 6 a^2 b^4 d^2 e^2
\end{aligned}$$

$$\begin{aligned}
& ^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2 \\
& *e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3 \\
& *e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^ \\
& 3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^ \\
& 2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c \\
& ^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e \\
& *(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 1 \\
& 6*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*1i - ((-(b^11*d^4 \\
& + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a \\
& ^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^ \\
& 3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - \\
& 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^ \\
& (1/2) - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b \\
& ^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c \\
& ^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5) \\
& ^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - \\
& b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(\\
& -(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 68 \\
& 0*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a \\
& ^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^ \\
& 2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^ \\
& 2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/ \\
& (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2 \\
& *c^3)))^{(1/4)}*(((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d \\
& *e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3 \\
& *c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - \\
& 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^ \\
& 2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6 \\
& *c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e \\
& - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 2 \\
& 00*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a \\
& ^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^ \\
& 3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 9 \\
& 6*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8*d + 4096*a^13*b^ \\
& 8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^ \\
& 7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e +
\end{aligned}$$

$$\begin{aligned}
& 262144*a^{17}*b*c^7*e) - x*(81920*a^{15}*b*c^8*d^2 - 49152*a^{16}*b*c^7*e^2 + 102 \\
& 4*a^{11}*b^9*c^4*d^2 - 13312*a^{12}*b^7*c^5*d^2 + 62464*a^{13}*b^5*c^6*d^2 - 1228 \\
& 80*a^{14}*b^3*c^7*d^2 + 1024*a^{13}*b^7*c^4*e^2 - 11264*a^{14}*b^5*c^5*e^2 + 4096 \\
& 0*a^{15}*b^3*c^6*e^2 - 65536*a^{16}*c^8*d*e - 2048*a^{12}*b^8*c^4*d*e + 24576*a^{11} \\
& 3*b^6*c^5*d*e - 102400*a^{14}*b^4*c^6*d*e + 163840*a^{15}*b^2*c^7*d*e))*(-(b^{11} \\
& *d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - \\
& 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d \\
& ^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2 \\
&)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6* \\
& a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4* \\
& b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^ \\
& 2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a \\
& *c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d* \\
& e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 \\
& + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + \\
& 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2 \\
& *c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1 \\
& /2)))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^1 \\
& 0*b^2*c^3)))^{(3/4)} - 64*a^{14}*c^7*e^5 - 128*a^{11}*b*c^9*d^5 + 192*a^{12}*c^9*d^ \\
& 4*e - 16*a^9*b^5*c^7*d^5 + 96*a^{10}*b^3*c^8*d^5 + 16*a^{13}*b^2*c^6*e^5 + 128* \\
& a^{13}*c^8*d^2*e^3 - 64*a^{10}*b^5*c^6*d^3*e^2 + 288*a^{11}*b^3*c^7*d^3*e^2 + 96* \\
& a^{11}*b^4*c^6*d^2*e^3 - 416*a^{12}*b^2*c^7*d^2*e^3 + 256*a^{13}*b*c^7*d*e^4 + 16 \\
& *a^9*b^6*c^6*d^4*e - 48*a^{10}*b^4*c^7*d^4*e - 112*a^{11}*b^2*c^8*d^4*e - 128*a \\
& ^{12}*b*c^8*d^3*e^2 - 64*a^{12}*b^3*c^6*d*e^4) - x*(8*a^{13}*c^7*e^6 - 8*a^{10}*c^1 \\
& 0*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^{11}*c^9*d^4*e^2 + 8*a^{12}*c^8*d^2*e^4 + 4*a^9 \\
& *b^4*c^7*d^4*e^2 + 16*a^{10}*b^2*c^8*d^4*e^2 - 16*a^{10}*b^3*c^7*d^3*e^3 + 28*a \\
& ^{11}*b^2*c^7*d^2*e^4 + 8*a^{10}*b*c^9*d^5*e - 24*a^{12}*b*c^7*d*e^5 - 8*a^9*b^3* \\
& c^8*d^5*e - 16*a^{11}*b*c^8*d^3*e^3))*(-(b^{11}*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3 \\
& *e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d \\
& ^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a \\
& ^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a* \\
& c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 \\
& - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4* \\
& d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^ \\
& 3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8* \\
& c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 2 \\
& 92*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a \\
& ^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6* \\
& b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b^2*c^2* \\
& d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1
\end{aligned}$$

$$\begin{aligned}
& c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 \\
& + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4* \\
& (- (4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366 \\
& *a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(\\
& - (4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b \\
& ^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^ \\
& 2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e \\
& ^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^ \\
& 3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 25 \\
& 6*a^10*b^2*c^3)))^{(3/4)} - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^5 + 192*a^12*c \\
& ^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13*b^2*c^6*e^5 + \\
& 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^3*c^7*d^3*e^2 \\
& + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^13*b*c^7*d*e^4 \\
& + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^2*c^8*d^4*e - \\
& 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c^7*e^6 - 8*a^1 \\
& 0*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + \\
& 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + \\
& 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9 \\
& *b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7* \\
& b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6* \\
& c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + \\
& 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9* \\
& c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2 \\
& *b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b \\
& ^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4 \\
& *(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2 \\
& *b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
&) - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - \\
& 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320 \\
& *a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b \\
& *c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^ \\
& 5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^ \\
& 11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)} + ((-(b^ \\
& 11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 \\
& - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2 \\
& *d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b \\
& ^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + \\
& 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(
\end{aligned}$$

$$\begin{aligned}
& 4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*((-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3))^{(1/4)}*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*
\end{aligned}$$

$$\begin{aligned}
& d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d \\
& *e^3 + 480a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^2e^3 - 16a^2b^3c^3d^3e * (\\
& - (4ac - b^2)^5)^{(1/2)} + 12a^3b^2c^2d^3e * (- (4ac - b^2)^5)^{(1/2)} + 18 * \\
& a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} - 8a^4b^2c^2d^3e * (- (4ac - b^2 \\
&)^5)^{(1/2)} / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - \\
& 256a^{10}b^2c^3))^{(3/4)} - 64a^{14}c^7e^5 - 128a^{11}b^2c^9d^5 + 192a^{12} \\
& *c^9d^4e - 16a^9b^5c^7d^5 + 96a^{10}b^3c^8d^5 + 16a^{13}b^2c^6e^5 \\
& + 128a^{13}c^8d^2e^3 - 64a^{10}b^5c^6d^3e^2 + 288a^{11}b^3c^7d^3e^2 \\
& + 96a^{11}b^4c^6d^2e^3 - 416a^{12}b^2c^7d^2e^3 + 256a^{13}b^2c^7d^2e^4 \\
& + 16a^9b^6c^6d^4e - 48a^{10}b^4c^7d^4e - 112a^{11}b^2c^8d^4e \\
& - 128a^{12}b^2c^8d^3e^2 - 64a^{12}b^3c^6d^4e^4 - x(8a^{13}c^7e^6 - 8a \\
& ^{10}c^{10}d^6 + 4a^9b^2c^9d^6 - 8a^{11}c^9d^4e^2 + 8a^{12}c^8d^2e^4 \\
& + 4a^9b^4c^7d^4e^2 + 16a^{10}b^2c^8d^4e^2 - 16a^{10}b^3c^7d^3e^3 \\
& + 28a^{11}b^2c^7d^2e^4 + 8a^{10}b^2c^9d^5e - 24a^{12}b^2c^7d^2e^5 - 8a \\
& ^9b^3c^8d^5e - 16a^{11}b^2c^8d^3e^3)) * (- (b^{11}d^4 + a^4b^7e^4 - b^6d^4 * \\
& (- (4ac - b^2)^5)^{(1/2)} - 112a^5b^2c^5d^4 - 11a^5b^5c^2e^4 - 48a^7 \\
& *b^2c^3e^4 + a^5c^2e^4 * (- (4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6 \\
& *c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 \\
& + 280a^4b^3c^4d^4 + a^3c^3d^4 * (- (4ac - b^2)^5)^{(1/2)} - a^4b^2e^4 * \\
& (- (4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a^2b^9 \\
& *c^2d^4 - 4a^2b^10d^3e - 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{(1/2)} - 6a \\
& ^2b^4d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5 \\
& *b^3c^3d^2e^2 - 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 5a^2b^4c^2d \\
& ^4 * (- (4ac - b^2)^5)^{(1/2)} + 4a^2b^5d^3e * (- (4ac - b^2)^5)^{(1/2)} + 56a \\
& ^2b^8c^2d^3e + 48a^4b^6c^2d^3e + 4a^3b^3d^3e * (- (4ac - b^2)^5)^{(1 \\
& /2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e \\
& - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e + 480a^6b^2c^4d^2e^2 + 3 \\
& 20a^6b^2c^3d^3e - 16a^2b^3c^3d^3e * (- (4ac - b^2)^5)^{(1/2)} + 12a^3 \\
& *b^2c^2d^3e * (- (4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2 * (- (4ac - b^2 \\
&)^5)^{(1/2)} - 8a^4b^2c^2d^3e * (- (4ac - b^2)^5)^{(1/2)} / (512(a^7b^8 + 256 * \\
& a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{(1/4)}) * (- (b \\
& ^{11}d^4 + a^4b^7e^4 - b^6d^4 * (- (4ac - b^2)^5)^{(1/2)} - 112a^5b^2c^5d^4 \\
& - 11a^5b^5c^2e^4 - 48a^7b^2c^3e^4 + a^5c^2e^4 * (- (4ac - b^2)^5)^{(1/2 \\
&)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2 \\
& *d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^4 * (- (4ac - \\
& b^2)^5)^{(1/2)} - a^4b^2e^4 * (- (4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + \\
& 6a^2b^9d^2e^2 - 15a^2b^9c^2d^4 - 4a^2b^10d^3e - 6a^2b^2c^2d^4 * (- \\
& (4ac - b^2)^5)^{(1/2)} - 6a^2b^4d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 366a \\
& ^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2 * (- (4ac - \\
& b^2)^5)^{(1/2)} + 5a^2b^4c^2d^4 * (- (4ac - b^2)^5)^{(1/2)} + 4a^2b^5d^3e * (- \\
& (4ac - b^2)^5)^{(1/2)} + 56a^2b^8c^2d^3e + 48a^4b^6c^2d^3e + 4a^3b^3 \\
& *d^3e^3 * (- (4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2 \\
& e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 \\
& + 480a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^2e^3 - 16a^2b^3c^3d^3e * (- (4 \\
& ac - b^2)^5)^{(1/2)} + 12a^3b^2c^2d^3e * (- (4ac - b^2)^5)^{(1/2)} + 18a^3 *
\end{aligned}$$

$$\begin{aligned}
& b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} \\
& / (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)} * 2i + 2*atan((((-(b^{11}*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)} * (((-(b^{11}*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})) / (512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(1/4)} * (262144*a^{17}*c^8*d + 4096*a^{13}*b^8*c^4*d - 53248*a^{14}*b^6*c^5*d + 245760*a^{15}*b^4*c^6*d - 458752*a^{16}*b^2*c^7*d - 4096*a^{14}*b^7*c^4*e + 49152*a^{15}*b^5*c^5*e - 196608*a^{16}*b^3*c^6*e + 262144*a^{17}*b*c^7*e) * 1i + x * (81920*a^{15}*b*c^8*d^2 - 49152*a^{16}*b*c^7*e^2 + 1024*a^{11}*b^9*c^4*d^2 - 13312*a^{12}*b^7*c^5*d^2 + 62464*a^{13}*b^5*c^6*d^2 - 122880*a^{14}*b^3*c^7*d^2 + 1024*a^{13}*b^7*c^4*e^2 - 11264*a^{14}*b^5*c^5*e^2 + 40960*a^{15}*b^3*c^6*e^2 - 65536*a^{16}*c^8*d*e - 2048*a^{12}*b^8*c^4*d*e + 24576*a^{13}*b^6*c^5*d*e - 102400*a^{14}*b^4*c^6*d*e + 163840*a^{15}*b^2*c^7*d*e) * (-(b^{11}*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a
\end{aligned}$$

$$\begin{aligned}
&^6c^5d^3e - 128a^7c^4d^4e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 \\
&+ 280a^4b^3c^4d^4 - a^3c^3d^4(-4ac - b^2)^5)^{(1/2)} + a^4b^2e^4 \\
&*(-4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9 \\
&c^4d^4 - 4ab^10d^3e + 6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2 \\
&b^4d^2e^2(-4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5 \\
&b^3c^3d^2e^2 + 6a^4c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 5ab^4c^* \\
&d^4(-4ac - b^2)^5)^{(1/2)} - 4ab^5d^3e(-4ac - b^2)^5)^{(1/2)} + 56a^2 \\
&b^8c^3d^3e + 48a^4b^6c^3d^3e - 4a^3b^3d^3e(-4ac - b^2)^5)^{(1/2)} - \\
&292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - \\
&640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + \\
&320a^6b^2c^3d^3e + 16a^2b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 12a^3 \\
&b^3c^2d^3e(-4ac - b^2)^5)^{(1/2)} - 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} \\
&+ 8a^4b^3c^2d^2e^2(-4ac - b^2)^5)^{(1/2)}/(512(a^7b^8 + 256 \\
&a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^10b^2c^3)))^{(3/4)}*i + \\
&64a^14c^7e^5 + 128a^11b^3c^9d^5 - 192a^12c^9d^4e + 16a^9b^5c^7 \\
&d^5 - 96a^10b^3c^8d^5 - 16a^13b^2c^6e^5 - 128a^13c^8d^2e^3 + 64 \\
&a^10b^5c^6d^3e^2 - 288a^11b^3c^7d^3e^2 - 96a^11b^4c^6d^2e^3 + \\
&416a^12b^2c^7d^2e^3 - 256a^13b^3c^7d^2e^4 - 16a^9b^6c^6d^4e + \\
&48a^10b^4c^7d^4e + 112a^11b^2c^8d^4e + 128a^12b^3c^8d^3e^2 + 6 \\
&4a^12b^3c^6d^4e^4)*i - x*(8a^13c^7e^6 - 8a^10c^10d^6 + 4a^9b^2c^9 \\
&d^6 - 8a^11c^9d^4e^2 + 8a^12c^8d^2e^4 + 4a^9b^4c^7d^4e^2 + \\
&16a^10b^2c^8d^4e^2 - 16a^10b^3c^7d^3e^3 + 28a^11b^2c^7d^2e^4 + \\
&8a^10b^3c^9d^5e - 24a^12b^3c^7d^2e^5 - 8a^9b^3c^8d^5e - 16a^11 \\
&b^3c^8d^3e^3)*(-b^11d^4 + a^4b^7e^4 + b^6d^4(-4ac - b^2)^5)^{(1/2)} - \\
&112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 - a^5c^4e^4(- \\
&-4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4 \\
&d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3 \\
&c^3d^4(-4ac - b^2)^5)^{(1/2)} + a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} \\
&+ 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c^4d^4 - 4ab^10d^3e + \\
&6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} + 6a^2b^4d^2e^2(-4ac - \\
&b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2 \\
&d^2e^2(-4ac - b^2)^5)^{(1/2)} - 5ab^4c^3d^4(-4ac - b^2)^5)^{(1/2)} - \\
&4ab^5d^3e(-4ac - b^2)^5)^{(1/2)} + 56a^2b^8c^3d^3e + 48a^4b^6 \\
&c^3d^3e - 4a^3b^3d^3e(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3 \\
&e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - \\
&200a^5b^4c^2d^3e + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e + 16 \\
&a^2b^3c^3d^3e(-4ac - b^2)^5)^{(1/2)} - 12a^3b^3c^2d^3e(-4ac - b^2)^5)^{(1/2)} \\
&- 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 8a^4b^3c^2d^2e^2(-4ac - \\
&b^2)^5)^{(1/2)}/(512(a^7b^8 + 256a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - \\
&256a^10b^2c^3)))^{(1/4)} - (((-b^11d^4 + a^4b^7e^4 + b^6d^4(-4ac - \\
&b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 - \\
&4a^5c^4e^4(-4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - \\
&128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - \\
&a^3c^3d^4(-4ac - b^2)^5)^{(1/2)} + a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} + \\
&40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c^4d^4 - 4ab^10d^3e - 15*
\end{aligned}$$

$$\begin{aligned}
& a^9 b^9 c^4 d^4 - 4 a^8 b^{10} d^3 e + 6 a^7 b^2 c^2 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} + \\
& 6 a^2 b^4 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} + 366 a^4 b^5 c^2 d^2 e^2 - 720 \\
& a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} - 5 a^8 b^4 \\
& c^4 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 4 a^8 b^5 d^3 e (-4 a^4 c - b^2)^5)^{(1/2)} + \\
& 56 a^2 b^8 c^3 d^3 e + 48 a^4 b^6 c^3 d^3 e - 4 a^3 b^3 d^3 e (-4 a^4 c - b^2)^5 \\
&)^{(1/2)} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c^2 d^2 e^2 + 680 a^4 b^4 c^3 d^3 \\
& e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^3 e + 480 a^6 b^3 c^4 d^2 e^2 \\
& + 320 a^6 b^2 c^3 d^3 e + 16 a^2 b^3 c^3 d^3 e (-4 a^4 c - b^2)^5)^{(1/2)} - 12 \\
& a^3 b^2 c^2 d^3 e (-4 a^4 c - b^2)^5)^{(1/2)} - 18 a^3 b^2 c^2 d^2 e^2 (-4 a^4 c - \\
& b^2)^5)^{(1/2)} + 8 a^4 b^3 c^2 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} / (512 (a^7 b^8 + \\
& 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - 256 a^{10} b^2 c^3))^{(1/4)} * (\\
& (-b^{11} d^4 + a^4 b^7 e^4 + b^6 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 112 a^5 b^3 c^5 \\
& d^4 - 11 a^5 b^5 c^3 e^4 - 48 a^7 b^3 c^3 e^4 - a^5 c^3 e^4 (-4 a^4 c - b^2)^5)^{(1/2)} \\
& - 4 a^3 b^8 d^3 e + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^3 e + 86 a^2 b^7 \\
& c^2 d^4 - 231 a^3 b^5 c^3 d^4 + 280 a^4 b^3 c^4 d^4 - a^3 c^3 d^4 (-4 a^4 c \\
& - b^2)^5)^{(1/2)} + a^4 b^2 e^4 (-4 a^4 c - b^2)^5)^{(1/2)} + 40 a^6 b^3 c^2 e^4 \\
& + 6 a^2 b^9 d^2 e^2 - 15 a^8 b^9 c^4 d^4 - 4 a^8 b^{10} d^3 e + 6 a^2 b^2 c^2 d^4 \\
& (-4 a^4 c - b^2)^5)^{(1/2)} + 6 a^2 b^4 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} + 3 \\
& 66 a^4 b^5 c^2 d^2 e^2 - 720 a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 (-4 a^4 c \\
& - b^2)^5)^{(1/2)} - 5 a^8 b^4 c^4 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 4 a^8 b^5 d^3 e \\
& (-4 a^4 c - b^2)^5)^{(1/2)} + 56 a^2 b^8 c^3 d^3 e + 48 a^4 b^6 c^3 d^3 e - 4 a^3 \\
& b^3 d^3 e (-4 a^4 c - b^2)^5)^{(1/2)} - 292 a^3 b^6 c^2 d^3 e - 78 a^3 b^7 c^2 \\
& d^2 e^2 + 680 a^4 b^4 c^3 d^3 e - 640 a^5 b^2 c^4 d^3 e - 200 a^5 b^4 c^2 d^3 \\
& e + 480 a^6 b^3 c^4 d^2 e^2 + 320 a^6 b^2 c^3 d^3 e + 16 a^2 b^3 c^3 d^3 e (- \\
& 4 a^4 c - b^2)^5)^{(1/2)} - 12 a^3 b^2 c^2 d^3 e (-4 a^4 c - b^2)^5)^{(1/2)} - 18 \\
& a^3 b^2 c^2 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} + 8 a^4 b^3 c^2 d^2 e^2 (-4 a^4 c - b^2 \\
&)^5)^{(1/2)} / (512 (a^7 b^8 + 256 a^{11} c^4 - 16 a^8 b^6 c + 96 a^9 b^4 c^2 - \\
& 256 a^{10} b^2 c^3))^{(1/4)} * (262144 a^{17} c^8 d + 4096 a^{13} b^8 c^4 d - 53248 \\
& a^{14} b^6 c^5 d + 245760 a^{15} b^4 c^6 d - 458752 a^{16} b^2 c^7 d - 4096 a^{14} \\
& b^7 c^4 e + 49152 a^{15} b^5 c^5 e - 196608 a^{16} b^3 c^6 e + 262144 a^{17} b^3 c^7 \\
& e) * i - x * (81920 a^{15} b^3 c^8 d^2 - 49152 a^{16} b^3 c^7 e^2 + 1024 a^{11} b^9 c^4 \\
& d^2 - 13312 a^{12} b^7 c^5 d^2 + 62464 a^{13} b^5 c^6 d^2 - 122880 a^{14} b^3 c^7 \\
& d^2 + 1024 a^{13} b^7 c^4 e^2 - 11264 a^{14} b^5 c^5 e^2 + 40960 a^{15} b^3 c^6 \\
& e^2 - 65536 a^{16} c^8 d e - 2048 a^{12} b^8 c^4 d e + 24576 a^{13} b^6 c^5 d e \\
& - 102400 a^{14} b^4 c^6 d e + 163840 a^{15} b^2 c^7 d e) * (-b^{11} d^4 + a^4 b^7 \\
& e^4 + b^6 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 112 a^5 b^3 c^5 d^4 - 11 a^5 b^5 c^5 \\
& e^4 - 48 a^7 b^3 c^3 e^4 - a^5 c^3 e^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 4 a^3 b^8 d^3 \\
& e^3 + 128 a^6 c^5 d^3 e - 128 a^7 c^4 d^3 e + 86 a^2 b^7 c^2 d^4 - 231 a^3 b^5 \\
& c^3 d^4 + 280 a^4 b^3 c^4 d^4 - a^3 c^3 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} + \\
& a^4 b^2 e^4 (-4 a^4 c - b^2)^5)^{(1/2)} + 40 a^6 b^3 c^2 e^4 + 6 a^2 b^9 d^2 e^2 \\
& - 15 a^8 b^9 c^4 d^4 - 4 a^8 b^{10} d^3 e + 6 a^2 b^2 c^2 d^4 (-4 a^4 c - b^2)^5 \\
&)^{(1/2)} + 6 a^2 b^4 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} + 366 a^4 b^5 c^2 d^2 e^2 \\
& - 720 a^5 b^3 c^3 d^2 e^2 + 6 a^4 c^2 d^2 e^2 (-4 a^4 c - b^2)^5)^{(1/2)} - \\
& 5 a^8 b^4 c^4 d^4 (-4 a^4 c - b^2)^5)^{(1/2)} - 4 a^8 b^5 d^3 e (-4 a^4 c - b^2)^5 \\
&)^{(1/2)} + 56 a^2 b^8 c^3 d^3 e + 48 a^4 b^6 c^3 d^3 e - 4 a^3 b^3 d^3 e (-4 a^4 c
\end{aligned}$$

$$\begin{aligned}
& - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(3/4)}*i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*i + x*(8*a^13*c^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)})/(((b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& \sqrt[1/2]{-12a^3bc^2d^3e(-4ac - b^2)^5} \sqrt[1/2]{-18a^3b^2cd^2e^2} \\
& * \sqrt[1/2]{(-4ac - b^2)^5} \sqrt[1/2]{8a^4b^2cd^3e^3} \sqrt[1/2]{(-4ac - b^2)^5} / (512 * \\
& (a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3) \\
&) \sqrt[1/4]{((-b^{11}d^4 + a^4b^7e^4 + b^6d^4(-4ac - b^2)^5) \sqrt[1/2]{-1} \\
& 12a^5b^2c^5d^4 - 11a^5b^5c^2e^4 - 48a^7b^3c^3e^4 - a^5c^2e^4(-4ac \\
& - b^2)^5) \sqrt[1/2]{-4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^2e^3} \\
& + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3 * \\
& d^4(-4ac - b^2)^5 \sqrt[1/2]{+ a^4b^2e^4(-4ac - b^2)^5} \sqrt[1/2]{+ 40a^6} \\
& b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a^2b^9c^2d^4 - 4a^2b^10d^3e + 6a^2} \\
& b^2c^2d^4(-4ac - b^2)^5 \sqrt[1/2]{+ 6a^2b^4d^2e^2(-4ac - b^2)^5} \\
&) \sqrt[1/2]{+ 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2} \\
& e^2(-4ac - b^2)^5 \sqrt[1/2]{- 5a^2b^4cd^4(-4ac - b^2)^5} \sqrt[1/2]{- 4a} \\
& ab^5d^3e(-4ac - b^2)^5 \sqrt[1/2]{+ 56a^2b^8cd^3e + 48a^4b^6cd *} \\
& e^3 - 4a^3b^3d^2e^3(-4ac - b^2)^5 \sqrt[1/2]{- 292a^3b^6c^2d^3e - 78} \\
& a^3b^7cd^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5} \\
& b^4c^2d^2e^3 + 480a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^2e^3 + 16a^2b^} \\
& 3c^3d^3e(-4ac - b^2)^5 \sqrt[1/2]{- 12a^3b^2cd^3e(-4ac - b^2)^5} \sqrt[1/2]{- 18a^3} \\
& b^2cd^2e^2(-4ac - b^2)^5 \sqrt[1/2]{+ 8a^4b^2cd^3e^3(-4ac - b^2)^5} \sqrt[1/2]{(512 *} \\
& (a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3) \\
&) \sqrt[1/4]{(262144a^{17}c^8d + 4096a^{13}b^8c^4} \\
& * d - 53248a^{14}b^6c^5d + 245760a^{15}b^4c^6d - 458752a^{16}b^2c^7d -} \\
& 4096a^{14}b^7c^4e + 49152a^{15}b^5c^5e - 196608a^{16}b^3c^6e + 26214} \\
& 4a^{17}b^2c^7e) * i + x(81920a^{15}b^3c^8d^2 - 49152a^{16}b^2c^7e^2 + 1024 *} \\
& a^{11}b^9c^4d^2 - 13312a^{12}b^7c^5d^2 + 62464a^{13}b^5c^6d^2 - 122880} \\
& * a^{14}b^3c^7d^2 + 1024a^{13}b^7c^4e^2 - 11264a^{14}b^5c^5e^2 + 40960 *} \\
& a^{15}b^3c^6e^2 - 65536a^{16}c^8d^2e - 2048a^{12}b^8c^4d^2e + 24576a^{13} *} \\
& b^6c^5d^2e - 102400a^{14}b^4c^6d^2e + 163840a^{15}b^2c^7d^2e) * (-b^{11}d} \\
& ^4 + a^4b^7e^4 + b^6d^4(-4ac - b^2)^5) \sqrt[1/2]{- 112a^5b^2c^5d^4 - 1} \\
& 1a^5b^5c^2e^4 - 48a^7b^3c^3e^4 - a^5c^2e^4(-4ac - b^2)^5) \sqrt[1/2]{- 4} \\
& * a^3b^8d^2e^3 + 128a^6c^5d^3e - 128a^7c^4d^2e^3 + 86a^2b^7c^2d^4} \\
& - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3d^4(-4ac - b^2)^} \\
& 5) \sqrt[1/2]{+ a^4b^2e^4(-4ac - b^2)^5} \sqrt[1/2]{+ 40a^6b^3c^2e^4 + 6a^2} \\
& b^9d^2e^2 - 15a^2b^9c^2d^4 - 4a^2b^10d^3e + 6a^2b^2c^2d^4(-4ac \\
& - b^2)^5) \sqrt[1/2]{+ 6a^2b^4d^2e^2(-4ac - b^2)^5} \sqrt[1/2]{+ 366a^4b^} \\
& 5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2e^2(-4ac - b^2)} \\
& ^5) \sqrt[1/2]{- 5a^2b^4cd^4(-4ac - b^2)^5} \sqrt[1/2]{- 4a^2b^5d^3e(-4ac} \\
& - b^2)^5) \sqrt[1/2]{+ 56a^2b^8cd^3e + 48a^4b^6cd^2e^3 - 4a^3b^3d^2e^} \\
& 3(-4ac - b^2)^5) \sqrt[1/2]{- 292a^3b^6c^2d^3e - 78a^3b^7cd^2e^2 +} \\
& 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^2e^3 + 48} \\
& 0a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^2e^3 + 16a^2b^3c^3d^3e(-4ac -} \\
& b^2)^5) \sqrt[1/2]{- 12a^3b^2cd^3e(-4ac - b^2)^5} \sqrt[1/2]{- 18a^3b^2cd} \\
& d^2e^2(-4ac - b^2)^5) \sqrt[1/2]{+ 8a^4b^2cd^3e^3(-4ac - b^2)^5} \sqrt[1/2]{(512 *} \\
& (a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3) \\
&) \sqrt[3/4]{* i + 64a^{14}c^7e^5 + 128a^{11}b^3c^9d^5 - 192a^{12}c^9d} \\
& ^4e + 16a^9b^5c^7d^5 - 96a^{10}b^3c^8d^5 - 16a^{13}b^2c^6e^5 - 128}
\end{aligned}$$

$$\begin{aligned}
& a^{13}c^8d^2e^3 + 64a^{10}b^5c^6d^3e^2 - 288a^{11}b^3c^7d^3e^2 - 96 \\
& a^{11}b^4c^6d^2e^3 + 416a^{12}b^2c^7d^2e^3 - 256a^{13}b^2c^7d^2e^4 - 1 \\
& 6a^9b^6c^6d^4e + 48a^{10}b^4c^7d^4e + 112a^{11}b^2c^8d^4e + 128a^{12}b^2c^8d^3e^2 + 64a^{12}b^3c^6d^4e^4) * i - x(8a^{13}c^7e^6 - 8a^{10} \\
& c^{10}d^6 + 4a^9b^2c^9d^6 - 8a^{11}c^9d^4e^2 + 8a^{12}c^8d^2e^4 + 4 \\
& a^9b^4c^7d^4e^2 + 16a^{10}b^2c^8d^4e^2 - 16a^{10}b^3c^7d^3e^3 + \\
& 28a^{11}b^2c^7d^2e^4 + 8a^{10}b^2c^9d^5e - 24a^{12}b^2c^7d^5e^5 - 8a^9b^3c^8d^5e - 16a^{11}b^2c^8d^3e^3) * (- (b^{11}d^4 + a^4b^7e^4 + b^6d^4 \\
& * (- (4ac - b^2)^5)^{1/2} - 112a^5b^2c^5d^4 - 11a^5b^5c^5e^4 - 48a^7b^2c^3e^4 - a^5c^4e^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 2 \\
& 80a^4b^3c^4d^4 - a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} + a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a^2b^9c^2d^4 - 4a^2b^10d^3e + 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^2 \\
& b^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 5a^2b^4c^2d^4 * (- (4ac - b^2)^5)^{1/2} - 4a^2b^5d^3e * (- (4ac - b^2)^5)^{1/2} + 56a^2b^8c^2d^3e + 48a^4b^6c^2d^3e - 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 6 \\
& 40a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^3e^3 + 16a^2b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 12a^3b^2c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 8a^4b^2c^2d^3e * (- (4ac - b^2)^5)^{1/2} / (512(a^7b^8 + 256a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3)))^{1/4} * i + ((- (b^{11}d^4 + a^4b^7e^4 + b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^2c^5d^4 - 11a^5b^5c^5e^4 - 48a^7b^2c^3e^4 - a^5c^4e^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} + a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a^2b^9c^2d^4 - 4a^2b^10d^3e + 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 6a^2b^2b^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 + 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 5a^2b^4c^2d^4 * (- (4ac - b^2)^5)^{1/2} - 4a^2b^5d^3e * (- (4ac - b^2)^5)^{1/2} + 56a^2b^8c^2d^3e + 48a^4b^6c^2d^3e - 4a^3b^3d^3e * (- (4ac - b^2)^5)^{1/2} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^2c^4d^2e^2 + 320a^6b^2c^3d^3e^3 + 16a^2b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 12a^3b^2c^2d^3e * (- (4ac - b^2)^5)^{1/2} - 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 8a^4b^2c^2d^3e * (- (4ac - b^2)^5)^{1/2} / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3)))^{1/4} * (((- (b^{11}d^4 + a^4b^7e^4 + b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^2c^5d^4 - 11a^5b^5c^5e^4 - 48a^7b^2c^3e^4 - a^5c^4e^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 - a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} + a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2}
\end{aligned}$$

$$\begin{aligned}
& ^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^{10} \\
& *d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(\\
& 4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + \\
& 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 4 \\
& 8*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6* \\
& c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4* \\
& d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e \\
& ^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4 \\
& *b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8* \\
& b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8*d + 409 \\
& 6*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^ \\
& 16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3 \\
& *c^6*e + 262144*a^17*b*c^7*e)*1i - x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c \\
& ^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^ \\
& 6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^ \\
& 5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e \\
& + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d \\
& *e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5 \\
& *b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b^2) \\
& ^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a \\
& ^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*(- \\
& (4*a*c - b^2)^5)^{(1/2)} + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3* \\
& c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^{10}*d^3*e + 6*a^2*b^2*c \\
& ^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2) \\
&) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(- \\
& (4*a*c - b^2)^5)^{(1/2)} - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a*b^5* \\
& d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 - \\
& 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b \\
& ^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4* \\
& c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*d^ \\
& 3*e*(-(4*a*c - b^2)^5)^{(1/2)} - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} \\
& - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 8*a^4*b*c*d*e^3*(-(4*a*c \\
& - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c \\
& ^2 - 256*a^10*b^2*c^3)))^{(3/4)}*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - \\
& 192*a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2 \\
& *c^6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^ \\
& 7*d^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b \\
& *c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^ \\
& 8*d^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i + x*(8*a^13*c^ \\
& 7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c \\
& ^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3* \\
& c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7* \\
& d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7
\end{aligned}$$

$$\begin{aligned}
& e^4 + b^6 d^4 (-4ac - b^2)^5)^{1/2} - 112a^5 b^5 c^5 d^4 - 11a^5 b^5 c^5 e^4 - 48a^7 b^3 c^3 e^4 - a^5 c^5 e^4 (-4ac - b^2)^5)^{1/2} - 4a^3 b^8 d^3 e^3 + 128a^6 c^5 d^3 e - 128a^7 c^4 d^3 e^3 + 86a^2 b^7 c^2 d^4 - 231a^3 b^5 c^3 d^4 + 280a^4 b^3 c^4 d^4 - a^3 c^3 d^4 (-4ac - b^2)^5)^{1/2} + a^4 b^2 e^4 (-4ac - b^2)^5)^{1/2} + 40a^6 b^3 c^2 e^4 + 6a^2 b^9 d^2 e^2 - 15a^2 b^9 c^2 d^4 - 4a^2 b^10 d^3 e + 6a^2 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} + 6a^2 b^4 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 366a^4 b^5 c^2 d^2 e^2 - 720a^5 b^3 c^3 d^2 e^2 + 6a^4 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} - 5a^2 b^4 c^2 d^4 (-4ac - b^2)^5)^{1/2} - 4a^2 b^5 d^3 e (-4ac - b^2)^5)^{1/2} + 56a^2 b^8 c^2 d^3 e + 48a^4 b^6 c^2 d^3 e - 4a^3 b^3 d^3 e^3 (-4ac - b^2)^5)^{1/2} - 292a^3 b^6 c^2 d^3 e - 78a^3 b^7 c^2 d^2 e^2 + 680a^4 b^4 c^3 d^3 e - 640a^5 b^2 c^4 d^3 e - 200a^5 b^4 c^2 d^3 e + 480a^6 b^3 c^4 d^2 e^2 + 320a^6 b^2 c^3 d^3 e + 16a^2 b^3 c^2 d^3 e (-4ac - b^2)^5)^{1/2} - 12a^3 b^3 c^2 d^3 e (-4ac - b^2)^5)^{1/2} - 18a^3 b^2 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 8a^4 b^3 c^2 d^2 e^3 (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^11 c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^10 b^2 c^3))^{1/4} * i) * (-b^11 d^4 + a^4 b^7 e^4 + b^6 d^4 (-4ac - b^2)^5)^{1/2} - 112a^5 b^5 c^5 d^4 - 11a^5 b^5 c^5 e^4 - 48a^7 b^3 c^3 e^4 - a^5 c^5 e^4 (-4ac - b^2)^5)^{1/2} - 4a^3 b^8 d^3 e^3 + 128a^6 c^5 d^3 e - 128a^7 c^4 d^3 e^3 + 86a^2 b^7 c^2 d^4 - 231a^3 b^5 c^3 d^4 + 280a^4 b^3 c^4 d^4 - a^3 c^3 d^4 (-4ac - b^2)^5)^{1/2} + a^4 b^2 e^4 (-4ac - b^2)^5)^{1/2} + 40a^6 b^3 c^2 e^4 + 6a^2 b^9 d^2 e^2 - 15a^2 b^9 c^2 d^4 - 4a^2 b^10 d^3 e + 6a^2 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} + 6a^2 b^4 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 366a^4 b^5 c^2 d^2 e^2 - 720a^5 b^3 c^3 d^2 e^2 + 6a^4 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} - 5a^2 b^4 c^2 d^4 (-4ac - b^2)^5)^{1/2} - 4a^2 b^5 d^3 e (-4ac - b^2)^5)^{1/2} + 56a^2 b^8 c^2 d^3 e + 48a^4 b^6 c^2 d^3 e - 4a^3 b^3 d^3 e^3 (-4ac - b^2)^5)^{1/2} - 292a^3 b^6 c^2 d^3 e - 78a^3 b^7 c^2 d^2 e^2 + 680a^4 b^4 c^3 d^3 e - 640a^5 b^2 c^4 d^3 e - 200a^5 b^4 c^2 d^3 e + 480a^6 b^3 c^4 d^2 e^2 + 320a^6 b^2 c^3 d^3 e + 16a^2 b^3 c^2 d^3 e (-4ac - b^2)^5)^{1/2} - 12a^3 b^3 c^2 d^3 e (-4ac - b^2)^5)^{1/2} - 18a^3 b^2 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 8a^4 b^3 c^2 d^2 e^3 (-4ac - b^2)^5)^{1/2} / (512(a^7 b^8 + 256a^11 c^4 - 16a^8 b^6 c + 96a^9 b^4 c^2 - 256a^10 b^2 c^3))^{1/4} + 2 \operatorname{atan}(\frac{(-b^11 d^4 + a^4 b^7 e^4 - b^6 d^4 (-4ac - b^2)^5)^{1/2} - 112a^5 b^5 c^5 d^4 - 11a^5 b^5 c^5 e^4 - 48a^7 b^3 c^3 e^4 + a^5 c^5 e^4 (-4ac - b^2)^5)^{1/2} - 4a^3 b^8 d^3 e^3 + 128a^6 c^5 d^3 e - 128a^7 c^4 d^3 e^3 + 86a^2 b^7 c^2 d^4 - 231a^3 b^5 c^3 d^4 + 280a^4 b^3 c^4 d^4 + a^3 c^3 d^4 (-4ac - b^2)^5)^{1/2} - a^4 b^2 e^4 (-4ac - b^2)^5)^{1/2} + 40a^6 b^3 c^2 e^4 + 6a^2 b^9 d^2 e^2 - 15a^2 b^9 c^2 d^4 - 4a^2 b^10 d^3 e - 6a^2 b^2 c^2 d^4 (-4ac - b^2)^5)^{1/2} - 6a^2 b^4 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 366a^4 b^5 c^2 d^2 e^2 - 720a^5 b^3 c^3 d^2 e^2 - 6a^4 c^2 d^2 e^2 (-4ac - b^2)^5)^{1/2} + 5a^2 b^4 c^2 d^4 (-4ac - b^2)^5)^{1/2} + 4a^2 b^5 d^3 e (-4ac - b^2)^5)^{1/2} + 56a^2 b^8 c^2 d^3 e + 48a^4 b^6 c^2 d^3 e + 4a^3 b^3 d^3 e^3 (-4ac - b^2)^5)^{1/2} - 292a^3 b^6 c^2 d^3 e - 78a^3 b^7 c^2 d^2 e^2 + 680a^4 b^4 c^3 d^3 e - 640a^5 b^2 c^4 d^3 e - 200a^5 b^4 c^2 d^3 e + 480a^6 b^3 c^4 d^2 e^2
\end{aligned}$$

$$\begin{aligned}
& + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12 \\
& *a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - \\
& b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^7*b^8 + \\
& 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*((\\
& (-b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^ \\
& 5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{ \\
& (1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^ \\
& 7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a* \\
& c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e \\
& ^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^ \\
& 4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 3 \\
& 66*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a \\
& *c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e \\
& *(- (4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3 \\
& *b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c* \\
& d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d \\
& *e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(- \\
& -(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18* \\
& a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2 \\
&)^5)^{(1/2)}/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - \\
& 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8*d + 4096*a^13*b^8*c^4*d - 53248* \\
& a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16*b^2*c^7*d - 4096*a^14* \\
& b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^6*e + 262144*a^17*b*c^ \\
& 7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7*e^2 + 1024*a^11*b^9*c^ \\
& 4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d^2 - 122880*a^14*b^3*c \\
& ^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e^2 + 40960*a^15*b^3*c^ \\
& 6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + 24576*a^13*b^6*c^5*d*e \\
& - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e))*(-(b^11*d^4 + a^4*b^ \\
& 7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c \\
& *e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d* \\
& e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3* \\
& b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - \\
& a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e \\
& ^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5) \\
& ^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e \\
& ^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{ \\
& (1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c \\
& - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^ \\
& 4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4 \\
& *d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1 \\
& /2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(- \\
& (4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)}/(512*(a^ \\
& 7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{ \\
& (3/4)}*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192*a^12*c^9*d^4*e + 16*a
\end{aligned}$$

$$\begin{aligned}
& ^9b^5c^7d^5 - 96a^{10}b^3c^8d^5 - 16a^{13}b^2c^6e^5 - 128a^{13}c^8d \\
& ^2e^3 + 64a^{10}b^5c^6d^3e^2 - 288a^{11}b^3c^7d^3e^2 - 96a^{11}b^4c \\
& ^6d^2e^3 + 416a^{12}b^2c^7d^2e^3 - 256a^{13}b^3c^7d^2e^4 - 16a^9b^6c \\
& ^6d^4e + 48a^{10}b^4c^7d^4e + 112a^{11}b^2c^8d^4e + 128a^{12}b^3c^8 \\
& d^3e^2 + 64a^{12}b^3c^6d^4e^4) * i - x * (8a^{13}c^7e^6 - 8a^{10}c^{10}d^6 + \\
& 4a^9b^2c^9d^6 - 8a^{11}c^9d^4e^2 + 8a^{12}c^8d^2e^4 + 4a^9b^4c^7 \\
& d^4e^2 + 16a^{10}b^2c^8d^4e^2 - 16a^{10}b^3c^7d^3e^3 + 28a^{11}b^2 \\
& c^7d^2e^4 + 8a^{10}b^3c^9d^5e - 24a^{12}b^3c^7d^2e^5 - 8a^9b^3c^8d^5 \\
& * e - 16a^{11}b^3c^8d^3e^3)) * (- (b^{11}d^4 + a^4b^7e^4 - b^6d^4 * (- (4ac - \\
& b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 + \\
& a^5c^4e^4 * (- (4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - \\
& 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c \\
& ^4d^4 + a^3c^3d^4 * (- (4ac - b^2)^5)^{(1/2)} - a^4b^2e^4 * (- (4ac - b^2 \\
&)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15a*b^9c*d^4 - 4a*a \\
& b^10d^3e - 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{(1/2)} - 6a^2b^4d^2e^2 \\
& * (- (4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e \\
& ^2 - 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 5a*b^4*c*d^4 * (- (4ac - \\
& b^2)^5)^{(1/2)} + 4a*b^5*d^3e * (- (4ac - b^2)^5)^{(1/2)} + 56a^2b^8c*d^3e \\
& + 48a^4b^6c*d^3e^3 + 4a^3b^3d^3e^3 * (- (4ac - b^2)^5)^{(1/2)} - 292a^3b \\
& ^6c^2d^3e - 78a^3b^7c*d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c \\
& ^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3 \\
& *d^3e^3 - 16a^2b^3c^3d^3e * (- (4ac - b^2)^5)^{(1/2)} + 12a^3b^3c^2d^3e * (- \\
& (4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} - 8 \\
& a^4b^3c^2d^2e^3 * (- (4ac - b^2)^5)^{(1/2)) / (512 * (a^7b^8 + 256a^{11}c^4 - 16a \\
& ^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{(1/4)} - ((- (b^{11}d^4 + a^4 \\
& *b^7e^4 - b^6d^4 * (- (4ac - b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 - 11a^5b^ \\
& 5c^4e^4 - 48a^7b^3c^3e^4 + a^5c^4e^4 * (- (4ac - b^2)^5)^{(1/2)} - 4a^3b^8 \\
& *d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a \\
& ^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^4 * (- (4ac - b^2)^5)^{(1/2)} \\
& - a^4b^2e^4 * (- (4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^ \\
& 2e^2 - 15a*b^9*c*d^4 - 4a*b^10*d^3e - 6a^2b^2c^2d^4 * (- (4ac - b^2) \\
& ^5)^{(1/2)} - 6a^2b^4d^2e^2 * (- (4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^ \\
& 2e^2 - 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{(1/2) \\
&) + 5a*b^4*c*d^4 * (- (4ac - b^2)^5)^{(1/2)} + 4a*b^5*d^3e * (- (4ac - b^2) \\
& ^5)^{(1/2)} + 56a^2b^8c*d^3e + 48a^4b^6c*d^3e^3 + 4a^3b^3d^3e^3 * (- (4a \\
& *c - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c*d^2e^2 + 680a^4 \\
& *b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^3c \\
& ^4d^2e^2 + 320a^6b^2c^3d^3e^3 - 16a^2b^3c^3d^3e * (- (4ac - b^2)^5) \\
& ^{(1/2)} + 12a^3b^3c^2d^3e * (- (4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2 \\
& * (- (4ac - b^2)^5)^{(1/2)} - 8a^4b^3c^2d^2e^3 * (- (4ac - b^2)^5)^{(1/2)) / (512 * \\
& (a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3) \\
&))^{(1/4)} * (((- (b^{11}d^4 + a^4b^7e^4 - b^6d^4 * (- (4ac - b^2)^5)^{(1/2)} - 1 \\
& 12a^5b^3c^5d^4 - 11a^5b^5c^4e^4 - 48a^7b^3c^3e^4 + a^5c^4e^4 * (- (4ac \\
& - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 \\
& + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^3e^3
\end{aligned}$$

$$\begin{aligned}
& d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - a^4 * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a^6 * b^3 * c^2 * e^4 + 6 * a^2 * b^9 * d^2 * e^2 - 15 * a * b^9 * c * d^4 - 4 * a * b^10 * d^3 * e - 6 * a^2 * b^2 * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 6 * a^2 * b^4 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} \\
&)^{(1/2)} + 366 * a^4 * b^5 * c^2 * d^2 * e^2 - 720 * a^5 * b^3 * c^3 * d^2 * e^2 - 6 * a^4 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 5 * a * b^4 * c * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * a * b^5 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 56 * a^2 * b^8 * c * d^3 * e + 48 * a^4 * b^6 * c * d * e^3 + 4 * a^3 * b^3 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 292 * a^3 * b^6 * c^2 * d^3 * e - 78 * a^3 * b^7 * c * d^2 * e^2 + 680 * a^4 * b^4 * c^3 * d^3 * e - 640 * a^5 * b^2 * c^4 * d^3 * e - 200 * a^5 * b^4 * c^2 * d * e^3 + 480 * a^6 * b * c^4 * d^2 * e^2 + 320 * a^6 * b^2 * c^3 * d * e^3 - 16 * a^2 * b^3 * c * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 12 * a^3 * b * c^2 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 18 * a^3 * b^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a^4 * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} / (512 * (a^7 * b^8 + 256 * a^11 * c^4 - 16 * a^8 * b^6 * c + 96 * a^9 * b^4 * c^2 - 256 * a^10 * b^2 * c^3))^{(1/4)} * (262144 * a^17 * c^8 * d + 4096 * a^13 * b^8 * c^4 * d - 53248 * a^14 * b^6 * c^5 * d + 245760 * a^15 * b^4 * c^6 * d - 458752 * a^16 * b^2 * c^7 * d - 4096 * a^14 * b^7 * c^4 * e + 49152 * a^15 * b^5 * c^5 * e - 196608 * a^16 * b^3 * c^6 * e + 26214 * a^17 * b * c^7 * e) * 1i - x * (81920 * a^15 * b * c^8 * d^2 - 49152 * a^16 * b * c^7 * e^2 + 1024 * a^11 * b^9 * c^4 * d^2 - 13312 * a^12 * b^7 * c^5 * d^2 + 62464 * a^13 * b^5 * c^6 * d^2 - 122880 * a^14 * b^3 * c^7 * d^2 + 1024 * a^13 * b^7 * c^4 * e^2 - 11264 * a^14 * b^5 * c^5 * e^2 + 40960 * a^15 * b^3 * c^6 * e^2 - 65536 * a^16 * c^8 * d * e - 2048 * a^12 * b^8 * c^4 * d * e + 24576 * a^13 * b^6 * c^5 * d * e - 102400 * a^14 * b^4 * c^6 * d * e + 163840 * a^15 * b^2 * c^7 * d * e) * (- (b^11 * d^4 + a^4 * b^7 * e^4 - b^6 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 112 * a^5 * b * c^5 * d^4 - 1 * a^5 * b^5 * c * e^4 - 48 * a^7 * b * c^3 * e^4 + a^5 * c * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 4 * a^3 * b^8 * d * e^3 + 128 * a^6 * c^5 * d^3 * e - 128 * a^7 * c^4 * d * e^3 + 86 * a^2 * b^7 * c^2 * d^4 - 231 * a^3 * b^5 * c^3 * d^4 + 280 * a^4 * b^3 * c^4 * d^4 + a^3 * c^3 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - a^4 * b^2 * e^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 40 * a^6 * b^3 * c^2 * e^4 + 6 * a^2 * b^9 * d^2 * e^2 - 15 * a * b^9 * c * d^4 - 4 * a * b^10 * d^3 * e - 6 * a^2 * b^2 * c^2 * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} - 6 * a^2 * b^4 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 366 * a^4 * b^5 * c^2 * d^2 * e^2 - 720 * a^5 * b^3 * c^3 * d^2 * e^2 - 6 * a^4 * c^2 * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} + 5 * a * b^4 * c * d^4 * (- (4 * a * c - b^2)^5)^{(1/2)} + 4 * a * b^5 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 56 * a^2 * b^8 * c * d^3 * e + 48 * a^4 * b^6 * c * d * e^3 + 4 * a^3 * b^3 * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} - 292 * a^3 * b^6 * c^2 * d^3 * e - 78 * a^3 * b^7 * c * d^2 * e^2 + 680 * a^4 * b^4 * c^3 * d^3 * e - 640 * a^5 * b^2 * c^4 * d^3 * e - 200 * a^5 * b^4 * c^2 * d * e^3 + 480 * a^6 * b * c^4 * d^2 * e^2 + 320 * a^6 * b^2 * c^3 * d * e^3 - 16 * a^2 * b^3 * c * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 12 * a^3 * b * c^2 * d^3 * e * (- (4 * a * c - b^2)^5)^{(1/2)} + 18 * a^3 * b^2 * c * d^2 * e^2 * (- (4 * a * c - b^2)^5)^{(1/2)} - 8 * a^4 * b * c * d * e^3 * (- (4 * a * c - b^2)^5)^{(1/2)} / (512 * (a^7 * b^8 + 256 * a^11 * c^4 - 16 * a^8 * b^6 * c + 96 * a^9 * b^4 * c^2 - 256 * a^10 * b^2 * c^3))^{(3/4)} * 1i + 64 * a^14 * c^7 * e^5 + 128 * a^11 * b * c^9 * d^5 - 192 * a^12 * c^9 * d^4 * e + 16 * a^9 * b^5 * c^7 * d^5 - 96 * a^10 * b^3 * c^8 * d^5 - 16 * a^13 * b^2 * c^6 * e^5 - 128 * a^13 * c^8 * d^2 * e^3 + 64 * a^10 * b^5 * c^6 * d^3 * e^2 - 288 * a^11 * b^3 * c^7 * d^3 * e^2 - 96 * a^11 * b^4 * c^6 * d^2 * e^3 + 416 * a^12 * b^2 * c^7 * d^2 * e^3 - 256 * a^13 * b * c^7 * d * e^4 - 16 * a^9 * b^6 * c^6 * d^4 * e + 48 * a^10 * b^4 * c^7 * d^4 * e + 112 * a^11 * b^2 * c^8 * d^4 * e + 128 * a^12 * b * c^8 * d^3 * e^2 + 64 * a^12 * b^3 * c^6 * d * e^4) * 1i + x * (8 * a^13 * c^7 * e^6 - 8 * a^10 * c^10 * d^6 + 4 * a^9 * b^2 * c^9 * d^6 - 8 * a^11 * c^9 * d^4 * e^2 + 8 * a^12 * c^8 * d^2 * e^4 + 4 * a^9 * b^4 * c^7 * d^4 * e^2 + 16 * a^10 * b^2 * c^8 * d^4 * e^2 - 16 * a^10 * b^3 * c^7 * d^3 * e^3 + 28 * a^11 * b^2 * c^7 * d^2 * e^4 + 8 * a^10 * b * c^9 * d^5 * e - 24 * a^12 * b * c^7 * d * e^5 - 8 * a^9 *
\end{aligned}$$

$$\begin{aligned}
& b^3c^8d^5e - 16a^{11}b^8c^8d^3e^3) * (- (b^{11}d^4 + a^4b^7e^4 - b^6d^4 \\
& * (- (4ac - b^2)^5)^{1/2} - 112a^5b^5c^5d^4 - 11a^5b^5c^5e^4 - 48a^7b \\
& * c^3e^4 + a^5c^5e^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^3e^3 + 128a^6c \\
& ^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 2 \\
& 80a^4b^3c^4d^4 + a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} - a^4b^2e^4 * (- \\
& (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c \\
& * d^4 - 4ab^{10}d^3e - 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} - 6a^2b \\
& ^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 - 720a^5b^3 \\
& c^3d^2e^2 - 6a^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 5ab^4c^2d^4 * \\
& (- (4ac - b^2)^5)^{1/2} + 4ab^5d^3e * (- (4ac - b^2)^5)^{1/2} + 56a^2b \\
& ^8c^2d^3e + 48a^4b^6c^2d^3e^3 + 4a^3b^3d^3e^3 * (- (4ac - b^2)^5)^{1/2} \\
& - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 6 \\
& 40a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^8c^4d^2e^2 + 320a \\
& ^6b^2c^3d^3e^3 - 16a^2b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} + 12a^3b^3c \\
& ^2d^3e * (- (4ac - b^2)^5)^{1/2} + 18a^3b^2c^2d^2e^2 * (- (4ac - b^2)^5 \\
&)^{1/2} - 8a^4b^3c^2d^2e^3 * (- (4ac - b^2)^5)^{1/2} / (512(a^7b^8 + 256a^1 \\
& 1c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^{10}b^2c^3))^{1/4} / (((- (b^1 \\
& 1d^4 + a^4b^7e^4 - b^6d^4 * (- (4ac - b^2)^5)^{1/2} - 112a^5b^5c^5d^4 \\
& - 11a^5b^5c^5e^4 - 48a^7b^5c^3e^4 + a^5c^5e^4 * (- (4ac - b^2)^5)^{1/2} \\
& - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 \\
& d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^4 * (- (4ac - b^ \\
& 2)^5)^{1/2} - a^4b^2e^4 * (- (4ac - b^2)^5)^{1/2} + 40a^6b^3c^2e^4 + 6 \\
& a^2b^9d^2e^2 - 15ab^9c^2d^4 - 4ab^{10}d^3e - 6a^2b^2c^2d^4 * (- (4 \\
& ac - b^2)^5)^{1/2} - 6a^2b^4d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 366a^4 \\
& b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2 * (- (4ac - b \\
& ^2)^5)^{1/2} + 5ab^4c^2d^4 * (- (4ac - b^2)^5)^{1/2} + 4ab^5d^3e * (- (4 \\
& ac - b^2)^5)^{1/2} + 56a^2b^8c^2d^3e + 48a^4b^6c^2d^3e^3 + 4a^3b^3d \\
& ^3e^3 * (- (4ac - b^2)^5)^{1/2} - 292a^3b^6c^2d^3e - 78a^3b^7c^2d^2e^ \\
& 2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + \\
& 480a^6b^8c^4d^2e^2 + 320a^6b^2c^3d^3e^3 - 16a^2b^3c^2d^3e * (- (4ac \\
& c - b^2)^5)^{1/2} + 12a^3b^3c^2d^3e * (- (4ac - b^2)^5)^{1/2} + 18a^3b^ \\
& 2c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} - 8a^4b^3c^2d^2e^3 * (- (4ac - b^2)^5)^{1/2} \\
&) / (512(a^7b^8 + 256a^{11}c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^ \\
& 10b^2c^3))^{1/4} * (((- (b^{11}d^4 + a^4b^7e^4 - b^6d^4 * (- (4ac - b^2)^5 \\
&)^{1/2} - 112a^5b^5c^5d^4 - 11a^5b^5c^5e^4 - 48a^7b^5c^3e^4 + a^5c^5e \\
& ^4 * (- (4ac - b^2)^5)^{1/2} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7 \\
& c^4d^3e^3 + 86a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 \\
& + a^3c^3d^4 * (- (4ac - b^2)^5)^{1/2} - a^4b^2e^4 * (- (4ac - b^2)^5)^{1 \\
& /2} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 - 15ab^9c^2d^4 - 4ab^{10}d^ \\
& 3e - 6a^2b^2c^2d^4 * (- (4ac - b^2)^5)^{1/2} - 6a^2b^4d^2e^2 * (- (4a \\
& c - b^2)^5)^{1/2} + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 - 6a \\
& ^4c^2d^2e^2 * (- (4ac - b^2)^5)^{1/2} + 5ab^4c^2d^4 * (- (4ac - b^2)^5 \\
&)^{1/2} + 4ab^5d^3e * (- (4ac - b^2)^5)^{1/2} + 56a^2b^8c^2d^3e + 48a \\
& ^4b^6c^2d^3e^3 + 4a^3b^3d^3e^3 * (- (4ac - b^2)^5)^{1/2} - 292a^3b^6c^2 \\
& d^3e - 78a^3b^7c^2d^2e^2 + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3
\end{aligned}$$

$$\begin{aligned}
& *e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 \\
& - 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c \\
& - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b* \\
& c*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6 \\
& *c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)}*(262144*a^17*c^8*d + 4096*a \\
& ^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d - 458752*a^16* \\
& b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 196608*a^16*b^3*c^ \\
& 6*e + 262144*a^17*b*c^7*e)*1i + x*(81920*a^15*b*c^8*d^2 - 49152*a^16*b*c^7* \\
& e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13*b^5*c^6*d \\
& ^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14*b^5*c^5*e \\
& ^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*c^4*d*e + \\
& 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^2*c^7*d*e) \\
&)*(-(b^11*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b* \\
& c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5 \\
&)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2* \\
& b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4* \\
& a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2 \\
& *e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2* \\
& d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + \\
& 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2*(-(4 \\
& *a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3 \\
& *e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a \\
& ^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7* \\
& c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2 \\
& *d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e \\
& *(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 1 \\
& 8*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3*(-(4*a*c - b \\
& ^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 \\
& - 256*a^10*b^2*c^3)))^{(3/4)}*1i + 64*a^14*c^7*e^5 + 128*a^11*b*c^9*d^5 - 192 \\
& *a^12*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^10*b^3*c^8*d^5 - 16*a^13*b^2*c^ \\
& 6*e^5 - 128*a^13*c^8*d^2*e^3 + 64*a^10*b^5*c^6*d^3*e^2 - 288*a^11*b^3*c^7*d \\
& ^3*e^2 - 96*a^11*b^4*c^6*d^2*e^3 + 416*a^12*b^2*c^7*d^2*e^3 - 256*a^13*b*c^ \\
& 7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^10*b^4*c^7*d^4*e + 112*a^11*b^2*c^8*d \\
& ^4*e + 128*a^12*b*c^8*d^3*e^2 + 64*a^12*b^3*c^6*d*e^4)*1i - x*(8*a^13*c^7*e \\
& ^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*c^8* \\
& d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3*c^7 \\
& *d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7*d*e \\
& ^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^7*e^ \\
& 4 - b^6*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 \\
& - 48*a^7*b*c^3*e^4 + a^5*c*e^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 \\
& + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5* \\
& c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4* \\
& b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - \\
& 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/ \\
& 2)} - 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 -
\end{aligned}$$

$$\begin{aligned}
& 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 5a \\
& *b^4c^4d^4(-4ac - b^2)^5)^{(1/2)} + 4a^5b^3d^3e^2(-4ac - b^2)^5)^{(1/2)} \\
& + 56a^2b^8c^3d^3e + 48a^4b^6c^3d^3e^3 + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^3d^2e^2 + 680a^4b^4c^3d^3e \\
& - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e^3 - 16a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& + 12a^3b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& / (512(a^7b^8 + 256a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^10b^2c^3))^{(1/4)} * i + ((-b^11d^4 + a^4b^7e^4 - b^6d^4(-4ac - b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 \\
& - 11a^5b^5c^3e^4 - 48a^7b^3c^3e^4 + a^5c^3e^4(-4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 8 \\
& 6a^2b^7c^2d^4 - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^4(-4ac - b^2)^5)^{(1/2)} - a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 \\
& + 6a^2b^9d^2e^2 - 15a^2b^9c^3d^4 - 4a^2b^10d^3e - 6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} - 6a^2b^4d^2e^2(-4ac - b^2)^5)^{(1/2)} \\
& + 366a^4b^5c^2d^2e^2 - 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 5a^2b^4c^3d^4(-4ac - b^2)^5)^{(1/2)} + 4a^2b^5d^3e^3 \\
& (-4ac - b^2)^5)^{(1/2)} + 56a^2b^8c^3d^3e + 48a^4b^6c^3d^3e^3 + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^3d^2e^2 \\
& + 680a^4b^4c^3d^3e - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e^3 - 16a^2b^3c^3d^3e^3 \\
& (-4ac - b^2)^5)^{(1/2)} + 12a^3b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& / (512(a^7b^8 + 256a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^10b^2c^3))^{(1/4)} * (((-b^11d^4 + a^4b^7e^4 - b^6d^4(-4ac - b^2)^5)^{(1/2)} - 112a^5b^3c^5d^4 \\
& - 11a^5b^5c^3e^4 - 48a^7b^3c^3e^4 + a^5c^3e^4(-4ac - b^2)^5)^{(1/2)} - 4a^3b^8d^3e^3 + 128a^6c^5d^3e - 128a^7c^4d^3e^3 + 86a^2b^7c^2d^4 \\
& - 231a^3b^5c^3d^4 + 280a^4b^3c^4d^4 + a^3c^3d^4(-4ac - b^2)^5)^{(1/2)} - a^4b^2e^4(-4ac - b^2)^5)^{(1/2)} + 40a^6b^3c^2e^4 + 6a^2b^9d^2e^2 \\
& - 15a^2b^9c^3d^4 - 4a^2b^10d^3e - 6a^2b^2c^2d^4(-4ac - b^2)^5)^{(1/2)} - 6a^2b^4d^2e^2(-4ac - b^2)^5)^{(1/2)} + 366a^4b^5c^2d^2e^2 \\
& - 720a^5b^3c^3d^2e^2 - 6a^4c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} + 5a^2b^4c^3d^4(-4ac - b^2)^5)^{(1/2)} + 4a^2b^5d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& + 56a^2b^8c^3d^3e + 48a^4b^6c^3d^3e^3 + 4a^3b^3d^3e^3(-4ac - b^2)^5)^{(1/2)} - 292a^3b^6c^2d^3e - 78a^3b^7c^3d^2e^2 + 680a^4b^4c^3d^3e \\
& - 640a^5b^2c^4d^3e - 200a^5b^4c^2d^3e^3 + 480a^6b^3c^4d^2e^2 + 320a^6b^2c^3d^3e^3 - 16a^2b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& + 12a^3b^3c^2d^3e^3(-4ac - b^2)^5)^{(1/2)} + 18a^3b^2c^2d^2e^2(-4ac - b^2)^5)^{(1/2)} - 8a^4b^3c^3d^3e^3(-4ac - b^2)^5)^{(1/2)} \\
& / (512(a^7b^8 + 256a^11c^4 - 16a^8b^6c + 96a^9b^4c^2 - 256a^10b^2c^3))^{(1/4)} * (262144a^17c^8d + 4096a^13b^8c^4d - 53248a^14b^6c^5d \\
& + 245760a^15b^4c^6d - 458752a^16b^2c^7d - 4096a^14b^7c^4e + 49152a^15b^5c^5e - 196608a^16b^3c^6e + 262144a^17b^3c^7e) * i - x(81920a^15b^3c^8d^2 - 491
\end{aligned}$$

$$\begin{aligned}
& 52*a^{16}*b*c^7*e^2 + 1024*a^{11}*b^9*c^4*d^2 - 13312*a^{12}*b^7*c^5*d^2 + 62464*a^{13}*b^5*c^6*d^2 - 122880*a^{14}*b^3*c^7*d^2 + 1024*a^{13}*b^7*c^4*e^2 - 11264*a^{14}*b^5*c^5*e^2 + 40960*a^{15}*b^3*c^6*e^2 - 65536*a^{16}*c^8*d*e - 2048*a^{12}*b^8*c^4*d*e + 24576*a^{13}*b^6*c^5*d*e - 102400*a^{14}*b^4*c^6*d*e + 163840*a^{15}*b^2*c^7*d*e) * (- (b^{11}*d^4 + a^4*b^7*e^4 - b^6*d^4 * (- (4*a*c - b^2)^5)^{(1/2)}) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3 * (- (4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3 * (- (4*a*c - b^2)^5)^{(1/2)}) / (512*(a^7*b^8 + 256*a^{11}*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^{10}*b^2*c^3))^{(3/4)} * i + 64*a^{14}*c^7*e^5 + 128*a^{11}*b*c^9*d^5 - 192*a^{12}*c^9*d^4*e + 16*a^9*b^5*c^7*d^5 - 96*a^{10}*b^3*c^8*d^5 - 16*a^{13}*b^2*c^6*e^5 - 128*a^{13}*c^8*d^2*e^3 + 64*a^{10}*b^5*c^6*d^3*e^2 - 288*a^{11}*b^3*c^7*d^3*e^2 - 96*a^{11}*b^4*c^6*d^2*e^3 + 416*a^{12}*b^2*c^7*d^2*e^3 - 256*a^{13}*b*c^7*d*e^4 - 16*a^9*b^6*c^6*d^4*e + 48*a^{10}*b^4*c^7*d^4*e + 112*a^{11}*b^2*c^8*d^4*e + 128*a^{12}*b*c^8*d^3*e^2 + 64*a^{12}*b^3*c^6*d*e^4) * i + x * (8*a^{13}*c^7*e^6 - 8*a^{10}*c^{10}*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^{11}*c^9*d^4*e^2 + 8*a^{12}*c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^{10}*b^2*c^8*d^4*e^2 - 16*a^{10}*b^3*c^7*d^3*e^3 + 28*a^{11}*b^2*c^7*d^2*e^4 + 8*a^{10}*b*c^9*d^5*e - 24*a^{12}*b*c^7*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^{11}*b*c^8*d^3*e^3) * (- (b^{11}*d^4 + a^4*b^7*e^4 - b^6*d^4 * (- (4*a*c - b^2)^5)^{(1/2)}) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 + a^3*c^3*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e - 6*a^2*b^2*c^2*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a^4*c^2*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4 * (- (4*a*c - b^2)^5)^{(1/2)} + 4*a*b^5*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3 + 4*a^3*b^3*d*e^3 * (- (4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e * (- (4*a*c - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2 * (- (4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c*d*e^3 * (- (4*a*c - b^2)^5)^{(1/2)})
\end{aligned}$$

$$\begin{aligned} &)/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b \\ & ^2*c^3)))^{(1/4)*1i}))*(-(b^{11}*d^4 + a^4*b^7*e^4 - b^6*d^4*(-(4*a*c - b^2)^5) \\ & ^{(1/2)} - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 + a^5*c*e^4 \\ & ^4*(-(4*a*c - b^2)^5)^{(1/2)} - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7* \\ & c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 \\ & + a^3*c^3*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - a^4*b^2*e^4*(-(4*a*c - b^2)^5)^{(1/2)} \\ & + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3 \\ & *e - 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^{(1/2)} - 6*a^2*b^4*d^2*e^2*(-(4*a* \\ & c - b^2)^5)^{(1/2)} + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 - 6*a \\ & ^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} + 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^{(1/2)} \\ & + 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 56*a^2*b^8*c*d^3*e + 48*a^4 \\ & *b^6*c*d*e^3 + 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^{(1/2)} - 292*a^3*b^6*c^2* \\ & d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3* \\ & e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 - \\ & 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^{(1/2)} + 12*a^3*b*c^2*d^3*e*(-(4*a*c \\ & - b^2)^5)^{(1/2)} + 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^{(1/2)} - 8*a^4*b*c \\ & *d*e^3*(-(4*a*c - b^2)^5)^{(1/2)})/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6* \\ & c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^{(1/4)} - d/(3*a*x^3) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)

[Out] Timed out

$$3.52 \quad \int \frac{x^4(1-x^4)}{1-x^4+x^8} dx$$

Optimal. Leaf size=278

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

[Out] $-x-1/12*\arctan((-2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}+1/12*\arctan((2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}-1/12*\arctan((-2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}+1/12*\arctan((2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}-1/24*\ln(1+x^2-x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}+1/24*\ln(1+x^2+x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}-1/24*\ln(1+x^2-x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}+1/24*\ln(1+x^2+x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1502, 1346, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] $-x - \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) + \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) + \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1502

```
Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(1-x^4)}{1-x^4+x^8} dx &= -x + \int \frac{1}{1-x^4+x^8} dx \\
&= -x + \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -x + \frac{\int \frac{\sqrt{3(2-\sqrt{3})} - (-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})} + (-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})} - (1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})} + (1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} \\
&= -x - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \dots \\
&= -x - \frac{\log\left(1 - \sqrt{2-\sqrt{3}}x + x^2\right)}{4\sqrt{6}} + \frac{\log\left(1 + \sqrt{2-\sqrt{3}}x + x^2\right)}{4\sqrt{6}} - \frac{\log\left(1 - \sqrt{2+\sqrt{3}}x + x^2\right)}{4\sqrt{6}} + \dots \\
&= -x - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \dots
\end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\log(x - \#1)}{2\#1^7 - \#1^3}\&\right] - x$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] -x + RootSum[1 - #1^4 + #1^8 &, Log[x - #1]/(-#1^3 + 2*#1^7) &]/4

fricas [A] time = 0.96, size = 218, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")

[Out] $-1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) + x^2 - \sqrt{x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1}*(\sqrt{3}*\sqrt{2}*x - 2)))/(3*x^2 - 2)) - 1/6*\sqrt{3}*\sqrt{2}*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) - x^2 - \sqrt{x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1}*(\sqrt{3}*\sqrt{2}*x + 2)))/(3*x^2 - 2)) + 1/24*\sqrt{3}*\sqrt{2}*\log(x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1) - 1/24*\sqrt{3}*\sqrt{2}*\log(x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1) - x$

giac [A] time = 0.45, size = 208, normalized size = 0.75

$$\frac{1}{12} \sqrt{6} \arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{12} \sqrt{6} \arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) + \frac{1}{12} \sqrt{6} \arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{12} \sqrt{6} \arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) + \frac{1}{24} \sqrt{6} \log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")

[Out] $1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - x$

maple [C] time = 0.01, size = 34, normalized size = 0.12

$$-x + \frac{\text{RootOf}(9_Z^4 + 1) \ln\left(3 \text{RootOf}(9_Z^4 + 1)^2 + 3 \text{RootOf}(9_Z^4 + 1)x + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x)

[Out] $-x+1/4*\sum(_R*\ln(3*_R^2+3*_R*x+x^2),_R=\text{RootOf}(9*_Z^4+1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x + \int \frac{1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")

[Out] -x + integrate(1/(x^8 - x^4 + 1), x)

mupad [B] time = 1.92, size = 56, normalized size = 0.20

$$-x + \sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i \right)}{\frac{2x^2}{3} - \frac{2}{3}i} \right) \left(-\frac{1}{12} - \frac{1}{12}i \right) + \sqrt{6} \operatorname{atan} \left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i \right)}{\frac{2x^2}{3} + \frac{2}{3}i} \right) \left(-\frac{1}{12} + \frac{1}{12}i \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)

[Out] - x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)

sympy [A] time = 0.23, size = 170, normalized size = 0.61

$$-x - \frac{\sqrt{6} \left(-2 \operatorname{atan} \left(\frac{\sqrt{6}x}{3} - \frac{1}{3} \right) - 2 \operatorname{atan} \left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3 \right) \right)}{24} - \frac{\sqrt{6} \left(-2 \operatorname{atan} \left(\frac{\sqrt{6}x}{3} + \frac{1}{3} \right) - 2 \operatorname{atan} \left(\sqrt{6}x^3 + 4x^2 - 2\sqrt{6}x + 3 \right) \right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)

[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24

$$3.53 \quad \int \frac{x^3(1-x^4)}{1-x^4+x^8} dx$$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 - x^4 + 1)$$

[Out] $-1/8*\ln(x^8-x^4+1)-1/12*\arctan(1/3*(-2*x^4+1)*3^{(1/2)})*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] $-\text{ArcTan}[(1 - 2*x^4)/\text{Sqrt}[3]]/(4*\text{Sqrt}[3]) - \text{Log}[1 - x^4 + x^8]/8$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In

$t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1468

$\text{Int}[(x_)^{(m_.)}*((a_) + (c_.)*(x_)^{(n2_.)} + (b_.)*(x_)^{(n_.)})^{(p_.)}*((d_) + (e_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3(1-x^4)}{1-x^4+x^8} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1-x}{1-x+x^2} dx, x, x^4 \right) \\ &= \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\ &= -\frac{1}{8} \log(1-x^4+x^8) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{8} \log(1-x^4+x^8) \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1} \left(\frac{2x^4-1}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 - x^4 + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8

fricas [A] time = 0.89, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^4 - 1) \right) - \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1), x, algorithm="fricas")

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) - 1/8*\log(x^8 - x^4 + 1)$

giac [A] time = 0.63, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")`

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) - 1/8*\log(x^8 - x^4 + 1)$

maple [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^8 - x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(-x^4+1)/(x^8-x^4+1),x)`

[Out] $-1/8*\ln(x^8-x^4+1)+1/12*3^{(1/2)}*\arctan(1/3*(2*x^4-1)*3^{(1/2)})$

maxima [A] time = 1.51, size = 32, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")`

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) - 1/8*\log(x^8 - x^4 + 1)$

mupad [B] time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)`

[Out] $-\log(x^8 - x^4 + 1)/8 - (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^4)/3))/12$

sympy [A] time = 0.15, size = 37, normalized size = 0.95

$$-\frac{\log(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)

[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12

$$3.54 \quad \int \frac{x^2(1-x^4)}{1-x^4+x^8} dx$$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right)$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A] time = 0.29, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2+\sqrt{3}}x + 1\right) + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2+\sqrt{3}}x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8), x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1279

Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1506

Int((((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q - b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x] / (q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +

$(c*d - e*q)*x^{(n/2)}, x]]/(q + r*x^{(n/2)} + c*x^n), x], x]] /; !LtQ[2*c*q - b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] \&\& EqQ[n2, 2*n] \&\& LtQ[b^2 - 4*a*c, 0] \&\& IntegerQ[m, n/2] \&\& LtQ[0, m, n] \&\& PosQ[a*c]$

Rubi steps

$$\begin{aligned} \int \frac{x^2(1-x^4)}{1-x^4+x^8} dx &= \frac{\int \frac{x^2(\sqrt{3}-2x^2)}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{x^2(\sqrt{3}+2x^2)}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\ &= \frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\ &= \frac{\int \frac{2\sqrt{2-\sqrt{3}}-(2-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+(2-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-(-2-\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+(-2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\ &= \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx + \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\ &= \frac{1}{8}\sqrt{\frac{2}{3}} - \frac{1}{\sqrt{3}} \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) \\ &= \frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1^4 \log(x - \#1) - \log(x - \#1)}{2\#1^5 - \#1}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] -1/4*RootSum[1 - #1^4 + #1^8 &, (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2*#1^5) &]

fricas [B] time = 1.19, size = 715, normalized size = 2.01

$$\frac{1}{48} \sqrt{6} \left(\sqrt{3} \sqrt{2} - 2 \sqrt{2} \right) \sqrt{\sqrt{3} + 2} \log \left(12x^2 + 2\sqrt{6} \left(2\sqrt{3} \sqrt{2}x - 3\sqrt{2}x \right) \sqrt{\sqrt{3} + 2} + 12 \right) - \frac{1}{48} \sqrt{6} \left(\sqrt{3} \sqrt{2} - 2 \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")

[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)

giac [A] time = 0.48, size = 253, normalized size = 0.71

$$-\frac{1}{24} \left(\sqrt{6} + 3 \sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} + 3 \sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} - 3 \sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} - 3 \sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2)))

6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)

maple [C] time = 0.01, size = 46, normalized size = 0.13

$$\frac{\left(\text{RootOf}\left(-Z^8 - Z^4 + 1\right)^6 - \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^2\right) \ln\left(-\text{RootOf}\left(-Z^8 - Z^4 + 1\right) + x\right)}{4\left(2\text{RootOf}\left(-Z^8 - Z^4 + 1\right)^7 - \text{RootOf}\left(-Z^8 - Z^4 + 1\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x)

[Out] -1/4*sum((R^6-R^2)/(2*R^7-R^3)*ln(-R+x),R=RootOf(-Z^8-Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(x^4 - 1)x^2}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")

[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)

mupad [B] time = 1.99, size = 248, normalized size = 0.70

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}}{2(-1+\sqrt{3}1i)} + \frac{\sqrt{3}x(8-\sqrt{3}8i)^{1/4}1i}{2(-1+\sqrt{3}1i)}\right)(8-\sqrt{3}8i)^{1/4}1i + \sqrt{3} \operatorname{atan}\left(\frac{x(8-\sqrt{3}8i)^{1/4}1i}{2(-1+\sqrt{3}1i)} - \frac{\sqrt{3}x(8-\sqrt{3}8i)^{1/4}}{2(-1+\sqrt{3}1i)}\right)(8-\sqrt{3}8i)^{1/4}1i}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)

[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4)) - (2^(3/4)*x)/(2*(3^(1/2)*1i - 1)^(3/4)))/12

$$\frac{(3/4)*3^{(1/2)*x*1i}/(2*(3^{(1/2)*1i + 1})^{(3/4)}))*(3^{(1/2)*1i + 1})^{(1/4)*1i}/12 - (2^{(3/4)*3^{(1/2)*atan((2^{(3/4)*x*1i})/(2*(3^{(1/2)*1i + 1})^{(3/4)})} + (2^{(3/4)*3^{(1/2)*x})/(2*(3^{(1/2)*1i + 1})^{(3/4)}))*(3^{(1/2)*1i + 1})^{(1/4)})/12$$

sympy [A] time = 3.16, size = 27, normalized size = 0.08

$$-\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log(442368t^7 - 384t^3 + x)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)

[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 384*_t**3 + x)))

$$3.55 \quad \int \frac{x(1-x^4)}{1-x^4+x^8} dx$$

Optimal. Leaf size=50

$$\frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

[Out] $-1/12*\ln(1+x^4-3^{(1/2)}*x^2)*3^{(1/2)}+1/12*\ln(1+x^4+3^{(1/2)}*x^2)*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1490, 1164, 628}

$$\frac{\log(x^4 + \sqrt{3}x^2 + 1)}{4\sqrt{3}} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] $-\text{Log}[1 - \text{Sqrt}[3]*x^2 + x^4]/(4*\text{Sqrt}[3]) + \text{Log}[1 + \text{Sqrt}[3]*x^2 + x^4]/(4*\text{Sqrt}[3])$

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rule 1490

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,

2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{x(1-x^4)}{1-x^4+x^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) \\ &= -\frac{\text{Subst} \left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} - \frac{\text{Subst} \left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2} dx, x, x^2 \right)}{4\sqrt{3}} \\ &= -\frac{\log(1-\sqrt{3}x^2+x^4)}{4\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{4\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.88

$$\frac{\log(x^4 + \sqrt{3}x^2 + 1) - \log(-x^4 + \sqrt{3}x^2 - 1)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8), x]

[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])

fricas [A] time = 1.06, size = 41, normalized size = 0.82

$$\frac{1}{12} \sqrt{3} \log \left(\frac{x^8 + 5x^4 + 2\sqrt{3}(x^6 + x^2) + 1}{x^8 - x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+1)/(x^8-x^4+1), x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))

giac [A] time = 0.44, size = 31, normalized size = 0.62

$$-\frac{1}{12} \sqrt{3} \log \left(\frac{x^2 - \sqrt{3} + \frac{1}{x^2}}{x^2 + \sqrt{3} + \frac{1}{x^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))

maple [A] time = 0.02, size = 39, normalized size = 0.78

$$-\frac{\sqrt{3} \ln(x^4 - \sqrt{3} x^2 + 1)}{12} + \frac{\sqrt{3} \ln(x^4 + \sqrt{3} x^2 + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^4+1)/(x^8-x^4+1),x)

[Out] -1/12*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/12*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(x^4 - 1)x}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")

[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)

mupad [B] time = 1.89, size = 20, normalized size = 0.40

$$\frac{\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3} x^2}{x^4+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)

[Out] (3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6

sympy [A] time = 0.13, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3} \log(x^4 - \sqrt{3} x^2 + 1)}{12} + \frac{\sqrt{3} \log(x^4 + \sqrt{3} x^2 + 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)

[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2 + 1)/12

$$3.56 \quad \int \frac{1-x^4}{1-x^4+x^8} dx$$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2\right)$$

[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))

Rubi [A] time = 0.22, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(1 - x^4 + x^8), x]

[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[

a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1169

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]

Rule 1421

Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{1-x^4+x^8} dx &= -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4} dx}{2\sqrt{3}} \\
&= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(2+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(2+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3(2-\sqrt{3})}} \\
&\quad + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(2+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3(2+\sqrt{3})}} \\
&= -\left(\frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx \right) - \frac{1}{8}\sqrt{\frac{1}{3}}(7-4\sqrt{3}) \int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx \\
&= \frac{1}{8}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}} \log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \\
&= -\frac{1}{4}\sqrt{\frac{1}{3}}(2+\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3}) \tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}}(2-\sqrt{3})
\end{aligned}$$

Mathematica [C] time = 0.01, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&, \frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(1 - x^4 + x^8), x]

[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#1^7) &]

fricas [B] time = 0.92, size = 715, normalized size = 2.01

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(12x^2+2\sqrt{6}\left(2\sqrt{3}\sqrt{2}x-3\sqrt{2}x\right)\sqrt{\sqrt{3}+2}+12\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqrt
(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6)
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqrt
(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6)
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqrt
(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6))*
(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
- 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
+ 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6))*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3)
+ 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6))*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
+ 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6))*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

giac [A] time = 0.54, size = 253, normalized size = 0.71

$$\frac{1}{24} \left(\sqrt{6} + 3\sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) + \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6)
+ sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

maple [C] time = 0.00, size = 44, normalized size = 0.12

$$\frac{\left(-\text{RootOf}\left(_Z^8 - _Z^4 + 1\right)^4 + 1\right) \ln\left(-\text{RootOf}\left(_Z^8 - _Z^4 + 1\right) + x\right)}{8 \text{RootOf}\left(_Z^8 - _Z^4 + 1\right)^7 - 4 \text{RootOf}\left(_Z^8 - _Z^4 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/(x^8-x^4+1),x)

[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4 - 1}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")

[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)

mupad [B] time = 0.00, size = 208, normalized size = 0.59

$$\frac{\sqrt{3} \operatorname{atan}\left(\frac{x}{(8-\sqrt{3}8i)^{1/4}} + \frac{\sqrt{3}xi}{(8-\sqrt{3}8i)^{1/4}}\right) (8-\sqrt{3}8i)^{1/4} \operatorname{li} \sqrt{3} \operatorname{atan}\left(\frac{xi}{(8-\sqrt{3}8i)^{1/4}} - \frac{\sqrt{3}x}{(8-\sqrt{3}8i)^{1/4}}\right) (8-\sqrt{3}8i)^{1/4} + 2^{3/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)

[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12

sympy [A] time = 3.23, size = 26, normalized size = 0.07

$$-\text{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_t + x)))
```

$$3.57 \quad \int \frac{1-x^4}{x(1-x^4+x^8)} dx$$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8} \log(x^8 - x^4 + 1) + \log(x)$$

[Out] $\ln(x) - 1/8 * \ln(x^8 - x^4 + 1) + 1/12 * \arctan(1/3 * (-2 * x^4 + 1) * 3^{(1/2)}) * 3^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{8} \log(x^8 - x^4 + 1) + \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - x^4)/(x*(1 - x^4 + x^8)), x]$

[Out] $\text{ArcTan}[(1 - 2*x^4)/\text{Sqrt}[3]]/(4*\text{Sqrt}[3]) + \text{Log}[x] - \text{Log}[1 - x^4 + x^8]/8$

Rule 204

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + (b \cdot x) + (c \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, x\} \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d + (e \cdot x))/(a + (b \cdot x) + (c \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1474

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{1-x^4}{x(1-x^4+x^8)} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{1-x}{x(1-x+x^2)} dx, x, x^4 \right) \\
 &= \frac{1}{4} \text{Subst} \left(\int \left(\frac{1}{x} - \frac{x}{1-x+x^2} \right) dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{4} \text{Subst} \left(\int \frac{x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) \\
 &= \log(x) - \frac{1}{8} \log(1-x^4+x^8) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\
 &= \frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} + \log(x) - \frac{1}{8} \log(1-x^4+x^8)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 44, normalized size = 1.07

$$\log(x) - \frac{1}{4} \text{RootSum} \left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^4 \log(x - \#1)}{2\#1^4 - 1} \& \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]

[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) &]/4

fricas [A] time = 0.79, size = 34, normalized size = 0.83

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log(x)

giac [A] time = 0.45, size = 38, normalized size = 0.93

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4*log(x^4)

maple [A] time = 0.01, size = 35, normalized size = 0.85

$$-\frac{\sqrt{3} \arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12} + \ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/x/(x^8-x^4+1),x)

[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))

maxima [A] time = 0.97, size = 38, normalized size = 0.93

$$-\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^4 - 1)\right) - \frac{1}{8} \log(x^8 - x^4 + 1) + \frac{1}{4} \log(x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")

[Out] $-1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) - 1/8*\log(x^8 - x^4 + 1) + 1/4*\log(x^4)$

mupad [B] time = 1.89, size = 36, normalized size = 0.88

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)

[Out] $\log(x) - \log(x^8 - x^4 + 1)/8 + (3^{(1/2)}*\operatorname{atan}(3^{(1/2)}/3 - (2*3^{(1/2)}*x^4)/3))/12$

sympy [A] time = 0.16, size = 41, normalized size = 1.00

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)

[Out] $\log(x) - \log(x**8 - x**4 + 1)/8 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**4/3 - \sqrt{3}/3)/12$

$$3.58 \quad \int \frac{1-x^4}{x^2(1-x^4+x^8)} dx$$

Optimal. Leaf size=280

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

[Out] $-1/x+1/12*\arctan((-2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}-1/12*\arctan((2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}+1/12*\arctan((-2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}-1/12*\arctan((2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}-1/24*\ln(1+x^2-x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}+1/24*\ln(1+x^2+x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*6^{(1/2)}-1/24*\ln(1+x^2-x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}+1/24*\ln(1+x^2+x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*6^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1504, 1372, 1169, 634, 618, 204, 628}

$$\frac{\log\left(x^2 - \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 - \sqrt{3}} x + 1\right)}{4\sqrt{6}} - \frac{\log\left(x^2 - \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}} + \frac{\log\left(x^2 + \sqrt{2 + \sqrt{3}} x + 1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]

[Out] $-x^{(-1)} + \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) + \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]]/(2*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) - \text{Log}[1 - \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6]) + \text{Log}[1 + \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2]/(4*\text{Sqrt}[6])$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m + n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c*d*(m + 2*n*(p + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x^2(1-x^4+x^8)} dx &= -\frac{1}{x} - \int \frac{x^6}{1-x^4+x^8} dx \\
&= -\frac{1}{x} + \frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} - \frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{1}{x} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-(1-\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+(1-\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2-\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+(1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{3}(2+\sqrt{3})} \\
&= -\frac{1}{x} - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{4\sqrt{6}} \\
&= -\frac{1}{x} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} \\
&= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&, \frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)), x]

[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) &]/4

fricas [A] time = 0.97, size = 224, normalized size = 0.80

$$4\sqrt{3}\sqrt{2}x\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)+4\sqrt{3}\sqrt{2}x\arctan\left(\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}(\sqrt{3}\sqrt{2}x-2)}{3x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{24}*(4*\sqrt{3}*\sqrt{2}*x*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) + x^2 - \sqrt{x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1)*(\sqrt{3}*\sqrt{2}*x - 2))/(3*x^2 - 2)) + 4*\sqrt{3}*\sqrt{2}*x*\arctan(-(\sqrt{3}*\sqrt{2}*(x^3 - x) - x^2 - \sqrt{x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1)*(\sqrt{3}*\sqrt{2}*x + 2))/(3*x^2 - 2)) + \sqrt{3}*\sqrt{2}*x*\log(x^4 + \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1) - \sqrt{3}*\sqrt{2}*x*\log(x^4 - \sqrt{3}*\sqrt{2}*(x^3 + x) + 3*x^2 + 1) - 24)/x$

giac [A] time = 0.58, size = 210, normalized size = 0.75

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")

[Out] $-1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} - \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} + \sqrt{2})/(\sqrt{6} + \sqrt{2})) - 1/12*\sqrt{6}*\arctan((4*x + \sqrt{6} + \sqrt{2})/(\sqrt{6} - \sqrt{2})) - 1/12*\sqrt{6}*\arctan((4*x - \sqrt{6} - \sqrt{2})/(\sqrt{6} - \sqrt{2})) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} + \sqrt{2}) + 1) + 1/24*\sqrt{6}*\log(x^2 + 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/24*\sqrt{6}*\log(x^2 - 1/2*x*(\sqrt{6} - \sqrt{2}) + 1) - 1/x$

maple [C] time = 0.01, size = 38, normalized size = 0.14

$$\frac{\text{RootOf}(9_Z^4 + 1)\ln\left(9\text{RootOf}(9_Z^4 + 1)^3 x - 3\text{RootOf}(9_Z^4 + 1)^2 + x^2\right)}{4} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/x^2/(x^8-x^4+1),x)

[Out] $-1/x - 1/4*\sum(_R*\ln(9*_R^3*x - 3*_R^2 + x^2), _R=\text{RootOf}(9*_Z^4 + 1))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{x} - \int \frac{x^6}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)

mupad [B] time = 1.86, size = 58, normalized size = 0.21

$$-\frac{1}{x} + \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} + \frac{1}{3}i\right)}{\frac{2x^2}{3} - \frac{2}{3}i}\right) \left(\frac{1}{12} - \frac{1}{12}i\right) + \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x \left(\frac{1}{3} - \frac{1}{3}i\right)}{\frac{2x^2}{3} + \frac{2}{3}i}\right) \left(\frac{1}{12} + \frac{1}{12}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)

[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) + 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) - 1/x

sympy [A] time = 0.23, size = 168, normalized size = 0.60

$$\frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} - \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 - 4x^2 + 2\sqrt{6}x - 3\right) \right)}{24} - \frac{\sqrt{6} \left(2 \operatorname{atan}\left(\frac{\sqrt{6}x}{3} + \frac{1}{3}\right) + 2 \operatorname{atan}\left(\sqrt{6}x^3 + 4x^2 + 2\sqrt{6}x - 3\right) \right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)

[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x + 1)/24 - 1/x

$$3.59 \quad \int \frac{1-x^4}{x^3(1-x^4+x^8)} dx$$

Optimal. Leaf size=89

$$-\frac{1}{2x^2} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) - \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3}) - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}}$$

[Out] $-1/2/x^2-1/4*\arctan(2*x^2-3^{(1/2)})-1/4*\arctan(2*x^2+3^{(1/2)})-1/24*\ln(1+x^4-3^{(1/2)}*x^2)*3^{(1/2)}+1/24*\ln(1+x^4+3^{(1/2)}*x^2)*3^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1490, 1281, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2} - \frac{\log(x^4 - \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{\log(x^4 + \sqrt{3}x^2 + 1)}{8\sqrt{3}} + \frac{1}{4} \tan^{-1}(\sqrt{3} - 2x^2) - \frac{1}{4} \tan^{-1}(2x^2 + \sqrt{3})$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)), x]

[Out] $-1/(2*x^2) + \text{ArcTan}[\text{Sqrt}[3] - 2*x^2]/4 - \text{ArcTan}[\text{Sqrt}[3] + 2*x^2]/4 - \text{Log}[1 - \text{Sqrt}[3]*x^2 + x^4]/(8*\text{Sqrt}[3]) + \text{Log}[1 + \text{Sqrt}[3]*x^2 + x^4]/(8*\text{Sqrt}[3])$

Rule 204

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1127

```
Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
- 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1164

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1281

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(
x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1490

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_))^(p_)*((d_) + (e
_)*(x_)^(n_))^(q_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x^3(1-x^4+x^8)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1-x^2}{x^2(1-x^2+x^4)} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} - \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} + \frac{1}{4} \text{Subst} \left(\int \frac{1-x^2}{1-x^2+x^4} dx, x, x^2 \right) - \frac{1}{4} \text{Subst} \left(\int \frac{1+x^2}{1-x^2+x^4} dx, x, x^2 \right) \\
&= -\frac{1}{2x^2} - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-\sqrt{3}x+x^2} dx, x, x^2 \right) - \frac{1}{8} \text{Subst} \left(\int \frac{1}{1+\sqrt{3}x+x^2} dx, x, x^2 \right) - \\
&= -\frac{1}{2x^2} - \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, -\sqrt{x} \right) \\
&= -\frac{1}{2x^2} + \frac{1}{4} \tan^{-1}(\sqrt{3}-2x^2) - \frac{1}{4} \tan^{-1}(\sqrt{3}+2x^2) - \frac{\log(1-\sqrt{3}x^2+x^4)}{8\sqrt{3}} + \frac{\log(1+\sqrt{3}x^2+x^4)}{8\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.55

$$-\frac{1}{4} \text{RootSum} \left[\#1^8 - \#1^4 + 1 \&, \frac{\#1^2 \log(x - \#1)}{2\#1^4 - 1} \& \right] - \frac{1}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)), x]

[Out] -1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 &, (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &]/4

fricas [B] time = 0.84, size = 188, normalized size = 2.11

$$4\sqrt{6}\sqrt{3}\sqrt{2}x^2 \arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2 + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4 + \sqrt{6}\sqrt{2}x^2 + 2 - \sqrt{3}}\right) + 4\sqrt{6}\sqrt{3}\sqrt{2}x^2 \arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2 + \frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4 - \sqrt{6}\sqrt{2}x^2 + 2 + \sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^3/(x^8-x^4+1), x, algorithm="fricas")

[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*

$\sqrt{2}x^2 \log(2x^4 + \sqrt{6}\sqrt{2}x^2 + 2) - \sqrt{6}\sqrt{2}x^2 \log(2x^4 - \sqrt{6}\sqrt{2}x^2 + 2) - 24/x^2$

giac [A] time = 0.53, size = 81, normalized size = 0.91

$$-\frac{1}{24} \sqrt{3} x^4 \log(x^4 + \sqrt{3} x^2 + 1) + \frac{1}{24} \sqrt{3} x^4 \log(x^4 - \sqrt{3} x^2 + 1) - \frac{1}{4} x^4 \arctan(2x^2 + \sqrt{3}) - \frac{1}{4} x^4 \arctan(2x^2 - \sqrt{3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")

[Out] -1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 - sqrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2 - sqrt(3)) - 1/2/x^2

maple [A] time = 0.01, size = 70, normalized size = 0.79

$$-\frac{\arctan(2x^2 - \sqrt{3})}{4} - \frac{\arctan(2x^2 + \sqrt{3})}{4} - \frac{\sqrt{3} \ln(x^4 - \sqrt{3} x^2 + 1)}{24} + \frac{\sqrt{3} \ln(x^4 + \sqrt{3} x^2 + 1)}{24} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/x^3/(x^8-x^4+1),x)

[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*3^(1/2)*ln(x^4-3^(1/2)*x^2+1)+1/24*3^(1/2)*ln(x^4+3^(1/2)*x^2+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2x^2} - \int \frac{x^5}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)

mupad [B] time = 0.10, size = 56, normalized size = 0.63

$$\operatorname{atan}\left(\frac{2x^2}{-1 + \sqrt{3} 1i}\right) \left(\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) + \operatorname{atan}\left(\frac{2x^2}{1 + \sqrt{3} 1i}\right) \left(-\frac{1}{4} + \frac{\sqrt{3} 1i}{12}\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)

[Out] $\operatorname{atan}\left(\frac{2x^2}{\sqrt{3}i - 1}\right) \cdot \left(\frac{\sqrt{3}i}{12} + \frac{1}{4}\right) + \operatorname{atan}\left(\frac{2x^2}{\sqrt{3}i + 1}\right) \cdot \left(\frac{\sqrt{3}i}{12} - \frac{1}{4}\right) - \frac{1}{2x^2}$

sympy [A] time = 0.23, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{24} + \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{24} - \frac{\operatorname{atan}(2x^2 - \sqrt{3})}{4} - \frac{\operatorname{atan}(2x^2 + \sqrt{3})}{4} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**4+1)/x**3/(x**8-x**4+1), x)`

[Out] $-\frac{\sqrt{3} \log(x^4 - \sqrt{3}x^2 + 1)}{24} + \frac{\sqrt{3} \log(x^4 + \sqrt{3}x^2 + 1)}{24} - \frac{\operatorname{atan}(2x^2 - \sqrt{3})}{4} - \frac{\operatorname{atan}(2x^2 + \sqrt{3})}{4} - \frac{1}{2x^2}$

$$3.60 \quad \int \frac{1-x^4}{x^4(1-x^4+x^8)} dx$$

Optimal. Leaf size=370

$$-\frac{1}{3x^3} + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}$$

[Out] $-1/3/x^3 - 1/4*\arctan((-2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*(1/2*2^{(1/2)}-1/6*6^{(1/2)})+1/4*\arctan((2*x+1/2*6^{(1/2)}-1/2*2^{(1/2)})/(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*(1/2*2^{(1/2)}-1/6*6^{(1/2)})-1/8*\ln(1+x^2-x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*(1/2*2^{(1/2)}-1/6*6^{(1/2)})+1/8*\ln(1+x^2+x*(1/2*6^{(1/2)}+1/2*2^{(1/2)}))*(1/2*2^{(1/2)}-1/6*6^{(1/2)})+1/4*\arctan((-2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*(1/2*2^{(1/2)}+1/6*6^{(1/2)})-1/4*\arctan((2*x+1/2*6^{(1/2)}+1/2*2^{(1/2)})/(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*(1/2*2^{(1/2)}+1/6*6^{(1/2)})+1/8*\ln(1+x^2-x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*(1/2*2^{(1/2)}+1/6*6^{(1/2)})-1/8*\ln(1+x^2+x*(1/2*6^{(1/2)}-1/2*2^{(1/2)}))*(1/2*2^{(1/2)}+1/6*6^{(1/2)})$

Rubi [A] time = 0.27, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3} + \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 - \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2+\sqrt{3})} \log\left(x^2 + \sqrt{2-\sqrt{3}}x + 1\right) - \frac{1}{8}\sqrt{\frac{1}{3}(2-\sqrt{3})}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)), x]

[Out] $-1/(3*x^3) - (\text{Sqrt}[(2 - \text{Sqrt}[3])/3]*\text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]])/4 + (\text{Sqrt}[(2 + \text{Sqrt}[3])/3]*\text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] - 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]])/4 + (\text{Sqrt}[(2 - \text{Sqrt}[3])/3]*\text{ArcTan}[(\text{Sqrt}[2 - \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 + \text{Sqrt}[3]]])/4 - (\text{Sqrt}[(2 + \text{Sqrt}[3])/3]*\text{ArcTan}[(\text{Sqrt}[2 + \text{Sqrt}[3]] + 2*x)/\text{Sqrt}[2 - \text{Sqrt}[3]]])/4 + (\text{Sqrt}[(2 + \text{Sqrt}[3])/3]*\text{Log}[1 - \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2])/8 - (\text{Sqrt}[(2 + \text{Sqrt}[3])/3]*\text{Log}[1 + \text{Sqrt}[2 - \text{Sqrt}[3]]*x + x^2])/8 - (\text{Sqrt}[(2 - \text{Sqrt}[3])/3]*\text{Log}[1 - \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2])/8 + (\text{Sqrt}[(2 - \text{Sqrt}[3])/3]*\text{Log}[1 + \text{Sqrt}[2 + \text{Sqrt}[3]]*x + x^2])/8$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (GtQ[(2*d)/e - b/c, 0] || (!LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2], 0]))
```

Rule 1164

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_)/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n)), x_Symbol] := W
```

```

ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
- n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]

```

Rule 1504

```

Int[((f_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^(n_.))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
&& EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]

```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x^4(1-x^4+x^8)} dx &= -\frac{1}{3x^3} - \frac{1}{3} \int \frac{3x^4}{1-x^4+x^8} dx \\
&= -\frac{1}{3x^3} - \int \frac{x^4}{1-x^4+x^8} dx \\
&= -\frac{1}{3x^3} - \frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} + \frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4} dx}{2\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} - \frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4} dx}{4\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} + \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2} dx}{8\sqrt{3}} \\
&= -\frac{1}{3x^3} + \frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2-\sqrt{3})} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3}(2+\sqrt{3})} \\
&= -\frac{1}{3x^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3}(2+\sqrt{3})} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3}(2-\sqrt{3})}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 47, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8 - \#1^4 + 1\&, \frac{\#1 \log(x - \#1)}{2\#1^4 - 1} \&\right] - \frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)), x]

[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) &] /4

fricas [B] time = 1.13, size = 608, normalized size = 1.64

$$8\sqrt{6}\sqrt{2}x^3\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2} + \frac{1}{6}\sqrt{6}\sqrt{2}\sqrt{2\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2} + 12x^2 + 12}\sqrt{\sqrt{3}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")

[Out] $\frac{1}{96} \cdot (8 \sqrt{6} \sqrt{2} x^3 \sqrt{\sqrt{3} + 2} \arctan(-\frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{\sqrt{3} + 2}) + \frac{1}{6} \sqrt{6} \sqrt{2} \sqrt{2 \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{\sqrt{3} + 2}} + 12 x^2 + 12) \sqrt{\sqrt{3} + 2} - \sqrt{3} - 2 + 8 \sqrt{6} \sqrt{2} x^3 \sqrt{\sqrt{3} + 2} \arctan(-\frac{1}{3} \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{\sqrt{3} + 2}) + \frac{1}{6} \sqrt{6} \sqrt{2} \sqrt{-2 \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{\sqrt{3} + 2}} + 12 x^2 + 12) \sqrt{\sqrt{3} + 2} + \sqrt{3} + 2 - 4 \sqrt{6} \sqrt{2} x^3 \sqrt{-4 \sqrt{3} + 8} \arctan(-\frac{1}{6} \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{-4 \sqrt{3} + 8}) + \frac{1}{12} \sqrt{6} \sqrt{2} \sqrt{\sqrt{6} \sqrt{3} \sqrt{2} \sqrt{-4 \sqrt{3} + 8}} + 12 x^2 + 12) \sqrt{-4 \sqrt{3} + 8} + \sqrt{3} - 2 - 4 \sqrt{6} \sqrt{2} x^3 \sqrt{-4 \sqrt{3} + 8} \arctan(-\frac{1}{6} \sqrt{6} \sqrt{3} \sqrt{2} \sqrt{-4 \sqrt{3} + 8}) + \frac{1}{12} \sqrt{6} \sqrt{2} \sqrt{-\sqrt{6} \sqrt{3} \sqrt{2} \sqrt{-4 \sqrt{3} + 8}} + 12 x^2 + 12) \sqrt{-4 \sqrt{3} + 8} - \sqrt{3} + 2 - 2 \sqrt{6} (\sqrt{3} \sqrt{2} x^3 - 2 \sqrt{2} x^3) \sqrt{\sqrt{3} + 2} \log(2 \sqrt{6} \sqrt{3} \sqrt{2} x \sqrt{\sqrt{3} + 2} + 12 x^2 + 12) + 2 \sqrt{6} (\sqrt{3} \sqrt{2} x^3 - 2 \sqrt{2} x^3) \sqrt{\sqrt{3} + 2} \log(-2 \sqrt{6} \sqrt{3} \sqrt{2} x \sqrt{\sqrt{3} + 2} + 12 x^2 + 12) - \sqrt{6} (\sqrt{3} \sqrt{2} x^3 + 2 \sqrt{2} x^3) \sqrt{-4 \sqrt{3} + 8} \log(\sqrt{6} \sqrt{3} \sqrt{2} x \sqrt{-4 \sqrt{3} + 8} + 12 x^2 + 12) + \sqrt{6} (\sqrt{3} \sqrt{2} x^3 + 2 \sqrt{2} x^3) \sqrt{-4 \sqrt{3} + 8} \log(-\sqrt{6} \sqrt{3} \sqrt{2} x \sqrt{-4 \sqrt{3} + 8} + 12 x^2 + 12) - 32) / x^3$

giac [A] time = 0.44, size = 258, normalized size = 0.70

$$-\frac{1}{24} \left(\sqrt{6} - 3\sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} - 3\sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2} \right) \arctan \left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right) - \frac{1}{24} \left(\sqrt{6} + 3\sqrt{2} \right) \arctan \left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}} \right) - \frac{1}{48} (\sqrt{6} - 3\sqrt{2}) \log(x^2 + \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1) + \frac{1}{48} (\sqrt{6} - 3\sqrt{2}) \log(x^2 - \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1) - \frac{1}{48} (\sqrt{6} + 3\sqrt{2}) \log(x^2 + \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1) + \frac{1}{48} (\sqrt{6} + 3\sqrt{2}) \log(x^2 - \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1) - \frac{1}{3} x^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")

[Out] $-\frac{1}{24} (\sqrt{6} - 3\sqrt{2}) \arctan((4x + \sqrt{6} - \sqrt{2}) / (\sqrt{6} + \sqrt{2})) - \frac{1}{24} (\sqrt{6} - 3\sqrt{2}) \arctan((4x - \sqrt{6} + \sqrt{2}) / (\sqrt{6} + \sqrt{2})) - \frac{1}{24} (\sqrt{6} + 3\sqrt{2}) \arctan((4x + \sqrt{6} + \sqrt{2}) / (\sqrt{6} - \sqrt{2})) - \frac{1}{24} (\sqrt{6} + 3\sqrt{2}) \arctan((4x - \sqrt{6} - \sqrt{2}) / (\sqrt{6} - \sqrt{2})) - \frac{1}{48} (\sqrt{6} - 3\sqrt{2}) \log(x^2 + \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1) + \frac{1}{48} (\sqrt{6} - 3\sqrt{2}) \log(x^2 - \frac{1}{2} x (\sqrt{6} + \sqrt{2}) + 1) - \frac{1}{48} (\sqrt{6} + 3\sqrt{2}) \log(x^2 + \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1) + \frac{1}{48} (\sqrt{6} + 3\sqrt{2}) \log(x^2 - \frac{1}{2} x (\sqrt{6} - \sqrt{2}) + 1) - \frac{1}{3} x^{-3}$

maple [C] time = 0.01, size = 46, normalized size = 0.12

$$-\frac{\text{RootOf}(-Z^8 - Z^4 + 1)^4 \ln(-\text{RootOf}(-Z^8 - Z^4 + 1) + x)}{4 \left(2 \text{RootOf}(-Z^8 - Z^4 + 1)^7 - \text{RootOf}(-Z^8 - Z^4 + 1)^3 \right)} - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)/x^4/(x^8-x^4+1),x)

[Out] -1/3/x^3-1/4*sum(1/(2*_R^7-_R^3)*_R^4*ln(-_R+x),_R=RootOf(_Z^8-_Z^4+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3x^3} - \int \frac{x^4}{x^8 - x^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")

[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)

mupad [B] time = 0.07, size = 479, normalized size = 1.29

$$-\frac{1}{3x^3} + \frac{\sqrt{3} \operatorname{atan} \left(\frac{x(8-\sqrt{3}8i)^{1/4}}{2 \left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}8i}1i}{4} + \frac{\sqrt{8-\sqrt{3}8i}}{4} \right)} + \frac{\sqrt{3}x(8-\sqrt{3}8i)^{1/4}1i}{2 \left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}8i}1i}{4} + \frac{\sqrt{8-\sqrt{3}8i}}{4} \right)} \right) (8-\sqrt{3}8i)^{1/4}1i}{12} + \frac{\sqrt{3} \operatorname{atan} \left(\frac{x(8-\sqrt{3}8i)^{1/4}}{2 \left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}8i}1i}{4} + \frac{\sqrt{8-\sqrt{3}8i}}{4} \right)} \right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)

[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/2)

$$4) \cdot i) / 12 - (2^{3/4} \cdot 3^{1/2} \cdot \operatorname{atan}((2^{3/4} \cdot x \cdot (3^{1/2} \cdot i + 1)^{1/4} \cdot i) / (2 \cdot ((2^{1/2} \cdot (3^{1/2} \cdot i + 1)^{1/2}) / 2 - (2^{1/2} \cdot 3^{1/2} \cdot (3^{1/2} \cdot i + 1)^{1/2} \cdot i) / 2)) + (2^{3/4} \cdot 3^{1/2} \cdot x \cdot (3^{1/2} \cdot i + 1)^{1/4}) / (2 \cdot ((2^{1/2} \cdot (3^{1/2} \cdot i + 1)^{1/2}) / 2 - (2^{1/2} \cdot 3^{1/2} \cdot (3^{1/2} \cdot i + 1)^{1/2} \cdot i) / 2)))) \cdot (3^{1/2} \cdot i + 1)^{1/4}) / 12$$

sympy [A] time = 3.17, size = 32, normalized size = 0.09

$$-\operatorname{RootSum}(5308416t^8 - 2304t^4 + 1, (t \mapsto t \log(-18432t^5 + 4t + x))) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)

[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*_t + x))) - 1/(3*x**3)

$$3.61 \quad \int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)} dx$$

Optimal. Leaf size=280

$$\frac{x^2(ad+be)}{2a^2e^2} + \frac{(a^2c^2d - 3ab^2cd + 2abc^2e + b^4d - b^3ce) \log(ax^2 + bx + c)}{2a^4(ad^2 - e(bd - ce))} + \frac{(5a^2bc^2d - 2a^2c^3e - 5ab^3cd + 4ab^2c^2e - b^4d + b^3ce)}{a^4\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))}$$

[Out] (a^2*d^2+b^2*e^2+a*e*(b*d-c*e))*x/a^3/e^3-1/2*(a*d+b*e)*x^2/a^2/e^2+1/3*x^3/a/e-d^5*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)*ln(a*x^2+b*x+c)/a^4/(a*d^2-e*(b*d-c*e))+(5*a^2*b*c^2*d-2*a^2*c^3*e-5*a*b^3*c*d+4*a*b^2*c^2*e+b^5*d-b^4*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.60, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{(a^2c^2d - 3ab^2cd + 2abc^2e - b^3ce + b^4d) \log(ax^2 + bx + c)}{2a^4(ad^2 - e(bd - ce))} + \frac{(5a^2bc^2d - 2a^2c^3e + 4ab^2c^2e - 5ab^3cd - b^4ce + b^5d)}{a^4\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[$b^2 - 4ac$, 0] && !NiceSqrtQ[$b^2 - 4ac$]

Rule 1569

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)} dx &= \int \frac{x^5}{(d + ex)(c + bx + ax^2)} dx \\
&= \int \left(\frac{a^2 d^2 + b^2 e^2 + ae(bd - ce)}{a^3 e^3} - \frac{(ad + be)x}{a^2 e^2} + \frac{x^2}{ae} + \frac{d^5}{e^3(-ad^2 + e(bd - ce))(d + ex)} \right) dx \\
&= \frac{(a^2 d^2 + b^2 e^2 + ae(bd - ce))x}{a^3 e^3} - \frac{(ad + be)x^2}{2a^2 e^2} + \frac{x^3}{3ae} - \frac{d^5 \log(d + ex)}{e^4(ad^2 - e(bd - ce))} + \frac{\int \frac{c(b^2 d^2 - 2abd^2 + a^2 d^2)}{d^2 + ed + e^2} dx}{e^4(ad^2 - e(bd - ce))} \\
&= \frac{(a^2 d^2 + b^2 e^2 + ae(bd - ce))x}{a^3 e^3} - \frac{(ad + be)x^2}{2a^2 e^2} + \frac{x^3}{3ae} - \frac{d^5 \log(d + ex)}{e^4(ad^2 - e(bd - ce))} + \frac{(b^4 d^2 - 2abd^2 + a^2 d^2)}{a^4 \sqrt{b^2 - 4ac}} \operatorname{Arctan} \left(\frac{d + ex}{\sqrt{b^2 - 4ac}} \right)
\end{aligned}$$

Mathematica [A] time = 0.23, size = 283, normalized size = 1.01

$$-\frac{x^2(ad + be)}{2a^2 e^2} + \frac{(a^2 c^2 d - 3ab^2 cd + 2abc^2 e + b^4 d - b^3 ce) \log(ax^2 + bx + c)}{2a^4(ad^2 - bde + ce^2)} + \frac{(5a^2 bc^2 d - 2a^2 c^3 e - 5ab^3 cd + 4ab^2 c^2 d - b^4 c^2 e)}{a^4 \sqrt{4ac - b^2}(-ad^2 + e(bd - ce))}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))

fricas [A] time = 80.22, size = 1027, normalized size = 3.67

$$\left[\frac{6(a^4 b^2 - 4a^5 c)d^5 \log(ex + d) - 2((a^4 b^2 - 4a^5 c)d^2 e^3 - (a^3 b^3 - 4a^4 bc)de^4 + (a^3 b^2 c - 4a^4 c^2)e^5)x^3 + 3((a^4 b^2 - 4a^5 c)d^2 e^3 - (a^3 b^3 - 4a^4 bc)de^4 + (a^3 b^2 c - 4a^4 c^2)e^5)}{a^4 \sqrt{4ac - b^2}(-ad^2 + e(bd - ce))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*\log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2* \\ & e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((\\ & a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a \\ & ^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 \\ & - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*\sqrt{b^2 - 4*a*c}*\log((2*a^2*x^2 \\ & + 2*a*b*x + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c}*(2*a*x + b))/(a*x^2 + b*x + c)) \\ & - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4 \\ & + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13* \\ & a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*\log(a*x^2 + b*x + c) \\ & /((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d \\ & *e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*\log(e* \\ & x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^ \\ & 3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - \\ & 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 - 6*((\\ & b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^ \\ & 5)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c)) \\ & - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4 \\ & + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13* \\ & a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*\log(a*x^2 + b*x + c) \\ & /((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d \\ & *e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)] \end{aligned}$$

giac [A] time = 0.38, size = 295, normalized size = 1.05

$$-\frac{d^5 \log(|xe + d|)}{ad^2e^4 - bde^5 + ce^6} + \frac{(b^4d - 3ab^2cd + a^2c^2d - b^3ce + 2abc^2e) \log(ax^2 + bx + c)}{2(a^5d^2 - a^4bde + a^4ce^2)} - \frac{(b^5d - 5ab^3cd + 5a^2bc^2d - b^4)}{(a^5d^2 - a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")

[Out]
$$\begin{aligned} & -d^5*\log(\text{abs}(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2 \\ & *c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*\log(a*x^2 + b*x + c)/(a^5*d^2 - a \\ & ^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4* \\ & a*b^2*c^2*e - 2*a^2*c^3*e)*\arctan((2*a*x + b)/\sqrt{-b^2 + 4*a*c})/((a^5*d^2 \\ & - a^4*b*d*e + a^4*c*e^2)*\sqrt{-b^2 + 4*a*c}) + 1/6*(2*a^2*x^3*e^2 - 3*a^2* \\ & d*x^2*e + 6*a^2*d^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x \\ & *e^2)*e^{(-3)}/a^3 \end{aligned}$$

maple [B] time = 0.01, size = 662, normalized size = 2.36

$$\frac{5b^2c^2d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a^2} + \frac{2c^3e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a^2} + \frac{5b^3cd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a^3} - \frac{4b^2c}{(ad^2 - deb + ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a+c/x^2+b/x)/(e*x+d),x)`

[Out] $\frac{1}{3}x^3/a/e-1/2/a/e^2x^2d-1/2/a^2/e^2x^2b+1/a/e^3d^2x+1/a^2/e^2b*d*x-1/a^2/e*c*x+1/a^3/e*b^2*x+1/2/(a*d^2-b*d*e+c*e^2)/a^2*\ln(a*x^2+b*x+c)*c^2*d-3/2/(a*d^2-b*d*e+c*e^2)/a^3*\ln(a*x^2+b*x+c)*b^2*c*d+1/(a*d^2-b*d*e+c*e^2)/a^3*\ln(a*x^2+b*x+c)*b*c^2*e+1/2/(a*d^2-b*d*e+c*e^2)/a^4*\ln(a*x^2+b*x+c)*b^4*d-1/2/(a*d^2-b*d*e+c*e^2)/a^4*\ln(a*x^2+b*x+c)*b^3*c*e-5/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d+2/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*e+5/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d-4/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*e-1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^5*d+1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*c*e-1/e^4*d^5/(a*d^2-b*d*e+c*e^2)*\ln(e*x+d)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 6.21, size = 2490, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((d+e*x)*(a+b/x+c/x^2)),x)`

[Out] $(\log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e^6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2$

$$\begin{aligned}
& - 4*a*c)^{(1/2)} + b^5*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + 2*a^3*c^3*d^3*e^4 - 4*a^4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^{(1/2)} + a*c^4*e^7*(b^2 - 4*a*c)^{(1/2)} + 2*a^5*d^7*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^{(1/2)} + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^2*c^3*e^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^{(1/2)} - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} - 15*a*b^3*c^2*d^2*e^5 + 15*a^2*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^{(1/2)} + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 5*a^2*b^3*c*d^3*e^4*x - 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)} - 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^{(1/2)} - a*b*c^3*e^7*x*(b^2 - 4*a*c)^{(1/2)} - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^{(1/2)} - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c*d^2*e^5*x + 9*a^2*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^{(1/2)} + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)}))*(b^5*d*(b^2 - 4*a*c)^{(1/2)} - b^6*d + 4*a^3*c^3*d + b^5*c*e - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d - b^4*c*e*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^3*c^2*e + 8*a^2*b*c^3*e - 2*a^2*c^3*e*(b^2 - 4*a*c)^{(1/2)} + 5*a^2*b*c^2*d*(b^2 - 4*a*c)^{(1/2)} + 4*a*b^2*c^2*e*(b^2 - 4*a*c)^{(1/2)} - 5*a*b^3*c*d*(b^2 - 4*a*c)^{(1/2)}))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 + a^4*b^3*d*e - 4*a^5*b*c*d*e)) - (d^5*log(d + e*x))/(c*e^6 + a*d^2*e^4 - b*d*e^5) - x*((b*d + c*e)/(a^2*e^2) - (a*d + b*e)^2/(a^3*e^3)) + (log(a^4*b^2*d^7 - 4*a^5*c*d^7 - b^3*c^3*e^7 + b^6*d^3*e^4 + 6*a^2*c^4*d*e^6 + 3*b^4*c^2*d*e^6 - 3*b^5*c*d^2*e^5 + 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2 - 4*a*c)^{(1/2)} + b^5*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} - 2*a^3*c^3*d^3*e^4 + 4*a^4*c^2*d^5*e^2 + 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^{(1/2)} + a*c^4*e^7*(b^2 - 4*a*c)^{(1/2)} + 2*a^5*d^7*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} - 8*a^5*c*d^6*e*x + 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^{(1/2)} - 12*a*b^2*c^3*d*e^6 - 6*a*b^4*c*d^3*e^4 - a*b^2*c^3*e^7*x + a*b^5*d^3*e^4*x + 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^{(1/2)} - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} + 15*a*b^3*c^2*d^2*e^5 - 15*a^2*b*c^3*d^2*e^5 - a^3*b^2*c*d^5*e^2 - a^3*b^3*d^5*e^2*x - 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^{(1/2)} + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} - 5*a^2*b^3*c*d^3*e^4*x + 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^{(1/2)} - a*b*c^3*e^7*x*(b^2 - 4*a*c)^{(1/2)} - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^{(1/2)} + 3*a*b^3*c^2*d*e^6*x - 3*a*b^4*c*d^2*e^5*x - 9*a^2*b*c^3*d*e^6*x + 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& 2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*b^2*c*d^3*e^4*x*(b^2 \\
& - 4*a*c)^{(1/2)}*(4*a^3*c^3*d - b^5*d*(b^2 - 4*a*c)^{(1/2)} - b^6*d + b^5*c*e \\
& - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d + b^4*c*e*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^3*c \\
& ^2*e + 8*a^2*b*c^3*e + 2*a^2*c^3*e*(b^2 - 4*a*c)^{(1/2)} - 5*a^2*b*c^2*d*(b^2 - 4* \\
& a*c)^{(1/2)} - 4*a*b^2*c^2*e*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^3*c*d*(b^2 - 4* \\
& a*c)^{(1/2)))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 \\
& + a^4*b^3*d*e - 4*a^5*b*c*d*e)) + x^3/(3*a*e) - (x^2*(a*d + b*e))/(2*a^2*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)

[Out] Timed out

$$3.62 \quad \int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)} dx$$

Optimal. Leaf size=218

$$\frac{(-2abcd + ac^2e + b^3d - b^2ce) \log(ax^2 + bx + c)}{2a^3(ad^2 - e(bd - ce))} - \frac{x(ad + be)}{a^2e^2} - \frac{(2a^2c^2d - 4ab^2cd + 3abc^2e + b^4d - b^3ce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}(ad^2 - e(bd - ce))}$$

[Out] $-(a*d+b*e)*x/a^2/e^2+1/2*x^2/a/e+d^4*\ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))-1/2*(-2*a*b*c*d+a*c^2*e+b^3*d-b^2*c*e)*\ln(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))-(2*a^2*c^2*d-4*a*b^2*c*d+3*a*b*c^2*e+b^4*d-b^3*c*e)*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/a^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{(-2abcd + ac^2e - b^2ce + b^3d) \log(ax^2 + bx + c)}{2a^3(ad^2 - e(bd - ce))} - \frac{(2a^2c^2d - 4ab^2cd + 3abc^2e - b^3ce + b^4d) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} x$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] $-(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(a^3*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*\operatorname{Log}[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*\operatorname{Log}[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)} dx &= \int \frac{x^4}{(d + ex)(c + bx + ax^2)} dx \\
&= \int \left(\frac{-ad - be}{a^2e^2} + \frac{x}{ae} + \frac{d^4}{e^2(ad^2 - e(bd - ce))(d + ex)} + \frac{-c(b^2d - acd - bce) - (b^3d - b^2cd - b^2ce + ac^2e)}{a^2(ad^2 - e(bd - ce))} \right) dx \\
&= -\frac{(ad + be)x}{a^2e^2} + \frac{x^2}{2ae} + \frac{d^4 \log(d + ex)}{e^3(ad^2 - e(bd - ce))} + \frac{\int \frac{-c(b^2d - acd - bce) - (b^3d - 2abcd - b^2ce + ac^2e)x}{c + bx + ax^2} dx}{a^2(ad^2 - e(bd - ce))} \\
&= -\frac{(ad + be)x}{a^2e^2} + \frac{x^2}{2ae} + \frac{d^4 \log(d + ex)}{e^3(ad^2 - e(bd - ce))} - \frac{(b^3d - 2abcd - b^2ce + ac^2e) \int \frac{b + 2ax}{c + bx + ax^2} dx}{2a^3(ad^2 - e(bd - ce))} \\
&= -\frac{(ad + be)x}{a^2e^2} + \frac{x^2}{2ae} + \frac{d^4 \log(d + ex)}{e^3(ad^2 - e(bd - ce))} - \frac{(b^3d - 2abcd - b^2ce + ac^2e) \log(c + bx + ax^2)}{2a^3(ad^2 - e(bd - ce))} \\
&= -\frac{(ad + be)x}{a^2e^2} + \frac{x^2}{2ae} - \frac{(b^4d - 4ab^2cd + 2a^2c^2d - b^3ce + 3abc^2e) \tanh^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right)}{a^3\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))} + \dots
\end{aligned}$$

Mathematica [A] time = 0.17, size = 218, normalized size = 1.00

$$\frac{(2abcd - ac^2e + b^3(-d) + b^2ce) \log(x(ax + b) + c)}{2a^3(ad^2 + e(ce - bd))} - \frac{x(ad + be)}{a^2e^2} + \frac{(2a^2c^2d - 4ab^2cd + 3abc^2e + b^4d - b^3ce) \tan^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right)}{a^3\sqrt{4ac - b^2}(ad^2 + e(ce - bd))}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))

fricas [A] time = 35.52, size = 798, normalized size = 3.66

$$\left[\frac{2(a^3b^2 - 4a^4c)d^4 \log(ex + d) + ((a^3b^2 - 4a^4c)d^2e^2 - (a^2b^3 - 4a^3bc)de^3 + (a^2b^2c - 4a^3c^2)e^4)x^2 + ((b^4 - 4ab^2c) \tanh^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right) - (b^3d - 2abcd - b^2ce + ac^2e) \log(c + bx + ax^2))}{a^3\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*\log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^2 \\ & - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4 - \\ & 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*\sqrt{b^2 - 4*a*c}* \\ & \log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c}*(2*a*x + b))/(a* \\ & x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a \\ & ^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2* \\ & b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*\log(a*x^2 + b*x + c)) \\ & /((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - \\ & 4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*\log(e*x + d) + ((a^3*b^2 - \\ & 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4) \\ & *x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)* \\ & \sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c)) - \\ & 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*e^3 + (a \\ & *b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d*e^3 - (b^4 \\ & *c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*\log(a*x^2 + b*x + c))/((a^4*b^2 - 4*a^5 \\ & *c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)] \end{aligned}$$

giac [A] time = 0.37, size = 224, normalized size = 1.03

$$\frac{d^4 \log(|xe + d|)}{ad^2e^3 - bde^4 + ce^5} - \frac{(b^3d - 2abcd - b^2ce + ac^2e) \log(ax^2 + bx + c)}{2(a^4d^2 - a^3bde + a^3ce^2)} + \frac{(b^4d - 4ab^2cd + 2a^2c^2d - b^3ce + 3abc^2e)}{(a^4d^2 - a^3bde + a^3ce^2)\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")

[Out]
$$\begin{aligned} & d^4*\log(\text{abs}(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c* \\ & d - b^2*c*e + a*c^2*e)*\log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^2) \\ & + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*\arctan((2* \\ & a*x + b)/\sqrt{-b^2 + 4*a*c})/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*\sqrt{-b^2 + \\ & 4*a*c}) + 1/2*(a*x^2*e - 2*a*d*x - 2*b*x*e)*e^{(-2)}/a^2 \end{aligned}$$

maple [B] time = 0.01, size = 512, normalized size = 2.35

$$\frac{2c^2d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2d^2 - deb + ce^2)\sqrt{4ac-b^2}} - \frac{4b^2cd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2d^2 - deb + ce^2)\sqrt{4ac-b^2}} + \frac{3bc^2e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a^2d^2 - deb + ce^2)\sqrt{4ac-b^2}} + \frac{b^4d}{(a^2d^2 - deb + ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x)

```
[Out] 1/2*x^2/a/e-1/a/e^2*x*d-1/a^2/e*b*x+1/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*d-1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*e-1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^3*d+1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^2*c*e+2/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d-4/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+3/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^2*e+1/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d-1/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*e+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.24, size = 2051, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*c^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^4*c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*c^2*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) + 2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^2*c^3*d*e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d^4*e^2 + a^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*
```

$$\begin{aligned}
& d^5 e^x (b^2 - 4ac)^{1/2} - 3ab^2 c^2 d e^5 x + 3ab^3 c d^2 e^4 x - 4 \\
& a^3 b c d^4 e^2 x + 3ab^2 c^2 d e^5 x (b^2 - 4ac)^{1/2} - 3ab^2 c d^2 e^4 x \\
& e^4 x (b^2 - 4ac)^{1/2} - 2a^2 b c d^3 e^3 x (b^2 - 4ac)^{1/2}) (b^4 d \\
& (b^2 - 4ac)^{1/2} - b^5 d + 4a^2 c^3 e + b^4 c e + 6ab^3 c d - b^3 c e \\
& e (b^2 - 4ac)^{1/2} - 8a^2 b c^2 d - 5ab^2 c^2 e + 2a^2 c^2 d (b^2 - \\
& 4ac)^{1/2} - 4ab^2 c d (b^2 - 4ac)^{1/2} + 3ab^2 c^2 e (b^2 - 4ac)^{1/2})) \\
& (2(4a^5 c d^2 - a^4 b^2 d^2 + 4a^4 c^2 e^2 - a^3 b^2 c e^2 + a^3 \\
& b^3 d e - 4a^4 b c d e)) + (\log(2a^4 c^4 e^6 - 4a^4 c d^6 + a^3 b^2 d^6 - \\
& b^2 c^3 e^6 + b^5 d^3 e^3 + 3b^3 c^2 d e^5 - 3b^4 c d^2 e^4 + b^4 d^3 e^3 \\
& 3(b^2 - 4ac)^{1/2} - 6a^2 c^3 d^2 e^4 + 4a^3 c^2 d^4 e^2 + a^3 b d^6 (\\
& b^2 - 4ac)^{1/2} - b c^3 e^6 (b^2 - 4ac)^{1/2} + 2a^4 d^6 x (b^2 - 4a \\
& c)^{1/2} - 9ab^2 c^3 d e^5 + a^2 c^2 d^3 e^3 (b^2 - 4ac)^{1/2} - a b c^3 \\
& e^6 x - 8a^4 c d^5 e^5 x - 3a^2 c^3 d e^5 (b^2 - 4ac)^{1/2} - 4a^3 c d^5 e^5 \\
& e (b^2 - 4ac)^{1/2} - a c^3 e^6 x (b^2 - 4ac)^{1/2} - 5ab^3 c d^3 e^3 \\
& + a b^4 d^3 e^3 x + 2a^3 b^2 d^5 e^5 x - 6a^2 c^3 d e^5 x + 3b^2 c^2 d e^5 \\
& 5(b^2 - 4ac)^{1/2} - 3b^3 c d^2 e^4 (b^2 - 4ac)^{1/2} + 12ab^2 c^2 d^2 \\
& e^4 + 5a^2 b c^2 d^3 e^3 - a^2 b^2 c d^4 e^2 - a^2 b^3 d^4 e^2 x + 2a^3 \\
& c^2 d^3 e^3 x + 6ab^2 c^2 d^2 e^4 (b^2 - 4ac)^{1/2} - 3ab^2 c d^3 e^3 \\
& 3(b^2 - 4ac)^{1/2} + a^2 b c d^4 e^2 (b^2 - 4ac)^{1/2} + a b^3 d^3 e^3 \\
& x (b^2 - 4ac)^{1/2} - 2a^3 c d^4 e^2 x (b^2 - 4ac)^{1/2} + 9a^2 b c^2 \\
& d^2 e^4 x - 4a^2 b^2 c d^3 e^3 x + a^2 b^2 d^4 e^2 x (b^2 - 4ac)^{1/2} \\
& + 3a^2 c^2 d^2 e^4 x (b^2 - 4ac)^{1/2} - 2a^3 b d^5 e^5 x (b^2 - 4ac)^{1/2} \\
& (1/2) + 3ab^2 c^2 d e^5 x - 3ab^3 c d^2 e^4 x + 4a^3 b c d^4 e^2 x + 3 \\
& a b c^2 d e^5 x (b^2 - 4ac)^{1/2} - 3ab^2 c d^2 e^4 x (b^2 - 4ac)^{1/2} \\
& (1/2) - 2a^2 b c d^3 e^3 x (b^2 - 4ac)^{1/2}) (b^5 d + b^4 d (b^2 - 4ac) \\
& ^{1/2} - 4a^2 c^3 e - b^4 c e - 6ab^3 c d - b^3 c e (b^2 - 4ac)^{1/2} \\
& + 8a^2 b c^2 d + 5ab^2 c^2 e + 2a^2 c^2 d (b^2 - 4ac)^{1/2} - 4ab^2 \\
& c d (b^2 - 4ac)^{1/2} + 3ab^2 c^2 e (b^2 - 4ac)^{1/2})) (2(4a^5 c d^2 \\
& - a^4 b^2 d^2 + 4a^4 c^2 e^2 - a^3 b^2 c e^2 + a^3 b^3 d e - 4a^4 b c d \\
& e)) + x^2 / (2ae) - (x(ad + be)) / (a^2 e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)

[Out] Timed out

$$3.63 \quad \int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)} dx$$

Optimal. Leaf size=176

$$\frac{(-acd + b^2d - bce) \log(ax^2 + bx + c)}{2a^2(ad^2 - e(bd - ce))} + \frac{(-3abcd + 2ac^2e + b^3d - b^2ce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} - \frac{d^3 \log(d+ex)}{e^2(ad^2 - e(bd - ce))} + \dots$$

[Out] x/a/e-d^3*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))+1/2*(-a*c*d+b^2*d-b*c*e)*ln(a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))+(-3*a*b*c*d+2*a*c^2*e+b^3*d-b^2*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.29, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{(-3abcd + 2ac^2e - b^2ce + b^3d) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} + \frac{(-acd + b^2d - bce) \log(ax^2 + bx + c)}{2a^2(ad^2 - e(bd - ce))} - \frac{d^3 \log(d+ex)}{e^2(ad^2 - e(bd - ce))} + \dots$$

Antiderivative was successfully verified.

[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)} dx &= \int \frac{x^3}{(d + ex)(c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{ae} + \frac{d^3}{e(-ad^2 + e(bd - ce))(d + ex)} + \frac{c(bd - ce) + (b^2d - acd - bce)x}{a(ad^2 - e(bd - ce))(c + bx + ax^2)} \right) dx \\
&= \frac{x}{ae} - \frac{d^3 \log(d + ex)}{e^2(ad^2 - e(bd - ce))} + \frac{\int \frac{c(bd - ce) + (b^2d - acd - bce)x}{c + bx + ax^2} dx}{a(ad^2 - bde + ce^2)} \\
&= \frac{x}{ae} - \frac{d^3 \log(d + ex)}{e^2(ad^2 - e(bd - ce))} + \frac{(b^2d - acd - bce) \int \frac{b + 2ax}{c + bx + ax^2} dx}{2a^2(ad^2 - e(bd - ce))} - \frac{(b^3d - 3abcd - b^2ce)}{2a^2(ad^2 - e(bd - ce))} \\
&= \frac{x}{ae} - \frac{d^3 \log(d + ex)}{e^2(ad^2 - e(bd - ce))} + \frac{(b^2d - acd - bce) \log(c + bx + ax^2)}{2a^2(ad^2 - e(bd - ce))} + \frac{(b^3d - 3abcd - b^2ce)}{2a^2(ad^2 - e(bd - ce))} \\
&= \frac{x}{ae} + \frac{(b^3d - 3abcd - b^2ce + 2ac^2e) \tanh^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right)}{a^2\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))} - \frac{d^3 \log(d + ex)}{e^2(ad^2 - e(bd - ce))} + \frac{(b^2ce)}{2a^2(ad^2 - e(bd - ce))}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 178, normalized size = 1.01

$$\frac{(-acd + b^2d - bce) \log(ax^2 + bx + c)}{2a^2(ad^2 - bde + ce^2)} + \frac{(-3abcd + 2ac^2e + b^3d - b^2ce) \tan^{-1}\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right)}{a^2\sqrt{4ac - b^2}(-ad^2 + bde - ce^2)} - \frac{d^3 \log(d + ex)}{e^2(ad^2 - bde + ce^2)} + \frac{x}{ae}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-a*d^2) + b*d*e - c*e^2) - (d^3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))

fricas [A] time = 10.87, size = 596, normalized size = 3.39

$$\left[\frac{2(a^2b^2 - 4a^3c)d^3 \log(ex + d) - ((b^3 - 3abc)de^2 - (b^2c - 2ac^2)e^3)\sqrt{b^2 - 4ac} \log\left(\frac{2a^2x^2 + 2abx + b^2 - 2ac + \sqrt{b^2 - 4ac}}{ax^2 + bx + c}\right)}{2((a^3b^2 - 4a^4c)d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*\log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*\sqrt{b^2 - 4*a*c}*\log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c}*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*\log(a*x^2 + b*x + c)] / ((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4), \\ & -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*\log(e*x + d) - 2*((b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*\log(a*x^2 + b*x + c)] / ((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)] \end{aligned}$$

giac [A] time = 0.40, size = 185, normalized size = 1.05

$$\frac{d^3 \log(|xe + d|)}{ad^2e^2 - bde^3 + ce^4} + \frac{xe^{(-1)}}{a} + \frac{(b^2d - acd - bce) \log(ax^2 + bx + c)}{2(a^3d^2 - a^2bde + a^2ce^2)} - \frac{(b^3d - 3abcd - b^2ce + 2ac^2e) \arctan\left(\frac{2ax}{\sqrt{-b^2}}\right)}{(a^3d^2 - a^2bde + a^2ce^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")

[Out]
$$\begin{aligned} & -d^3*\log(\text{abs}(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^{(-1)}/a + 1/2*(b^2*d - a*c*d - b*c*e)*\log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2) \\ & - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*\arctan((2*a*x + b)/\sqrt{-b^2 + 4*a*c}) / ((a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*\sqrt{-b^2 + 4*a*c}) \end{aligned}$$

maple [B] time = 0.01, size = 388, normalized size = 2.20

$$\frac{3bcd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a} - \frac{2c^2e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a} - \frac{b^3d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a^2} + \frac{b^2ce \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+c/x^2+b/x)/(e*x+d),x)

[Out]
$$\begin{aligned} & x/a/e - 1/2/(a*d^2 - b*d*e + c*e^2)/a*\ln(a*x^2 + b*x + c)*c*d + 1/2/(a*d^2 - b*d*e + c*e^2)/a^2*\ln(a*x^2 + b*x + c)*b^2*d - 1/2/(a*d^2 - b*d*e + c*e^2)/a^2*\ln(a*x^2 + b*x + c)*b*c*e \\ & + 3/(a*d^2 - b*d*e + c*e^2)/a/(4*a*c - b^2)^{(1/2)}*\arctan((2*a*x + b)/(4*a*c - b^2)^{(1/2)})*b*c*d - 2/(a*d^2 - b*d*e + c*e^2)/a/(4*a*c - b^2)^{(1/2)}*\arctan((2*a*x + b)/(4*a*c - b^2)^{(1/2)})*e*c^2 - 1/(a*d^2 - b*d*e + c*e^2)/a^2/(4*a*c - b^2)^{(1/2)}*\arctan((2*a*x + b)/(4*a*c - b^2)^{(1/2)})*e*c^2 \end{aligned}$$

```
*x+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*
arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*l
n(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 4.34, size = 1367, normalized size = 7.77

$$\frac{x \ln\left(c^3 e^5 \sqrt{b^2 - 4ac} - bc^3 e^5 - 4a^3 c d^5 + a^2 b^2 d^5 + b^4 d^3 e^2 + 3b^2 c^2 d e^4 - 3b^3 c d^2 e^3 - b^3 d^3 e^2 \sqrt{b^2 - 4ac}\right)}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2*
b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a^2*
b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*
e*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2)
+ 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2*d^2*
e^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^
2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 2*a^
2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a
*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2)
))*b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d
+ 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2 - 4*a*c)^(1/2)
) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c
^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (log(a^2*b^2*d^5 -
b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^4*d^3*e^2 + 3*b^
2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2
*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2 - 4*a*c)^(1/2) +
2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^2*c*d^4*e*(b^2 - 4*
```


$$\begin{aligned}
& a*c)^{(1/2)} + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^{(1/2)} + 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x + 3*a*c^2*d^2*e^3*(b^2 - 4*a*c)^{(1/2)} - 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^{(1/2)} + 6*a^2*c^2*d^2*e^3*x + 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*c*d^3*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c^2*d*e^4*x - a*b*c*d^3*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^{(1/2)} + 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^{(1/2)} - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x - 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^{(1/2))}*(b^4*d + b^3*d*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d + 4*a*b*c^2*e + 2*a*c^2*e*(b^2 - 4*a*c)^{(1/2)} - b^2*c*e*(b^2 - 4*a*c)^{(1/2)} - 3*a*b*c*d*(b^2 - 4*a*c)^{(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (d^3*log(d + e*x))/(c*e^4 + a*d^2*e^2 - b*d*e^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)

[Out] Timed out

$$3.64 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)} dx$$

Optimal. Leaf size=149

$$-\frac{(-2acd + b^2d - bce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} + \frac{d^2 \log(d+ex)}{e(ad^2 - bde + ce^2)} - \frac{(bd - ce) \log(ax^2 + bx + c)}{2a(ad^2 - e(bd - ce))}$$

[Out] $d^2 \ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2*(b*d-c*e)*\ln(a*x^2+b*x+c)/a/(a*d^2-e*(b*d-c*e))-(-2*a*c*d+b^2*d-b*c*e)*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/a/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{(-2acd + b^2d - bce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} + \frac{d^2 \log(d+ex)}{e(ad^2 - bde + ce^2)} - \frac{(bd - ce) \log(ax^2 + bx + c)}{2a(ad^2 - e(bd - ce))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] $-(((b^2*d - 2*a*c*d - b*c*e)*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(a*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*\operatorname{Log}[d + e*x])/(e*(a*d^2 - b*d*e + c*e^2)) - ((b*d - c*e)*\operatorname{Log}[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e)))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1445

Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^p_.)*((d_.) + (e_.)*(x_)^(n_.))^q_.), x_Symbol] := Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)} dx &= \int \frac{x^2}{(d + ex)(c + bx + ax^2)} dx \\
&= \int \left(\frac{d^2}{(ad^2 - e(bd - ce))(d + ex)} + \frac{-cd - (bd - ce)x}{(ad^2 - e(bd - ce))(c + bx + ax^2)} \right) dx \\
&= \frac{d^2 \log(d + ex)}{e(ad^2 - bde + ce^2)} + \frac{\int \frac{-cd - (bd - ce)x}{c + bx + ax^2} dx}{ad^2 - e(bd - ce)} \\
&= \frac{d^2 \log(d + ex)}{e(ad^2 - bde + ce^2)} - \frac{(bd - ce) \int \frac{b + 2ax}{c + bx + ax^2} dx}{2a(ad^2 - e(bd - ce))} + \frac{(b^2d - 2acd - bce) \int \frac{1}{c + bx + ax^2} dx}{2a(ad^2 - e(bd - ce))} \\
&= \frac{d^2 \log(d + ex)}{e(ad^2 - bde + ce^2)} - \frac{(bd - ce) \log(c + bx + ax^2)}{2a(ad^2 - e(bd - ce))} - \frac{(b^2d - 2acd - bce) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - b^2} dx\right)}{a(ad^2 - e(bd - ce))} \\
&= -\frac{(b^2d - 2acd - bce) \tanh^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right)}{a\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))} + \frac{d^2 \log(d + ex)}{e(ad^2 - bde + ce^2)} - \frac{(bd - ce) \log(c + bx + ax^2)}{2a(ad^2 - e(bd - ce))}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 132, normalized size = 0.89

$$\frac{\sqrt{4ac - b^2} \left(e(bd - ce) \log(x(ax + b) + c) - 2ad^2 \log(d + ex) \right) + 2e \left(2acd + b^2(-d) + bce \right) \tan^{-1} \left(\frac{2ax + b}{\sqrt{4ac - b^2}} \right)}{2ae\sqrt{4ac - b^2} (ad^2 + e(ce - bd))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]

[Out] -1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))

fricas [A] time = 3.57, size = 405, normalized size = 2.72

$$\left[\frac{2(ab^2 - 4a^2c)d^2 \log(ex + d) + (bce^2 - (b^2 - 2ac)de)\sqrt{b^2 - 4ac} \log\left(\frac{2a^2x^2 + 2abx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2ax + b)}{ax^2 + bx + c}\right) - ((b^3 - 4a^2b^2 - 4a^3c)d^2e - (ab^3 - 4a^2bc)de^2 + (ab^2c - 4a^2c^2)e^3)}{2((a^2b^2 - 4a^3c)d^2e - (ab^3 - 4a^2bc)de^2 + (ab^2c - 4a^2c^2)e^3)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")

[Out] $\left[\frac{1}{2} \left(2 \left(a b^2 - 4 a^2 c \right) d^2 \log(e x + d) + \left(b^2 c e^2 - \left(b^2 - 2 a c \right) d e \right) \sqrt{b^2 - 4 a c} \log\left(\frac{2 a^2 x^2 + 2 a b x + b^2 - 2 a c + \sqrt{b^2 - 4 a c}}{a x^2 + b x + c}\right) - \left(b^3 - 4 a b c \right) d e - \left(b^2 c - 4 a c^2 \right) e^2 \right) \log(a x^2 + b x + c) \right] / \left(\left(a^2 b^2 - 4 a^3 c \right) d^2 e - \left(a b^3 - 4 a^2 b c \right) d e^2 + \left(a b^2 c - 4 a^2 c^2 \right) e^3 \right), \frac{1}{2} \left(2 \left(a b^2 - 4 a^2 c \right) d^2 \log(e x + d) + 2 \left(b^2 c e^2 - \left(b^2 - 2 a c \right) d e \right) \sqrt{-b^2 + 4 a c} \arctan\left(\frac{-\sqrt{-b^2 + 4 a c} \left(2 a x + b \right)}{b^2 - 4 a c}\right) - \left(b^3 - 4 a b c \right) d e - \left(b^2 c - 4 a c^2 \right) e^2 \right) \log(a x^2 + b x + c) \right] / \left(\left(a^2 b^2 - 4 a^3 c \right) d^2 e - \left(a b^3 - 4 a^2 b c \right) d e^2 + \left(a b^2 c - 4 a^2 c^2 \right) e^3 \right) \right]$

giac [A] time = 0.37, size = 149, normalized size = 1.00

$$\frac{d^2 \log(|x e + d|)}{a d^2 e - b d e^2 + c e^3} - \frac{(b d - c e) \log(a x^2 + b x + c)}{2(a^2 d^2 - a b d e + a c e^2)} + \frac{(b^2 d - 2 a c d - b c e) \arctan\left(\frac{2 a x + b}{\sqrt{-b^2 + 4 a c}}\right)}{(a^2 d^2 - a b d e + a c e^2) \sqrt{-b^2 + 4 a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")

[Out] $d^2 \log(\text{abs}(x e + d)) / (a d^2 e - b d e^2 + c e^3) - \frac{1}{2} (b d - c e) \log(a x^2 + b x + c) / (a^2 d^2 - a b d e + a c e^2) + (b^2 d - 2 a c d - b c e) \arctan\left(\frac{2 a x + b}{\sqrt{-b^2 + 4 a c}}\right) / \left((a^2 d^2 - a b d e + a c e^2) \sqrt{-b^2 + 4 a c} \right)$

maple [A] time = 0.01, size = 275, normalized size = 1.85

$$\frac{b^2 d \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2) \sqrt{4 a c - b^2} a} - \frac{b c e \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2) \sqrt{4 a c - b^2} a} - \frac{2 c d \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2) \sqrt{4 a c - b^2} a} - \frac{b d \ln(a x^2 + b x + c)}{2(a d^2 - d e b + c e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/(e*x+d),x)

[Out] $-1/2 / (a d^2 - b d e + c e^2) / a \ln(a x^2 + b x + c) * b d + 1/2 / (a d^2 - b d e + c e^2) / a \ln(a x^2 + b x + c) * c e - 2 / (a d^2 - b d e + c e^2) / (4 a c - b^2)^{(1/2)} * \arctan\left(\frac{2 a x + b}{(4 a c - b^2)^{(1/2)}}\right) * c d + 1 / (a d^2 - b d e + c e^2) / (4 a c - b^2)^{(1/2)} * \arctan\left(\frac{2 a x + b}{(4 a c - b^2)^{(1/2)}}\right) / a b^2 d - 1 / (a d^2 - b d e + c e^2) / (4 a c - b^2)^{(1/2)} * \arctan\left(\frac{2 a x + b}{(4 a c - b^2)^{(1/2)}}\right) / a b c e + d^2 \ln(e x + d) / e / (a d^2 - b d e + c e^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 3.67, size = 966, normalized size = 6.48

$$\frac{d^2 \ln(d+ex)}{ad^2e - bde^2 + ce^3} \ln\left(\frac{ab^2d^4 - 2c^3e^4 - 4a^2cd^4 + b^3d^3e + c^2e^4x\sqrt{b^2 - 4ac} + 10ac^2d^2e^2 - 4b^2cd^2e^2 - b^3}{ad^2e - bde^2 + ce^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)*(a + b/x + c/x^2)),x)

[Out] $(d^2 \log(d + ex)) / (c e^3 + a d^2 e - b d e^2) - (\log(a b^2 d^4 - 2 c^3 e^4 - 4 a^2 c d^4 + b^3 d^3 e + c^2 e^4 x \sqrt{b^2 - 4 a c} + 10 a c^2 d^2 e^2 - 4 b^2 c d^2 e^2 - b^3) - 4 a^2 c d^4 + b^3 d^3 e + c^2 e^4 x (b^2 - 4 a c)^{1/2} + 10 a c^2 d^2 e^2 - 4 b^2 c d^2 e^2 - b^3) / (c e^3 + a d^2 e - b d e^2) + 10 a c^2 d^2 e^2 - 4 b^2 c d^2 e^2 - b^3) / (c e^3 + a d^2 e - b d e^2) + 3 b^3 c^2 d^2 e^3 - b^3 c^2 e^4 x + b^2 d^3 e (b^2 - 4 a c)^{1/2} + 3 c^2 d e^3 (b^2 - 4 a c)^{1/2} + 2 a^2 d^4 x (b^2 - 4 a c)^{1/2} + 3 a b^2 d^3 e x + 6 a c^2 d e^3 x - 10 a^2 c d^3 e x - 2 b^3 c d^2 e^2 (b^2 - 4 a c)^{1/2} - 3 a b^3 c d^3 e + b^2 d^2 e^2 x (b^2 - 4 a c)^{1/2} - 5 a^3 c d^3 e (b^2 - 4 a c)^{1/2} - a b^3 d^3 e x (b^2 - 4 a c)^{1/2} + a b^3 c d^2 e^2 x - 5 a^3 c d^2 e^2 x (b^2 - 4 a c)^{1/2}) * (e((b^2 c)/2 - 2 a c^2 + (b c (b^2 - 4 a c)^{1/2}))/2) - (b^3 d)/2 - (b^2 d (b^2 - 4 a c)^{1/2})/2 + a c d (b^2 - 4 a c)^{1/2} + 2 a b c d) / (4 a^3 c d^2 - a^2 b^2 d^2 + 4 a^2 c^2 e^2 + a b^3 d e - a b^2 c e^2 - 4 a^2 b c d e) + (\log(2 c^3 e^4 - a b^2 d^4 + 4 a^2 c d^4 - b^3 d^3 e + c^2 e^4 x (b^2 - 4 a c)^{1/2} - 10 a c^2 d^2 e^2 + 4 b^2 c d^2 e^2 + b^3 d^2 e^2 x + a b d^4 (b^2 - 4 a c)^{1/2} - 3 b^3 c^2 d e^3 + b c^2 e^4 x + b^2 d^3 e (b^2 - 4 a c)^{1/2} + 3 c^2 d e^3 (b^2 - 4 a c)^{1/2} + 2 a^2 d^4 x (b^2 - 4 a c)^{1/2} - 3 a b^2 d^3 e x - 6 a c^2 d e^3 x + 10 a^2 c d^3 e x - 2 b^3 c d^2 e^2 (b^2 - 4 a c)^{1/2} + 3 a b^3 c d^3 e + b^2 d^2 e^2 x (b^2 - 4 a c)^{1/2} - 5 a^3 c d^3 e (b^2 - 4 a c)^{1/2} - a b^3 d^3 e x (b^2 - 4 a c)^{1/2} - a b^3 c d^2 e^2 x - 5 a^3 c d^2 e^2 x (b^2 - 4 a c)^{1/2}) * ((b^3 d)/2 + e(2 a c^2 - (b^2 c)/2 + (b c (b^2 - 4 a c)^{1/2}))/2) - (b^2 d (b^2 - 4 a c)^{1/2})/2 + a c d (b^2 - 4 a c)^{1/2} - 2 a b c d) / (4 a^3 c d^2 - a^2 b^2 d^2 + 4 a^2 c^2 e^2 + a b^3 d e - a b^2 c e^2 - 4 a^2 b c d e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

$$3.65 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x(d+ex)} dx$$

Optimal. Leaf size=124

$$\frac{(bd - 2ce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{d \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))} - \frac{d \log(d + ex)}{ad^2 - e(bd - ce)}$$

[Out] $-d*\ln(e*x+d)/(a*d^2-e*(b*d-c*e))+1/2*d*\ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))+$
 $(b*d-2*c*e)*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/(a*d^2-e*(b*d-c*e))/(-4*a$
 $*c+b^2)^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{(bd - 2ce) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{d \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))} - \frac{d \log(d + ex)}{ad^2 - e(bd - ce)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]

[Out] $((b*d - 2*c*e)*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(\operatorname{Sqrt}[b^2 - 4*a*c]*($
 $a*d^2 - e*(b*d - c*e))) - (d*\operatorname{Log}[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*\operatorname{Log}$
 $[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 800

Int[(((d_.) + (e_.)*(x_)^m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 1569

Int[(x_)^m_*((a_.) + (b_.)*(x_)^mn_) + (c_.)*(x_)^mn2_)^p_)*((d_) + (e_.)*(x_)^n_)^q_, x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x(d+ex)} dx &= \int \frac{x}{(d+ex)(c+bx+ax^2)} dx \\
&= \int \left(\frac{de}{(-ad^2 + e(bd - ce))(d+ex)} + \frac{ce+adx}{(ad^2 - e(bd - ce))(c+bx+ax^2)} \right) dx \\
&= -\frac{d \log(d+ex)}{ad^2 - bde + ce^2} + \frac{\int \frac{ce+adx}{c+bx+ax^2} dx}{ad^2 - e(bd - ce)} \\
&= -\frac{d \log(d+ex)}{ad^2 - bde + ce^2} + \frac{d \int \frac{b+2ax}{c+bx+ax^2} dx}{2(ad^2 - bde + ce^2)} + \frac{(-bd + 2ce) \int \frac{1}{c+bx+ax^2} dx}{2(ad^2 - e(bd - ce))} \\
&= -\frac{d \log(d+ex)}{ad^2 - bde + ce^2} + \frac{d \log(c+bx+ax^2)}{2(ad^2 - bde + ce^2)} + \frac{(bd - 2ce) \text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b+ax\right)}{ad^2 - e(bd - ce)} \\
&= \frac{(bd - 2ce) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} - \frac{d \log(d+ex)}{ad^2 - bde + ce^2} + \frac{d \log(c+bx+ax^2)}{2(ad^2 - bde + ce^2)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.86

$$\frac{d\sqrt{4ac - b^2} (2 \log(d + ex) - \log(x(ax + b) + c)) + 2(bd - 2ce) \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac - b^2} (e(bd - ce) - ad^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]

[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e)))

fricas [A] time = 1.49, size = 305, normalized size = 2.46

$$\left[\frac{(b^2 - 4ac)d \log(ax^2 + bx + c) - 2(b^2 - 4ac)d \log(ex + d) - \sqrt{b^2 - 4ac}(bd - 2ce) \log\left(\frac{2a^2x^2 + 2abx + b^2 - 2ac - \sqrt{b^2 - 4ac}}{ax^2 + bx + c}\right)}{2((ab^2 - 4a^2c)d^2 - (b^3 - 4abc)de + (b^2c - 4ac^2)e^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")

[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]

giac [A] time = 0.39, size = 127, normalized size = 1.02

$$-\frac{de \log(|xe + d|)}{ad^2e - bde^2 + ce^3} + \frac{d \log(ax^2 + bx + c)}{2(ad^2 - bde + ce^2)} - \frac{(bd - 2ce) \arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{(ad^2 - bde + ce^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")

[Out] -d*e*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))

maple [A] time = 0.01, size = 169, normalized size = 1.36

$$-\frac{bd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}} + \frac{2ce \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}} - \frac{d \ln(ex + d)}{ad^2 - deb + ce^2} + \frac{d \ln(ax^2 + bx + c)}{2ad^2 - 2deb + 2ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x)

[Out] 1/2/(a*d^2-b*d*e+c*e^2)*d*ln(a*x^2+b*x+c)-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*d+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c*e-d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 3.41, size = 801, normalized size = 6.46

$$\ln \left(a e x - \frac{\left(d \left(\frac{b \sqrt{b^2 - 4ac}}{2} - 2ac + \frac{b^2}{2} \right) - c e \sqrt{b^2 - 4ac} \right) \left(x (d a^2 e + b a e^2) + \frac{\left(d \left(\frac{b \sqrt{b^2 - 4ac}}{2} - 2ac + \frac{b^2}{2} \right) - c e \sqrt{b^2 - 4ac} \right) (x (2a^3 d^2 e - 2a^2 b d e^2 - 6ca^2 e^3 + 2ab^2 e^3) + a b^2 d^2 e^2 - 2a^2 c d^2 + a b^2 d^2 + 4abcde - 4ac^2 e^2 - b^3 de + b^2 ce^2)}{-4a^2 c d^2 + a b^2 d^2 + 4abcde - 4ac^2 e^2 - b^3 de + b^2 ce^2}}{-4a^2 c d^2 + a b^2 d^2 + 4abcde - 4ac^2 e^2 - b^3 de + b^2 ce^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)

[Out] $(\log(a e x - ((d * ((b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - 2 * a * c + b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) * (x * (a * b * e^2 + a^2 * d * e) + ((d * ((b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - 2 * a * c + b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) * (x * (2 * a * b^2 * e^3 - 6 * a^2 * c * e^3 + 2 * a^3 * d^2 * e - 2 * a^2 * b * d * e^2) + a * b * c * e^3 + a * b^2 * d * e^2 + a^2 * b * d^2 * e - 8 * a^2 * c * d * e^2)) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) + a * c * e^2 + a * b * d * e) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) * (d * ((b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - 2 * a * c + b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) - (\log(((d * (2 * a * c + (b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) * (x * (a * b * e^2 + a^2 * d * e) - ((d * (2 * a * c + (b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) * (x * (2 * a * b^2 * e^3 - 6 * a^2 * c * e^3 + 2 * a^3 * d^2 * e - 2 * a^2 * b * d * e^2) + a * b * c * e^3 + a * b^2 * d * e^2 + a^2 * b * d^2 * e - 8 * a^2 * c * d * e^2)) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) + a * c * e^2 + a * b * d * e) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) + a * e * x) * (d * (2 * a * c + (b * (b^2 - 4 * a * c)^{(1/2)})) / 2 - b^2 / 2) - c * e * (b^2 - 4 * a * c)^{(1/2)}) / (a * b^2 * d^2 - 4 * a^2 * c * d^2 - 4 * a * c^2 * e^2 + b^2 * c * e^2 - b^3 * d * e + 4 * a * b * c * d * e) - (d * \log(d + e * x)) / (a * d^2 + c * e^2 - b * d * e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)

[Out] Timed out

$$3.66 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^2(d+ex)} dx$$

Optimal. Leaf size=123

$$-\frac{(2ad - be) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} - \frac{e \log(ax^2 + bx + c)}{2(ad^2 - bde + ce^2)} + \frac{e \log(d + ex)}{ad^2 - bde + ce^2}$$

[Out] $e \ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*\ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)-(2*a*d-b*e)*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1569, 705, 31, 634, 618, 206, 628}

$$-\frac{(2ad - be) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} - \frac{e \log(ax^2 + bx + c)}{2(ad^2 - bde + ce^2)} + \frac{e \log(d + ex)}{ad^2 - bde + ce^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)), x]

[Out] $-(((2*a*d - b*e)*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(\operatorname{Sqrt}[b^2 - 4*a*c] * (a*d^2 - e*(b*d - c*e)))) + (e*\operatorname{Log}[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*\operatorname{Log}[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 705

Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 1569

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^2(d+ex)} dx &= \int \frac{1}{(d+ex)(c+bx+ax^2)} dx \\
&= \frac{e^2 \int \frac{1}{d+ex} dx}{ad^2 - bde + ce^2} + \frac{\int \frac{ad-be-ax}{c+bx+ax^2} dx}{ad^2 - e(bd - ce)} \\
&= \frac{e \log(d+ex)}{ad^2 - bde + ce^2} - \frac{e \int \frac{b+2ax}{c+bx+ax^2} dx}{2(ad^2 - bde + ce^2)} + \frac{(2ad - be) \int \frac{1}{c+bx+ax^2} dx}{2(ad^2 - e(bd - ce))} \\
&= \frac{e \log(d+ex)}{ad^2 - bde + ce^2} - \frac{e \log(c+bx+ax^2)}{2(ad^2 - bde + ce^2)} - \frac{(2ad - be) \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b\right)}{ad^2 - e(bd - ce)} \\
&= -\frac{(2ad - be) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{e \log(d+ex)}{ad^2 - bde + ce^2} - \frac{e \log(c+bx+ax^2)}{2(ad^2 - bde + ce^2)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 105, normalized size = 0.85

$$\frac{e\sqrt{4ac - b^2} (\log(x(ax + b) + c) - 2 \log(d + ex)) + (2be - 4ad) \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac - b^2} (e(bd - ce) - ad^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]

[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e)))

fricas [A] time = 1.45, size = 305, normalized size = 2.48

$$\left[\frac{(b^2 - 4ac)e \log(ax^2 + bx + c) - 2(b^2 - 4ac)e \log(ex + d) + \sqrt{b^2 - 4ac} (2ad - be) \log\left(\frac{2a^2x^2 + 2abx + b^2 - 2ac + \sqrt{b^2 - 4ac}}{ax^2 + bx + c}\right)}{2((ab^2 - 4a^2c)d^2 - (b^3 - 4abc)de + (b^2c - 4ac^2)e^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")

[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c

+ sqrt(b^2 - 4*a*c)*(2*a*x + b)/(a*x^2 + b*x + c))/((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]

giac [A] time = 0.34, size = 126, normalized size = 1.02

$$-\frac{e \log(ax^2 + bx + c)}{2(ad^2 - bde + ce^2)} + \frac{e^2 \log(|xe + d|)}{ad^2e - bde^2 + ce^3} + \frac{(2ad - be) \arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{(ad^2 - bde + ce^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")

[Out] -1/2*e*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))

maple [A] time = 0.01, size = 168, normalized size = 1.37

$$\frac{2ad \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}} - \frac{be \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}} + \frac{e \ln(ex + d)}{ad^2 - deb + ce^2} - \frac{e \ln(ax^2 + bx + c)}{2(ad^2 - deb + ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x)

[Out] -1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*e+e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 3.82, size = 521, normalized size = 4.24

$$\ln \left(\frac{3a^2 e^2 x + abe^2 + a^2 de - \frac{ae \left(\frac{b^2 e}{2} - 2ace + ad \sqrt{b^2 - 4ac} - \frac{be \sqrt{b^2 - 4ac}}{2} \right) (2xa^2 d^2 + abd^2 - 2xabde - 8cade - 6cxa e^2 + b^2 de + 2xb^2 e^2 + (4ac - b^2)(ad^2 - bde + ce^2))}{(4ac - b^2)(ad^2 - bde + ce^2)}}{-4a^2 cd^2 + ab^2 d^2 + 4abcde - 4ac^2 e^2 - b^3 de + b^2 ce^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)`

[Out] $(\log(3a^2e^2x + abe^2 + a^2de - (ae*((b^2e)/2 - 2ac*e + a*d*(b^2 - 4ac)^{(1/2)} - (b*e*(b^2 - 4ac)^{(1/2)))/2)*(2a^2d^2x + 2b^2e^2x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6a*c*e^2x - 8a*c*d*e - 2a*b*d*e*x))/((4ac - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2ac + (b*(b^2 - 4ac)^{(1/2)}/2 - b^2/2) - a*d*(b^2 - 4ac)^{(1/2)))/(a*b^2*d^2 - 4a^2*c*d^2 - 4a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4a*b*c*d*e) - (\log(3a^2e^2x + abe^2 + a^2de - (ae*((b^2e)/2 - 2ac*e - a*d*(b^2 - 4ac)^{(1/2)} + (b*e*(b^2 - 4ac)^{(1/2)}/2)*(2a^2d^2x + 2b^2e^2x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6a*c*e^2x - 8a*c*d*e - 2a*b*d*e*x))/((4ac - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*((b*(b^2 - 4ac)^{(1/2)}/2 - 2ac + b^2/2) - a*d*(b^2 - 4ac)^{(1/2)))/(a*b^2*d^2 - 4a^2*c*d^2 - 4a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4a*b*c*d*e) + (e*\log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)`

[Out] Timed out

$$3.67 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^3(d+ex)} dx$$

Optimal. Leaf size=158

$$\frac{(abd + 2ace + b^2(-e)) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} - \frac{e^2 \log(d+ex)}{d(ad^2 - bde + ce^2)} - \frac{(ad - be) \log(ax^2 + bx + c)}{2c(ad^2 - e(bd - ce))} + \frac{\log(x)}{cd}$$

[Out] ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2*(a*d-b*e)*ln(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))+(a*b*d+2*a*c*e-b^2*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.27, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{(abd + 2ace + b^2(-e)) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac} (ad^2 - e(bd - ce))} - \frac{e^2 \log(d+ex)}{d(ad^2 - e(bd - ce))} - \frac{(ad - be) \log(ax^2 + bx + c)}{2c(ad^2 - e(bd - ce))} + \frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]

[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e)))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 893

Int[((d_.) + (e_.)*(x_)^m)*((f_.) + (g_.)*(x_)^n)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p, x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 1569

Int[(x_)^m*((a_.) + (b_.)*(x_)^mn) + (c_.)*(x_)^(mn2))^p, x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^3(d+ex)} dx &= \int \frac{1}{x(d+ex)(c+bx+ax^2)} dx \\
&= \int \left(\frac{1}{cdx} + \frac{e^3}{d(-ad^2 + e(bd - ce))(d+ex)} + \frac{b^2e - a(bd + ce) - a(ad - be)x}{c(ad^2 - e(bd - ce))(c+bx+ax^2)} \right) dx \\
&= \frac{\log(x)}{cd} - \frac{e^2 \log(d+ex)}{d(ad^2 - e(bd - ce))} + \frac{\int \frac{b^2e - a(bd + ce) - a(ad - be)x}{c+bx+ax^2} dx}{c(ad^2 - bde + ce^2)} \\
&= \frac{\log(x)}{cd} - \frac{e^2 \log(d+ex)}{d(ad^2 - e(bd - ce))} + \frac{(-abd + b^2e - 2ace) \int \frac{1}{c+bx+ax^2} dx}{2c(ad^2 - bde + ce^2)} - \frac{(ad - be) \int \frac{1}{c+bx+ax^2} dx}{2c(ad^2 - bde + ce^2)} \\
&= \frac{\log(x)}{cd} - \frac{e^2 \log(d+ex)}{d(ad^2 - e(bd - ce))} - \frac{(ad - be) \log(c+bx+ax^2)}{2c(ad^2 - e(bd - ce))} - \frac{(-abd + b^2e - 2ace)}{2c(ad^2 - bde + ce^2)} \\
&= \frac{(abd - b^2e + 2ace) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}(ad^2 - bde + ce^2)} + \frac{\log(x)}{cd} - \frac{e^2 \log(d+ex)}{d(ad^2 - e(bd - ce))} - \frac{(ad - be)}{2c(ad^2 - bde + ce^2)}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 152, normalized size = 0.96

$$\frac{\sqrt{4ac - b^2} \left(-2 \log(x) (ad^2 + e(ce - bd)) + d(ad - be) \log(x(ax + b) + c) + 2ce^2 \log(d + ex) \right) + 2d(abd + 2ace + b^2e - 2ace)}{2cd\sqrt{4ac - b^2} (ad^2 + e(ce - bd))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]

[Out] -1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d + e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a*d^2 + e*(-(b*d) + c*e)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 164, normalized size = 1.04

$$\frac{(ad - be) \log(ax^2 + bx + c)}{2(acd^2 - bcde + c^2e^2)} - \frac{e^3 \log(|xe + d|)}{ad^3e - bd^2e^2 + cde^3} - \frac{(abd - b^2e + 2ace) \arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{(acd^2 - bcde + c^2e^2)\sqrt{-b^2 + 4ac}} + \frac{\log(|x|)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")

[Out] $-1/2*(a*d - b*e)*\log(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - e^3*\log(\text{abs}(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d - b^2*e + 2*a*c*e)*\arctan((2*a*x + b)/\text{sqrt}(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*\text{sqrt}(-b^2 + 4*a*c)) + \log(\text{abs}(x))/(c*d)$

maple [A] time = 0.01, size = 285, normalized size = 1.80

$$\frac{abd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}c} - \frac{2ae \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}} + \frac{b^2e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac - b^2}c} - \frac{ad \ln(ax^2 + bx + c)}{2(ad^2 - deb + ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x)

[Out] $-1/2/(a*d^2-b*d*e+c*e^2)/c*a*\ln(a*x^2+b*x+c)*d+1/2/(a*d^2-b*d*e+c*e^2)/c*\ln(a*x^2+b*x+c)*b*e-1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d-2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*e+1/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*e+\ln(x)/c/d-e^2*\ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

$$\begin{aligned}
& - 4ac)^{1/2} - 13ab^3c^2d^2e^3 + 21a^2b^3c^3d^2e^3 - 10a^3c^3d^2e^3x + 6ab^3cd^3e^2(b^2 - 4ac)^{1/2} - 6a^2b^2cd^4e(b^2 \\
& - 4ac)^{1/2} + 6ab^4d^3e^2x(b^2 - 4ac)^{1/2} - 6a^2b^3d^4ex(b^2 - 4ac)^{1/2} + 4a^2c^3de^4x(b^2 - 4ac)^{1/2} + 32a^2b^3cd^3e^2x \\
& - 35a^3b^3c^2d^3e^2x + 7ab^2c^2d^2e^3(b^2 - 4ac)^{1/2} - 13a^2b^3c^2d^3e^2(b^2 - 4ac)^{1/2} + 9a^3c^2d^3e^2x(b^2 - 4 \\
& ac)^{1/2} + 27a^2b^2c^2d^2e^3x + 4ab^3c^3de^4(b^2 - 4ac)^{1/2} - 4ab^3c^3e^5x(b^2 - 4ac)^{1/2} - b^4cd^4e^4x(b^2 - 4ac)^{1/2} \\
& - 5ab^3c^2de^4x - 14ab^4cd^2e^3x + 4a^2b^3c^3de^4x - 26a^3b^2cd^4ex + 14a^3b^3cd^4ex(b^2 - 4ac)^{1/2} + 3ab^2c^2de^4x(b^2 - 4ac)^{1/2} \\
& + 10ab^3cd^2e^3x(b^2 - 4ac)^{1/2} - 13a^2b^3c^2d^2e^3x(b^2 - 4ac)^{1/2} - 20a^2b^2cd^3e^2x(b^2 - 4ac)^{1/2}) * ((b^3e)/2 + d(2a^2c - (ab^2)/2 + (ab(b^2 - 4ac)^{1/2}))/2) \\
& - (b^2e(b^2 - 4ac)^{1/2})/2 + ac*e*(b^2 - 4ac)^{1/2} - 2ab*c*e)/(4a^3c^3e^2 + 4a^2c^2d^2 - b^2c^2e^2 + b^3cd*e - ab^2cd^2 - 4ab^3c^2de) - (e^2*log(d + ex))/(a*d^3 - b*d^2e + c*d*e^2) + log(x)/(c*d)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)

[Out] Timed out

$$3.68 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^4(d+ex)} dx$$

Optimal. Leaf size=193

$$\frac{(2a^2cd - ab(bd + 3ce) + b^3e) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (abd + ace + b^2(-e)) \log(ax^2 + bx + c) + \frac{e^3 \log(d + ex)}{d^2(ad^2 - e(bd - ce))}}{c^2\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{(abd + ace + b^2(-e)) \log(ax^2 + bx + c)}{2c^2(ad^2 - e(bd - ce))} + \frac{e^3 \log(d + ex)}{d^2(ad^2 - e(bd - ce))}$$

[Out] $-1/c/d/x - (b*d+c*e)*\ln(x)/c^2/d^2+e^3*\ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))+1/2*(a*b*d+a*c*e-b^2*e)*\ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))+(2*a^2*c*d+b^3*e-a*b*(b*d+3*c*e))*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/c^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{(2a^2cd - ab(bd + 3ce) + b^3e) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (abd + ace + b^2(-e)) \log(ax^2 + bx + c) + \frac{e^3 \log(d + ex)}{d^2(ad^2 - e(bd - ce))}}{c^2\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} + \frac{(abd + ace + b^2(-e)) \log(ax^2 + bx + c)}{2c^2(ad^2 - e(bd - ce))} + \frac{e^3 \log(d + ex)}{d^2(ad^2 - e(bd - ce))}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)), x]`

[Out] $-(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(c^2*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*d + c*e)*\operatorname{Log}[x])/(c^2*d^2) + (e^3*\operatorname{Log}[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))) + ((a*b*d - b^2*e + a*c*e)*\operatorname{Log}[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e)))$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628


```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (b_)*(x_)
+ (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1569

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^4 (d + ex)} dx &= \int \frac{1}{x^2 (d + ex) (c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{cdx^2} + \frac{-bd - ce}{c^2 d^2 x} + \frac{e^4}{d^2 (ad^2 - e(bd - ce)) (d + ex)} + \frac{-a^2 cd - b^3 e + ab(bd + 2ce)}{c^2 (ad^2 - e(bd - ce))} \right) dx \\
&= -\frac{1}{cdx} - \frac{(bd + ce) \log(x)}{c^2 d^2} + \frac{e^3 \log(d + ex)}{d^2 (ad^2 - e(bd - ce))} + \frac{\int \frac{-a^2 cd - b^3 e + ab(bd + 2ce) + a(abd - b^2 e + ace)}{c + bx + ax^2} dx}{c^2 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cdx} - \frac{(bd + ce) \log(x)}{c^2 d^2} + \frac{e^3 \log(d + ex)}{d^2 (ad^2 - e(bd - ce))} + \frac{(abd - b^2 e + ace) \int \frac{b + 2ax}{c + bx + ax^2} dx}{2c^2 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cdx} - \frac{(bd + ce) \log(x)}{c^2 d^2} + \frac{e^3 \log(d + ex)}{d^2 (ad^2 - e(bd - ce))} + \frac{(abd - b^2 e + ace) \log(c + bx)}{2c^2 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cdx} + \frac{(2a^2 cd + b^3 e - ab(bd + 3ce)) \tanh^{-1}\left(\frac{b + 2ax}{\sqrt{b^2 - 4ac}}\right)}{c^2 \sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} - \frac{(bd + ce) \log(x)}{c^2 d^2} + \frac{e^3 \log(d + ex)}{d^2 (ad^2 - e(bd - ce))}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 194, normalized size = 1.01

$$\frac{(2a^2 cd - ab(bd + 3ce) + b^3 e) \tan^{-1}\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right)}{c^2 \sqrt{4ac - b^2} (e(bd - ce) - ad^2)} + \frac{(abd + ace + b^2(-e)) \log(x(ax + b) + c)}{2c^2 (ad^2 + e(ce - bd))} + \frac{e^3 \log(d + ex)}{ad^4 + d^2 e(ce - bd)} - \frac{\log(x)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]

[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) - ((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d) + c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*(-(b*d) + c*e)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.34, size = 210, normalized size = 1.09

$$\frac{(abd - b^2e + ace) \log(ax^2 + bx + c)}{2(ac^2d^2 - bc^2de + c^3e^2)} + \frac{e^4 \log(|xe + d|)}{ad^4e - bd^3e^2 + cd^2e^3} + \frac{(ab^2d - 2a^2cd - b^3e + 3abce) \arctan\left(\frac{2ax+b}{\sqrt{-b^2+4ac}}\right)}{(ac^2d^2 - bc^2de + c^3e^2)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")

[Out] 1/2*(a*b*d - b^2*e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c^3*e^2) + e^4*log(abs(x*e + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + (a*b^2*d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c) - (b*d + c*e)*log(abs(x))/(c^2*d^2) - 1/(c*d*x)

maple [B] time = 0.01, size = 412, normalized size = 2.13

$$\frac{2a^2d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c} + \frac{ab^2d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c^2} + \frac{3abe \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c} - \frac{b^3e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x)

[Out] 1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*e-1/2/(a*d^2-b*d*e+c*e^2)/c^2*ln(a*x^2+b*x+c)*b^2*e-2/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*d+1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d+3/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*e-1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e-1/c/d/x-1/c^2/d*ln(x)*b-1/c/d^2*ln(x)*e+e^3/(a*d^2-b*d*e+c*e^2)/d^2*ln(e*x+d)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help

elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is $4ac - b^2$ positive or negative?

mupad [B] time = 20.39, size = 2388, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^4*(d + e*x)*(a + b/x + c/x^2)), x)$

[Out] $(e^3 \log(d + ex)) / (a^4 d^4 + c^2 d^2 e^2 - b^2 d^3 e) + (\log((a^4 e^4 x) / (c^2 d^2 - ((a^4 d^4 + b^4 e^4 + 2a^2 c^2 e^4 + 2a^3 c d^2 e^2 - 4ab^2 c e^4 + 2a^2 b^2 c d e^3)) / (c^2 d^2) - ((a^2 b^2 d^4 - 4ac^3 e^4 - a^3 c d^4 + b^2 c^2 e^4 + b^4 d^2 e^2 + 4a^2 c^2 d^2 e^2 - 2ab^3 d^3 e + b^3 c d e^3 - 4ab^2 c d e^3 + 5a^2 b^2 c d^3 e - 5ab^2 c d^2 e^2)) / (cd) + (a^2 b^2 d^3 e + 12a^2 c^2 d e^3 - 8ab^2 c^2 e^4 + a^3 c d^3 e - 11ab^2 c d e^3 + 8a^2 b^2 c d^2 e^2)) / (cd) + (a^2 b^4 e + b^3 e (b^2 - 4ac)^{1/2} + 4a^2 c^2 e - ab^3 d + 4a^2 b^2 c d - 5ab^2 c e - ab^2 d (b^2 - 4ac)^{1/2} + 2a^2 c d (b^2 - 4ac)^{1/2} - 3ab^2 c e (b^2 - 4ac)^{1/2})) * (4a^2 c^2 d^3 e + b^2 c^2 d e^3 + b^3 c d^2 e^2 + 2a^2 b^2 d^4 x + 2b^2 c^2 e^4 x + 2b^4 d^2 e^2 x + a^2 b^2 c d^4 - 4ac^3 d e^3 - 6a^3 c d^4 x - 8a^2 c^3 e^4 x - 2ab^2 c d^3 e - 4ab^3 d^3 e x - 2b^3 c d e^3 x - 3ab^2 c^2 d^2 e^2 - 6a^2 c^2 d^2 e^2 x + 8ab^2 c^2 d e^3 x + 14a^2 b^2 c d^3 e x - 6ab^2 c^2 d^2 e^2 x)) / (2c^2 (4ac - b^2) (a^2 d^2 + c^2 e^2 - b^2 d e)) * (b^4 e + b^3 e (b^2 - 4ac)^{1/2} + 4a^2 c^2 e - ab^3 d + 4a^2 b^2 c d - 5ab^2 c e - ab^2 d (b^2 - 4ac)^{1/2} + 2a^2 c d (b^2 - 4ac)^{1/2} - 3ab^2 c e (b^2 - 4ac)^{1/2})) / (2c^2 (4ac - b^2) (a^2 d^2 + c^2 e^2 - b^2 d e)) * (b^4 e + b^3 e (b^2 - 4ac)^{1/2} + 4a^2 c^2 e - ab^3 d + 4a^2 b^2 c d - 5ab^2 c e - ab^2 d (b^2 - 4ac)^{1/2} + 2a^2 c d (b^2 - 4ac)^{1/2} - 3ab^2 c e (b^2 - 4ac)^{1/2})) / (2c^2 (4ac - b^2) (a^2 d^2 + c^2 e^2 - b^2 d e)) * (b^4 e + b^3 e (b^2 - 4ac)^{1/2} + 4a^2 c^2 e - ab^3 d + 4a^2 b^2 c d - 5ab^2 c e - ab^2 d (b^2 - 4ac)^{1/2} + 2a^2 c d (b^2 - 4ac)^{1/2} - 3ab^2 c e (b^2 - 4ac)^{1/2})) / (2(4ac^4 e^2 + 4a^2 c^3 d^2 - b^2 c^3 e^2 - ab^2 c^2 d^2 + b^3 c^2 d e - 4ab^2 c^3 d e)) + (\log((a^4 e^4 x) / (c^2 d^2 - ((a^4 d^4 + b^4 e^4 + 2a^2 c^2 e^4 + 2a^3 c d^2 e^2 - 4ab^2 c e^4 + 2a^2 b^2 c d e^3)) / (c^2 d^2) - ((a^2 b^2 d^4 - 4ac^3 e^4 - a^3 c d^4 + b^2 c^2 e^4 + b^4 d^2 e^2 + 4a^2 c^2 d^2 e^2 - 2ab^3 d^3 e + b^3 c d e^3 - 4ab^2 c d e^3 + 5a^2 b^2 c d^3 e - 5ab^2 c d^2 e^2)) / (cd) + (a^2 b^2 d^3 e + 12a^2 c^2 d e^3 - 8ab^2 c^2 e^4 + a^3 c d^3 e - 11ab^2 c d e^3 + 8a^2 b^2 c d^2 e^2)) / (cd) + (a^2 b^4 e - b^3 e (b^2 - 4ac)^{1/2} + 4a^2 c^2 e - ab^3 d + 4a^2 b^2 c d - 5ab^2 c e + ab^2 d (b^2 - 4ac)^{1/2} - 2a^2 c d (b^2 - 4ac)^{1/2} + 3ab^2 c e (b^2 - 4ac)^{1/2})) * (4$

$$\begin{aligned} & *a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2 \\ & *e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8 \\ & *a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b* \\ & c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x \\ & - 6*a*b^2*c*d^2*e^2*x)/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))*(b^4 \\ & *e - b^3*e*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a* \\ & b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*c*d*(b^2 - 4*a*c)^{(1/2)} + 3*a \\ & *b*c*e*(b^2 - 4*a*c)^{(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) \\ & + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2)*(b^4*e - b \\ & ^3*e*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c* \\ & e + a*b^2*d*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*c*d*(b^2 - 4*a*c)^{(1/2)} + 3*a*b*c*e \\ & *(b^2 - 4*a*c)^{(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))*(b^4* \\ & e - b^3*e*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b \\ & ^2*c*e + a*b^2*d*(b^2 - 4*a*c)^{(1/2)} - 2*a^2*c*d*(b^2 - 4*a*c)^{(1/2)} + 3*a* \\ & b*c*e*(b^2 - 4*a*c)^{(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 - \\ & a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - 1/(c*d*x) - (\log(x)*(b*d + \\ & c*e))/(c^2*d^2) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)

[Out] Timed out

$$3.69 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^5(d+ex)} dx$$

Optimal. Leaf size=252

$$\frac{(a^2cd - ab(bd + 2ce) + b^3e) \log(ax^2 + bx + c)}{2c^3(ad^2 - e(bd - ce))} - \frac{(a^2c(3bd + 2ce) - ab^2(bd + 4ce) + b^4e) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} + \frac{\log(x)}{1}$$

[Out] $-1/2/c/d/x^2+(b*d+c*e)/c^2/d^2/x+(b^2*d^2+b*c*d*e-c*(a*d^2-c*e^2))*\ln(x)/c^3/d^3-e^4*\ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c*d+b^3*e-a*b*(b*d+2*c*e))*\ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))-(b^4*e+a^2*c*(3*b*d+2*c*e)-a*b^2*(b*d+4*c*e))*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/c^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.43, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{(a^2cd - ab(bd + 2ce) + b^3e) \log(ax^2 + bx + c)}{2c^3(ad^2 - e(bd - ce))} - \frac{(a^2c(3bd + 2ce) - ab^2(bd + 4ce) + b^4e) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}(ad^2 - e(bd - ce))} + \frac{\log(x)}{1}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]

[Out] $-1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e) - a*b^2*(b*d + 4*c*e))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(c^3*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*e^2))*\operatorname{Log}[x])/(c^3*d^3) - (e^4*\operatorname{Log}[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*\operatorname{Log}[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e)))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 893

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (b_)*(x_)
+ (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
x)^n(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)

Rule 1569

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^5 (d + ex)} dx &= \int \frac{1}{x^3 (d + ex) (c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{cdx^3} + \frac{-bd - ce}{c^2 d^2 x^2} + \frac{b^2 d^2 + bcde - c(ad^2 - ce^2)}{c^3 d^3 x} + \frac{e^5}{d^3 (-ad^2 + e(bd - ce)) (d + ex)} \right) dx \\
&= -\frac{1}{2cdx^2} + \frac{bd + ce}{c^2 d^2 x} + \frac{(b^2 d^2 + bcde - c(ad^2 - ce^2)) \log(x)}{c^3 d^3} - \frac{e^4 \log(d + ex)}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{2cdx^2} + \frac{bd + ce}{c^2 d^2 x} + \frac{(b^2 d^2 + bcde - c(ad^2 - ce^2)) \log(x)}{c^3 d^3} - \frac{e^4 \log(d + ex)}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{2cdx^2} + \frac{bd + ce}{c^2 d^2 x} + \frac{(b^2 d^2 + bcde - c(ad^2 - ce^2)) \log(x)}{c^3 d^3} - \frac{e^4 \log(d + ex)}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{2cdx^2} + \frac{bd + ce}{c^2 d^2 x} - \frac{(b^4 e + a^2 c(3bd + 2ce) - ab^2 (bd + 4ce)) \tanh^{-1} \left(\frac{b+2ax}{\sqrt{b^2-4ac}} \right)}{c^3 \sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))} +
\end{aligned}$$

Mathematica [A] time = 0.22, size = 252, normalized size = 1.00

$$\frac{(a^2 cd - ab(bd + 2ce) + b^3 e) \log(x(ax + b) + c)}{2c^3 (ad^2 + e(ce - bd))} - \frac{(a^2 c(3bd + 2ce) - ab^2 (bd + 4ce) + b^4 e) \tan^{-1} \left(\frac{2ax+b}{\sqrt{4ac-b^2}} \right)}{c^3 \sqrt{4ac - b^2} (e(bd - ce) - ad^2)} + \frac{\log(x)}{d^3 (ad^2 - e(bd - ce))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)), x]

[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]/(c^3*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d) + c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e)))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.35, size = 279, normalized size = 1.11

$$\frac{(ab^2d - a^2cd - b^3e + 2abce) \log(ax^2 + bx + c)}{2(ac^3d^2 - bc^3de + c^4e^2)} - \frac{e^5 \log(|xe + d|)}{ad^5e - bd^4e^2 + cd^3e^3} - \frac{(ab^3d - 3a^2bcd - b^4e + 4ab^2ce - 2a^2c^2d)}{(ac^3d^2 - bc^3de + c^4e^2)\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")

[Out]
$$-1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*\log(a*x^2 + b*x + c)/(a*c^3*d^2 - b*c^3*d*e + c^4*e^2) - e^5*\log(\text{abs}(x*e + d))/(a*d^5*e - b*d^4*e^2 + c*d^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*\arctan((2*a*x + b)/\sqrt{-b^2 + 4*a*c})/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*\sqrt{-b^2 + 4*a*c}) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*\log(\text{abs}(x))/(c^3*d^3 - 3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)$$

maple [B] time = 0.01, size = 562, normalized size = 2.23

$$\frac{3a^2bd \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c^2} + \frac{2a^2e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c} - \frac{ab^3d \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)\sqrt{4ac-b^2}c^3} - \frac{4ab^2e}{(ad^2 - deb + ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x)

[Out]
$$1/2/(a*d^2-b*d*e+c*e^2)/c^2*a^2*\ln(a*x^2+b*x+c)*d-1/2/(a*d^2-b*d*e+c*e^2)/c^3*a*\ln(a*x^2+b*x+c)*b^2*d-1/(a*d^2-b*d*e+c*e^2)/c^2*a*\ln(a*x^2+b*x+c)*b*e+1/2/(a*d^2-b*d*e+c*e^2)/c^3*\ln(a*x^2+b*x+c)*b^3*e+3/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d+2/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d-4/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e+1/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*\arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/c/d/x^2+1/c^2/d/x*b+1/c/d^2/x*e-1/c^2/d*\ln(x)*a+1/c^3/d*\ln(x)*b^2+1/c^2/d^2*\ln(x)*b*e+1/c/d^3*\ln(x)*e^2-e^4/(a*d^2-b*d*e+c*e^2)/d^3*\ln(e*x+d)$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 26.16, size = 3530, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*e
*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e +
b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^
4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2
*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2)))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*
a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4
*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*
a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e
^3 + 10*a^2*b^2*c*d^3*e^2)))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^(1/2) - b
^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) -
5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c
*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))*(4*a^2*c^2*d^3*e
+ b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4
*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x -
2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6
*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2
*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c
)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*
c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^
3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(a^3*b^3*d^6 + b^3*c^3*e^6
+ b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*e + 2*b^4*c^2*d*e^5 + 2*b^5*c
*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3*a*b*c^4*e^6 + 9*a^2*b^2*c^2*
d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^3 - 9*a*b^3*c^2*d^2*e^4 + 7*a
^2*b*c^3*d^2*e^4))/(c^4*d^4) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4*c^
2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^4*e
^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b^4*
c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^
5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) - 5
```

$$\begin{aligned}
& a^2 b^2 c d - 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2} / (2 c^3 (4 a c - b^2) (a d^2 + c e^2 - b d e)) - (a^5 e^5 x) / (c^3 d^3) (b^4 e (b^2 - 4 a c)^{1/2} - b^5 e + 4 a^3 c^2 d + a b^4 d + 6 a b^3 c e - a b^3 d (b^2 - 4 a c)^{1/2} - 5 a^2 b^2 c d - 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2}) / (2 (4 a c^5 e^2 + 4 a^2 c^4 d^2 - b^2 c^4 e^2 - a b^2 c^3 d^2 + b^3 c^3 d e - 4 a b c^4 d e)) - (e^4 \log(d + e x)) / (a d^5 + c d^3 e^2 - b d^4 e) - (\log(((a e (a^3 b^3 d^6 + b^3 c^3 e^6 + b^6 d^3 e^3 + 4 a^2 c^4 d e^5 + a^4 c^2 d^5 e + 2 b^4 c^2 d e^5 + 2 b^5 c d^2 e^4 - 4 a^3 c^3 d^3 e^3 - a^4 b c d^6 - 3 a b c^4 e^6 + 9 a^2 b^2 c^2 d^3 e^3 - 8 a b^2 c^3 d e^5 - 6 a b^4 c d^3 e^3 - 9 a b^3 c^2 d^2 e^4 + 7 a^2 b c^3 d^2 e^4)) / (c^4 d^4) - ((a e (a^2 b^3 d^5 - 4 a c^4 e^5 + b^2 c^3 e^5 + b^5 d^3 e^2 - 3 a^3 c^2 d^4 e + b^3 c^2 d e^4 + b^4 c d^2 e^3 + 4 a^2 c^3 d^2 e^3 - 2 a^3 b c d^5 - 2 a b^4 d^4 e - 4 a b c^3 d e^4 - 6 a b^3 c d^3 e^2 + 7 a^2 b^2 c d^4 e - 5 a b^2 c^2 d^2 e^3 + 8 a^2 b c^2 d^3 e^2)) / (c^2 d^2) + (a e x (2 a^3 b^2 d^5 - 3 a^4 c d^5 + 2 b^3 c^2 e^5 + 2 b^5 d^2 e^3 - 2 a b^4 d^3 e^2 - 2 a^2 b^3 d^4 e + 8 a^2 c^3 d e^4 - 8 a^3 c^2 d^3 e^2 - 8 a b c^3 e^5 + b^4 c d e^4 + 4 a^3 b c d^4 e - 6 a b^2 c^2 d e^4 - 12 a b^3 c d^2 e^3 + 16 a^2 b c^2 d^2 e^3 + 10 a^2 b^2 c d^3 e^2)) / (c^2 d^2) + (a e (b^5 e + b^4 e (b^2 - 4 a c)^{1/2} - 4 a^3 c^2 d - a b^4 d - 6 a b^3 c e - a b^3 d (b^2 - 4 a c)^{1/2} + 5 a^2 b^2 c d + 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2})) * (4 a^2 c^2 d^3 e + b^2 c^2 d e^3 + b^3 c d^2 e^2 + 2 a^2 b^2 d^4 x + 2 b^2 c^2 e^4 x + 2 b^4 d^2 e^2 x + a^2 b c d^4 - 4 a c^3 d e^3 - 6 a^3 c d^4 x - 8 a c^3 e^4 x - 2 a b^2 c d^3 e - 4 a b^3 d^3 e x - 2 b^3 c d e^3 x - 3 a b c^2 d^2 e^2 - 6 a^2 c^2 d^2 e^2 x + 8 a b c^2 d e^3 x + 14 a^2 b c d^3 e x - 6 a b^2 c d^2 e^2 x)) / (2 c^3 (4 a c - b^2) (a d^2 + c e^2 - b d e))) * (b^5 e + b^4 e (b^2 - 4 a c)^{1/2} - 4 a^3 c^2 d - a b^4 d - 6 a b^3 c e - a b^3 d (b^2 - 4 a c)^{1/2} + 5 a^2 b^2 c d + 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2})) / (2 c^3 (4 a c - b^2) (a d^2 + c e^2 - b d e)) + (a e x (a^4 b^2 d^6 + 2 a^2 c^4 e^6 + b^4 c^2 e^6 + b^6 d^2 e^4 - 4 a b^2 c^3 e^6 - 6 a^3 c^3 d^2 e^4 + 2 a^4 c^2 d^4 e^2 + 2 b^5 c d e^5 + 11 a^2 b^2 c^2 d^2 e^4 - 10 a b^3 c^2 d e^5 - 6 a b^4 c d^2 e^4 + 10 a^2 b c^3 d e^5)) / (c^4 d^4) * (b^5 e + b^4 e (b^2 - 4 a c)^{1/2} - 4 a^3 c^2 d - a b^4 d - 6 a b^3 c e - a b^3 d (b^2 - 4 a c)^{1/2} + 5 a^2 b^2 c d + 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2})) / (2 c^3 (4 a c - b^2) (a d^2 + c e^2 - b d e)) + (a^4 e^4 (b^2 d^2 + c^2 e^2 - a c d^2 + b c d e)) / (c^4 d^4) - (a^5 e^5 x) / (c^3 d^3) * (b^5 e + b^4 e (b^2 - 4 a c)^{1/2} - 4 a^3 c^2 d - a b^4 d - 6 a b^3 c e - a b^3 d (b^2 - 4 a c)^{1/2} + 5 a^2 b^2 c d + 8 a^2 b c^2 e + 2 a^2 c^2 e (b^2 - 4 a c)^{1/2} + 3 a^2 b c d (b^2 - 4 a c)^{1/2} - 4 a b^2 c e (b^2 - 4 a c)^{1/2})) / (2 (4 a c^5 e^2 + 4 a^2 c^4 d^2 - b^2 c^4 e^2 - a b^2 c^3 d^2 + b^3 c^3 d e - 4 a b c^4 d e)) - (1 / (2 c d) - (x (b d + c e)) / (c^2 d^2)) / x^2 + (\log(x) (c^2 e^2 - d^2 (a
\end{aligned}$$

```
*c - b^2) + b*c*d*e))/(c^3*d^3)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

$$3.70 \quad \int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$$

Optimal. Leaf size=343

$$\frac{(-b^2c(3ad^2 - ce^2) + 4abc^2de + ac^2(ad^2 - ce^2) + b^4d^2 - 2b^3cde) \log(ax^2 + bx + c) - \frac{x(2ad + be)}{a^2e^3} + \frac{(-4a^2c^3de - b^3d^2)}{2a^3(ad^2 - e(bd - ce))^2}}{2a^3(ad^2 - e(bd - ce))^2}$$

[Out] $-(2ad+be)x/a^2/e^3+1/2x^2/a/e^2+d^5/e^4/(ad^2-e(bd-ce))/(e^4x+d^4(3ad^2-e(4bd-5ce))\ln(e^4x+d^5/(ad^2-e(bd-ce))^2+1/2(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\ln(ax^2+bx+c)/a^3/(ad^2-e(bd-ce))^2+(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+a^2c^3d^2+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2))\operatorname{arctanh}((2ax+b)/(-4ac+b^2)^{1/2}))/a^3/(ad^2-e(bd-ce))^2/(-4ac+b^2)^{1/2}$

Rubi [A] time = 0.91, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{(-b^2c(3ad^2 - ce^2) + 4abc^2de + ac^2(ad^2 - ce^2) - 2b^3cde + b^4d^2) \log(ax^2 + bx + c) - \frac{(-4a^2c^3de + 8ab^2c^2de - b^3d^2)}{2a^3(ad^2 - e(bd - ce))^2}}{2a^3(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] $-\frac{((2ad+be)x)/(a^2e^3)+x^2/(2ae^2)+d^5/(e^4(ad^2-e(bd-ce))*(d+ex))+((b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)-b^3c(5ad^2-ce^2))\operatorname{ArcTanh}[(b+2ax)/\sqrt{b^2-4ac}])/(a^3\sqrt{b^2-4ac}(ad^2-e(bd-ce))^2)+(d^4(3ad^2-e(4bd-5ce))\operatorname{Log}[d+ex])/(e^4(ad^2-e(bd-ce))^2)+((b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)-b^2c(3ad^2-ce^2))\operatorname{Log}[c+bx+ax^2])/(2a^3(ad^2-e(bd-ce))^2)}{2a^3(ad^2-e(bd-ce))^2}$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q_.), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_)^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d + ex)^2} dx &= \int \frac{x^5}{(d + ex)^2 (c + bx + ax^2)} dx \\
&= \int \left(\frac{-2ad - be}{a^2 e^3} + \frac{x}{ae^2} + \frac{d^5}{e^3 (-ad^2 + e(bd - ce)) (d + ex)^2} + \frac{d^4 (3ad^2 - e(4bd - 5ce))}{e^3 (ad^2 - e(bd - ce))^2} \right) dx \\
&= -\frac{(2ad + be)x}{a^2 e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4 (ad^2 - e(bd - ce)) (d + ex)} + \frac{d^4 (3ad^2 - e(4bd - 5ce))}{e^4 (ad^2 - e(bd - ce))^2} \\
&= -\frac{(2ad + be)x}{a^2 e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4 (ad^2 - e(bd - ce)) (d + ex)} + \frac{d^4 (3ad^2 - e(4bd - 5ce))}{e^4 (ad^2 - e(bd - ce))^2} \\
&= -\frac{(2ad + be)x}{a^2 e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4 (ad^2 - e(bd - ce)) (d + ex)} + \frac{d^4 (3ad^2 - e(4bd - 5ce))}{e^4 (ad^2 - e(bd - ce))^2} \\
&= -\frac{(2ad + be)x}{a^2 e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(b^5 d^2 - 2b^4 cde + 8ab^2 c^2 d)}{e^4 (ad^2 - e(bd - ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 338, normalized size = 0.99

$$\frac{(b^2 c (ce^2 - 3ad^2) + 4abc^2 de + ac^2 (ad^2 - ce^2) + b^4 d^2 - 2b^3 cde) \log(x(ax + b) + c) - \frac{x(2ad + be)}{a^2 e^3} (-4a^2 c^3 de + b^5 d^2 - 2b^4 cde + 8ab^2 c^2 d)}{2a^3 (ad^2 + e ce - bd)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]

[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.42, size = 565, normalized size = 1.65

$$\frac{d^5 e^4}{(ad^2 e^8 - bde^9 + ce^{10})(xe + d)} + \frac{(b^5 d^2 e^2 - 5 ab^3 cd^2 e^2 + 5 a^2 bc^2 d^2 e^2 - 2 b^4 cde^3 + 8 ab^2 c^2 de^3 - 4 a^2 c^3 de^3 + b^3 c^2 e^4 - \dots)}{(a^5 d^4 - 2 a^4 bd^3 e + a^3 b^2 d^2 e^2 + 2 a^4 cd^2 e^2 - 2 a^3 bcde^3 \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")

[Out] $d^5 e^4 / ((a d^2 e^8 - b d e^9 + c e^{10})(x e + d)) + (b^5 d^2 e^2 - 5 a b^3 c d^2 e^2 + 5 a^2 b c^2 d^2 e^2 - 2 b^4 c d e^3 + 8 a b^2 c^2 d e^3 - 4 a^2 c^3 d e^3 + b^3 c^2 e^4 - 3 a^3 b c^2 d e^3 + b^3 c^2 e^4 - 3 a b^3 c^3 e^4) \arctan(-2 a d - 2 a d^2 / (x e + d) - b e + 2 b d e / (x e + d) - 2 c e^2 / (x e + d)) e^{-1} / \sqrt{-b^2 + 4 a c} e^{-2} / ((a^5 d^4 - 2 a^4 b d^3 e + a^3 b^2 d^2 e^2 + 2 a^4 c d^2 e^2 - 2 a^3 b c d e^3 + a^3 c^2 e^4) \sqrt{-b^2 + 4 a c}) + 1/2 (a^2 - 2 (3 a^2 d e + a b e^2)) e^{-1} / (x e + d) (x e + d)^2 e^{-4} / a^3 + 1/2 (b^4 d^2 - 3 a b^2 c d^2 + a^2 c^2 d^2 - 2 b^3 c d e + 4 a b^2 c^2 d e + b^2 c^2 e^2 - a c^3 e^2) \log(-a + 2 a d / (x e + d) - a d^2 / (x e + d)^2 - b e / (x e + d) + b d e / (x e + d)^2 - c e^2 / (x e + d)^2) / (a^5 d^4 - 2 a^4 b d^3 e + a^3 b^2 d^2 e^2 + 2 a^4 c d^2 e^2 - 2 a^3 b c d e^3 + a^3 c^2 e^4) - (3 a^2 d^2 + 2 a b d e + b^2 e^2 - a c e^2) e^{-4} \log(\text{abs}(x e + d)) e^{-1} / (x e + d)^2 / a^3$

maple [B] time = 0.01, size = 943, normalized size = 2.75

$$-\frac{5b^3 c^2 d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a} + \frac{4c^3 d e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a} + \frac{5b^3 c d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a^2} - \frac{8b^2 c^2}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x)

[Out] $1/2 x^2 / a e^{-2} - 2/a e^{-3} x d - 1/a^2 e^{-2} b x + 1/2 / (a d^2 - b d e + c e^2)^2 / a \ln(a x^2 + b x + c) c^2 d^2 - 3/2 / (a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) b^2 c d^2 + 2 / (a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) b^2 c^2 d e - 1/2 / (a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) c^3 e^2 + 1/2 / (a d^2 - b d e + c e^2)^2 / a^3 \ln(a x^2 + b x + c) b^4 d^2 - 1 / (a d^2 - b d e + c e^2)^2 / a^3 \ln(a x^2 + b x + c) b^3 c d e + 1/2 / (a d^2 - b d e + c e^2)^2 / a^3 \ln(a x^2 + b x + c) b^2 c^2 e^2 - 5 / (a d^2 - b d e + c e^2)^2 / a / (4$

$$\begin{aligned} & *a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b*c^2*d^2+4/(a*d^2-b*d* \\ & e+c*e^2)^2/a/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*c^3*d*e+ \\ & 5/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *b^3*c*d^2-8/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a* \\ & x+b)/(4*a*c-b^2)^{(1/2)})*b^2*c^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^{(1/2)} \\ & *arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b*c^3*e^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^{(1/2)} \\ & *arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^5*d^2+2/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^{(1/2)} \\ & *arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^4*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^{(1/2)} \\ & *arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*c^2*e^2+3/e^4*d^6/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*a-4/ \\ & e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*b+5/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*c+1/e^4*d^5/(a*d^2-b*d*e+c*e^2)/(e*x+d) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 8.04, size = 3503, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)

[Out] $(\log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4 + b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (\log(12*a^5*c*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b^3*d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^{(1/2)} + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^{(1/2)} + b*c^4*e^8*(b^2 - 4*a*c)^{(1/2)} - 6*a^5*d^8*x*(b^2 - 4*a*c)^{(1/2)} + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^{(1/2)} + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a*c)^{(1/2)} + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^{(1/2)} + a*b*c^4*e^8*x + 24*a^5*c*d^7*e*x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^{(1/2)} + 12*a^4*c*d^7*e*(b^2 - 4*a*c)^{(1/2)} + a*c^4*e^8*x*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^4*c*d^4*e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x + 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^{(1/2)} - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^{(1/2)} - 4*b^4*c*d^7*e*(b^2 - 4*a*c)^{(1/2)})$

$$\begin{aligned}
& 3e^{5*(b^2 - 4ac)^{(1/2)} - 24ab^2c^3d^2e^6 + 20ab^3c^2d^3e^5 - 20a^2b^3c^3d^3e^5 - 4a^2b^3c^3d^5e^3 + 16a^3b^3c^2d^5e^3 - 2a^3b^2c^2d^6e^2 - 4a^2b^4d^5e^3x + 11a^3b^3d^6e^2x - 8a^3c^3d^3e^5x + 40a^4c^2d^5e^3x - 12ab^3c^3d^2e^6*(b^2 - 4ac)^{(1/2)} - 4ab^3c^3d^4e^4*(b^2 - 4ac)^{(1/2)} - 24a^3b^3c^3d^6e^2*(b^2 - 4ac)^{(1/2)} + ab^4d^4e^4x*(b^2 - 4ac)^{(1/2)} - 4a^4c^3d^6e^2x*(b^2 - 4ac)^{(1/2)} \\
&) + 6ab^3c^2d^2e^6x - 18a^2b^3c^3d^2e^6x - 15a^3b^3c^2d^4e^4x + 6a^3b^2c^2d^5e^3x + 12ab^2c^2d^3e^5*(b^2 - 4ac)^{(1/2)} - 2a^2b^3c^2d^4e^4*(b^2 - 4ac)^{(1/2)} + 4a^2b^2c^2d^5e^3*(b^2 - 4ac)^{(1/2)} + 4a^2b^3d^5e^3x*(b^2 - 4ac)^{(1/2)} - 11a^3b^2d^6e^2x*(b^2 - 4ac)^{(1/2)} - 6a^2c^3d^2e^6x*(b^2 - 4ac)^{(1/2)} + 11a^3c^2d^4e^4x*(b^2 - 4ac)^{(1/2)} + 16a^2b^2c^2d^3e^5x + 14a^4b^7d^7e^7x*(b^2 - 4ac)^{(1/2)} - 4ab^2c^3d^7e^7x - 4ab^4c^3d^3e^5x - 44a^4b^3c^3d^6e^2x - 4ab^3c^3d^7e^7x*(b^2 - 4ac)^{(1/2)} - 4ab^3c^3d^3e^5x*(b^2 - 4ac)^{(1/2)} + 2a^3b^3c^3d^5e^3x*(b^2 - 4ac)^{(1/2)} + 6ab^2c^2d^2e^6x*(b^2 - 4ac)^{(1/2)} + 8a^2b^3c^2d^3e^5x*(b^2 - 4ac)^{(1/2)} - 8a^2b^2c^2d^4e^4x*(b^2 - 4ac)^{(1/2)}*(b^6d^2 + b^5d^2*(b^2 - 4ac)^{(1/2)} - 4a^3c^3d^2 + 4a^2c^4e^2 + b^4c^2e^2 - 5ab^2c^3e^2 + b^3c^2e^2*(b^2 - 4ac)^{(1/2)} - 2b^5c^3d^2e^2 + 13a^2b^2c^2d^2 - 7ab^4c^3d^2 + 12ab^3c^2d^2e^2 - 16a^2b^3c^3d^2e^2 - 5ab^3c^3d^2*(b^2 - 4ac)^{(1/2)} - 3ab^3c^3e^2*(b^2 - 4ac)^{(1/2)} - 4a^2c^3d^2e^2*(b^2 - 4ac)^{(1/2)} + 5a^2b^3c^2d^2*(b^2 - 4ac)^{(1/2)} - 2b^4c^3d^2e^2*(b^2 - 4ac)^{(1/2)} + 8ab^2c^2d^2e^2*(b^2 - 4ac)^{(1/2)))/(2*(4a^6c^3d^4 - a^5b^2d^4 + 4a^4c^3e^4 + 2a^4b^3d^3e - a^3b^2c^2e^4 - a^3b^4d^2e^2 + 8a^5c^2d^2e^2 - 8a^5b^3c^3d^3e + 2a^3b^3c^3d^2e^3 - 8a^4b^3c^2d^2e^3 + 2a^4b^2c^2d^2e^2)) - (\log(2a^5c^5e^8 - 12a^5c^3d^8 + 3a^4b^2d^8 - b^2c^4e^8 - b^6d^4e^4 - 4a^3b^3d^7e + 4b^3c^3d^7e^7 + 4b^5c^3d^3e^5 + b^5d^4e^4*(b^2 - 4ac)^{(1/2)} - 12a^2c^4d^2e^6 + 22a^3c^3d^4e^4 - 8a^4c^2d^6e^2 - 6b^4c^2d^2e^6 - 3a^4b^4d^8*(b^2 - 4ac)^{(1/2)} + b^3c^4e^8*(b^2 - 4ac)^{(1/2)} - 6a^5d^8x*(b^2 - 4ac)^{(1/2)} - 12ab^3c^4d^7e^7 + 16a^4b^3c^4d^7e - 4a^2c^3d^3e^5*(b^2 - 4ac)^{(1/2)} + 20a^3c^2d^5e^3*(b^2 - 4ac)^{(1/2)} + 6b^3c^2d^2e^6*(b^2 - 4ac)^{(1/2)} - ab^3c^4e^8x - 24a^5c^3d^7e^7x - 14a^2b^2c^2d^4e^4 + 4a^4c^4d^7e^7*(b^2 - 4ac)^{(1/2)} + 12a^4c^3d^7e^7*(b^2 - 4ac)^{(1/2)} + a^4c^4e^8x*(b^2 - 4ac)^{(1/2)} + 6ab^4c^4d^4e^4 - ab^5d^4e^4x + 6a^4b^2d^7e^7x - 8a^2c^4d^7e^7x + 4a^3b^2d^7e^7*(b^2 - 4ac)^{(1/2)} - 4b^2c^3d^7e^7*(b^2 - 4ac)^{(1/2)} - 4b^4c^3d^3e^5*(b^2 - 4ac)^{(1/2)} + 24ab^2c^3d^2e^6 - 20ab^3c^2d^3e^5 + 20a^2b^3c^3d^3e^5 + 4a^2b^3c^3d^5e^3 - 16a^3b^3c^2d^5e^3 + 2a^3b^2c^2d^6e^2 + 4a^2b^4d^5e^3x - 11a^3b^3d^6e^2x + 8a^3c^3d^3e^5x - 40a^4c^2d^5e^3x - 12ab^3c^3d^2e^6*(b^2 - 4ac)^{(1/2)} - 4ab^3c^3d^4e^4*(b^2 - 4ac)^{(1/2)} - 24a^3b^3c^3d^6e^2*(b^2 - 4ac)^{(1/2)} + ab^4d^4e^4x*(b^2 - 4ac)^{(1/2)} - 4a^4c^3d^6e^2x*(b^2 - 4ac)^{(1/2)} - 6ab^3c^2d^2e^6x + 18a^2b^3c^3d^2e^6x + 15a^3b^3c^2d^4e^4x - 6a^3b^2c^2d^5e^3x + 12ab^2c^2d^3e^5*(b^2 - 4ac)^{(1/2)} - 2a^2b^3c^2d^4e^4*(b^2 - 4ac)^{(1/2)} + 4a^2b^2c^2d^4e^4*(b^2 - 4ac)^{(1/2)} + 4a^2b^2c^2d^4e^4*(b^2 - 4ac)^{(1/2)} + 4a^2b^2c^2d^4e^4*(b^2 - 4ac)^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
& ^5e^3(b^2 - 4ac)^{1/2} + 4a^2b^3d^5e^3x(b^2 - 4ac)^{1/2} - 11a^3b^2d^6e^2x(b^2 - 4ac)^{1/2} - 6a^2c^3d^2e^6x(b^2 - 4ac)^{1/2} \\
& + 11a^3c^2d^4e^4x(b^2 - 4ac)^{1/2} - 16a^2b^2c^2d^3e^5x + 14a^4bd^7e^5x(b^2 - 4ac)^{1/2} + 4a^2b^2c^3de^7x + 4a^2b^4c^3d^3e^5x \\
& + 44a^4b^2c^3d^6e^2x - 4a^2b^2c^3de^7x(b^2 - 4ac)^{1/2} - 4a^2b^3c^3d^3e^5x(b^2 - 4ac)^{1/2} + 2a^3b^2c^3d^5e^3x(b^2 - 4ac)^{1/2} \\
& + 6a^2b^2c^2d^2e^6x(b^2 - 4ac)^{1/2} + 8a^2b^2c^2d^3e^5x(b^2 - 4ac)^{1/2} - 8a^2b^2c^2d^4e^4x(b^2 - 4ac)^{1/2} \\
& + (b^6d^2 - b^5d^2)(b^2 - 4ac)^{1/2} - 4a^3c^3d^2 + 4a^2c^4e^2 + b^4c^2e^2 - 5a^2b^2c^3e^2 - b^3c^2e^2(b^2 - 4ac)^{1/2} \\
& - 2b^5c^2de + 13a^2b^2c^2d^2 - 7a^2b^4c^2d^2 + 12a^2b^3c^2de - 16a^2b^2c^3de + 5a^2b^3cd^2(b^2 - 4ac)^{1/2} \\
& + 3a^2b^3c^3e^2(b^2 - 4ac)^{1/2} + 4a^2c^3de(b^2 - 4ac)^{1/2} - 5a^2b^2c^2d^2(b^2 - 4ac)^{1/2} + 2b^4c^2de(b^2 - 4ac)^{1/2} \\
& - 8a^2b^2c^2de(b^2 - 4ac)^{1/2}))/((2(4a^6cd^4 - a^5b^2d^4 + 4a^4c^3e^4 + 2a^4b^3d^3e - a^3b^2c^2e^4 - a^3b^4d^2e^2 + 8a^5c^2d^2e^2 - 8a^5b^2cd^3e + 2a^3b^3cd^2e^3 - 8a^4b^2c^2de^3 + 2a^4b^2cd^2e^2)) + x^2/(2ae^2) - (x(b^2e^2 + 2ade)))/(a^2e^4) + (a^2d^5)/(e(a^2de^3 + a^2e^4x)(ad^2 + ce^2 - bde))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)

[Out] Timed out

$$3.71 \quad \int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$$

Optimal. Leaf size=274

$$\frac{(bd - ce)(-2acd + b^2d - bce) \log(ax^2 + bx + c) \left(-b^2c(4ad^2 - ce^2) + 6abc^2de + 2ac^2(ad^2 - ce^2) + b^4d^2 - 2b^3cde\right)}{2a^2(ad^2 - e(bd - ce))^2} - \frac{a^2\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2}{2a^2(ad^2 - e(bd - ce))^2}$$

[Out] x/a/e^2-d^4/e^3/(a*d^2-e*(b*d-c*e))/(e*x+d)-d^3*(2*a*d^2-e*(3*b*d-4*c*e))*ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))^2-1/2*(b*d-c*e)*(-2*a*c*d+b^2*d-b*c*e)*ln(a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))^2-(b^4*d^2-2*b^3*c*d*e+6*a*b*c^2*d*e+2*a*c^2*(a*d^2-c*e^2)-b^2*c*(4*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.56, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-b^2c(4ad^2 - ce^2) + 6abc^2de + 2ac^2(ad^2 - ce^2) - 2b^3cde + b^4d^2\right) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) (bd - ce)(-2acd + b^2d - bce)}{a^2\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2} - \frac{2a^2(ad^2 - e(bd - ce))^2}{a^2\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]]/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Dist}[\frac{2*c*d - b*e}{2*c}, \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[\frac{b + 2*c*x}{a + b*x + c*x^2}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1569

$\text{Int}[x^{(m_.)}*((a_.) + (b_.)*(x_.)^{(mn_.)} + (c_.)*(x_.)^{(mn2_.)})^{(p_.)}*((d_.) + (e_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \text{Int}[x^{(m - 2*n*p)}*(d + e*x^n)^q*(c + b*x^n + a*x^{(2*n)})^p, x] /;$ $\text{FreeQ}[\{a, b, c, d, e, m, n, q\}, x] \&\& \text{EqQ}[mn, -n] \&\& \text{EqQ}[mn2, 2*mn] \&\& \text{IntegerQ}[p]$

Rule 1628

$\text{Int}[(Pq_)*((d_.) + (e_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, m\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{IGtQ}[p, -2]$

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx &= \int \frac{x^4}{(d+ex)^2(c+bx+ax^2)} dx \\
&= \int \left(\frac{1}{ae^2} + \frac{d^4}{e^2(ad^2 - e(bd - ce))(d+ex)^2} + \frac{d^3(-2ad^2 + e(3bd - 4ce))}{e^2(ad^2 - e(bd - ce))^2(d+ex)} + \frac{-c(b^2 - ce)}{e^2(ad^2 - e(bd - ce))^2(d+ex)} \right) dx \\
&= \frac{x}{ae^2} - \frac{d^4}{e^3(ad^2 - e(bd - ce))(d+ex)} - \frac{d^3(2ad^2 - e(3bd - 4ce)) \log(d+ex)}{e^3(ad^2 - e(bd - ce))^2} + \frac{\int \frac{-c}{e^2(ad^2 - e(bd - ce))^2} dx}{e^3(ad^2 - e(bd - ce))^2} \\
&= \frac{x}{ae^2} - \frac{d^4}{e^3(ad^2 - e(bd - ce))(d+ex)} - \frac{d^3(2ad^2 - e(3bd - 4ce)) \log(d+ex)}{e^3(ad^2 - e(bd - ce))^2} - \frac{(bd - ce)}{e^3(ad^2 - e(bd - ce))^2} \\
&= \frac{x}{ae^2} - \frac{d^4}{e^3(ad^2 - e(bd - ce))(d+ex)} - \frac{d^3(2ad^2 - e(3bd - 4ce)) \log(d+ex)}{e^3(ad^2 - e(bd - ce))^2} - \frac{(bd - ce)}{e^3(ad^2 - e(bd - ce))^2} \\
&= \frac{x}{ae^2} - \frac{d^4}{e^3(ad^2 - e(bd - ce))(d+ex)} - \frac{(b^4d^2 - 2b^3cde + 6abc^2de + 2ac^2(ad^2 - ce^2)) \log(d+ex)}{a^2\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2} - \frac{(bd - ce)}{e^3(ad^2 - e(bd - ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 269, normalized size = 0.98

$$\frac{(bd - ce)(2acd + b^2(-d) + bce) \log(x(ax + b) + c)}{2a^2(ad^2 + e(ce - bd))^2} + \frac{(b^2c(ce^2 - 4ad^2) + 6abc^2de + 2ac^2(ad^2 - ce^2) + b^4d^2 - 2b^3cde)}{a^2\sqrt{4ac - b^2}(ad^2 + e(ce - bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*c*e)*Log[c + x*(b + a*x)]/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [B] time = 103.93, size = 2139, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*\sqrt{b^2 - 4*a*c}*\log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*\log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*\log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x), -1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 2*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^4*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*e^6)*x^2 + 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 3*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 2*a*c^3)*d*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 3*a*b*c^2)*d*e^5 + (b^2*c^2 - 2*a*c^3)*e^6)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^3*b^2 - 4*a^4*c)*d^5*e - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^3 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^4 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^5)*x + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2*e^4 + (b^3*c^2 - 4*a*b*c^3)*d*e^5 + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*e^4 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e^5 + (b^3*c^2 - 4*a*b*c^3)*e^6)*x)*\log(a*x^2 + b*x + c) + 2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 3*(a^2*b^3 - 4*a^3*b*c)*d^5*e + 4*(a^2*b^2*c - 4*a^3*c^2)*d^4*e^2 + (2*(a^3*b^2 - 4*a^4*c)*d^5*e - 3*(a^2*b^3 - 4*a^3*b*c)*d^4*e^2 + 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^3)*x)*\log(e*x + d))/((a^4*b^2 - 4*a^5*c)*d^5*e^3 - 2*(a^3*b^3 - 4*a^4*b*c)*d^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e^6 + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^7 + ((a^4*b^2 - 4*a^5*c)*d^4*e^4 - 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^5 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^7 + (a^2*b^2*c^2 - 4*a^3*c^3)*e^8)*x)]$$

giac [A] time = 0.40, size = 476, normalized size = 1.74

$$\frac{d^4 e^3 (b^4 d^2 e^2 - 4 a b^2 c d^2 e^2 + 2 a^2 c^2 d^2 e^2 - 2 b^3 c d e^3 + 6 a b c^2 d e^3 + b^2 c^2 e^4 - 2 a c^3 e^4) \arctan\left(\frac{ad^2 e^6 - b d e^7 + c e^8}{(x e + d)}\right)}{(a^4 d^4 - 2 a^3 b d^3 e + a^2 b^2 d^2 e^2 + 2 a^3 c d^2 e^2 - 2 a^2 b c d e^3 + a^2 c^2 e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")

[Out]
$$-d^4 e^3 / ((a d^2 e^6 - b d e^7 + c e^8) (x e + d)) - (b^4 d^2 e^2 - 4 a b^2 c d^2 e^2 + 2 a^2 c^2 d^2 e^2 - 2 b^3 c d e^3 + 6 a b c^2 d e^3 + b^2 c^2 e^4 - 2 a c^3 e^4) \arctan\left(\frac{2 a d - 2 a d^2 / (x e + d) - b e + 2 b d e / (x e + d) - 2 c e^2 / (x e + d)}{e^{-1} / \sqrt{-b^2 + 4 a c}}\right) e^{-2} / ((a^4 d^4 - 2 a^3 b d^3 e + a^2 b^2 d^2 e^2 + 2 a^3 c d^2 e^2 - 2 a^2 b c d e^3 + a^2 c^2 e^4) \sqrt{-b^2 + 4 a c}) + (x e + d) e^{-3} / a - 1/2 (b^3 d^2 - 2 a b^2 c d^2 - 2 b^2 c d e + 2 a c^2 d e + b c^2 e^2) \log(-a + 2 a d / (x e + d) - a d^2 / (x e + d)^2 - b e / (x e + d) + b d e / (x e + d)^2 - c e^2 / (x e + d)^2) / (a^4 d^4 - 2 a^3 b d^3 e + a^2 b^2 d^2 e^2 + 2 a^3 c d^2 e^2 - 2 a^2 b c d e^3 + a^2 c^2 e^4) + (2 a d + b e) e^{-3} \log(\text{abs}(x e + d) e^{-1} / (x e + d)^2) / a^2$$

maple [B] time = 0.01, size = 765, normalized size = 2.79

$$\frac{4 b^2 c d^2 \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2)^2 \sqrt{4 a c - b^2} a} + \frac{6 b c^2 d e \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2)^2 \sqrt{4 a c - b^2} a} - \frac{2 c^3 e^2 \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{(a d^2 - d e b + c e^2)^2 \sqrt{4 a c - b^2} a} + \frac{b^4 d^2}{(a d^2 - d e b + c e^2)^2 \sqrt{4 a c - b^2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x)

[Out]
$$x/a e^2 + 1/(a d^2 - b d e + c e^2)^2 / a \ln(a x^2 + b x + c) b c d^2 - 1/(a d^2 - b d e + c e^2)^2 / a \ln(a x^2 + b x + c) c^2 d e - 1/2 / (a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) b^3 d^2 + 1/(a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) b^2 c d e - 1/2 / (a d^2 - b d e + c e^2)^2 / a^2 \ln(a x^2 + b x + c) b c^2 e^2 + 2 / (a d^2 - b d e + c e^2)^2 / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) c^2 d^2 - 4 / (a d^2 - b d e + c e^2)^2 / a / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^2 c d^2 + 6 / (a d^2 - b d e + c e^2)^2 / a / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^2 c d^2 + 6 / (a d^2 - b d e + c e^2)^2 / a / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^2 c d^2 + 6 / (4 a c - b^2)^{1/2} c^3 e^2 + 1 / (a d^2 - b d e + c e^2)^2 / a^2 / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^4 d^2 - 2 / (a d^2 - b d e + c e^2)^2 / a^2 / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^3 c d e + 1 / (a d^2 - b d e + c e^2)^2 / a^2 / (4 a c - b^2)^{1/2} \arctan((2 a x + b) / (4 a c - b^2)^{1/2}) b^2 c^2 e^2 - 1 / e^3 d^4 / (a d^2 - b d e + c e^2) / (e x + d) - 2 / e^3 d^5 / (a d^2 - b d e + c e^2)^2 \ln(a x^2 + b x + c)$$

$e*x+d)*a+3/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*b-4/e*d^3/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation may help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 6.00, size = 2495, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)

[Out]
$$\begin{aligned} & x/(a*e^2) - (\log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 + a \\ & ^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) + (\\ & \log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^{(1/2)} - 2*a^3*b^2*d^7 + \\ & b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^4*d^ \\ & 4*e^3*(b^2 - 4*a*c)^{(1/2)} - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6*b^3* \\ & c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^{(1/ \\ & 2)} - 4*a^4*d^7*x*(b^2 - 4*a*c)^{(1/2)} - 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^ \\ & 3*(b^2 - 4*a*c)^{(1/2)} + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^{(1/2)} + 16*a^4*c*d^ \\ & 6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^{(1/2)} - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^{(1/ \\ & 2)} - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x - 4*a^3*b^2 \\ & *d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^{(1/2)} - 6*a*c^3*d^2*e^5*(b^2 - 4*a \\ & *c)^{(1/2)} - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} + 20*a*b^2*c^2*d^3*e^4 + 17 \\ & *a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x - 12*a^2*c^3 \\ & *d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^{(1/2)} - \\ & 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^{(1/2)} + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^{(\\ & 1/2)} - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 6*a*b^2*c^2*d^2*e^5*x - 4*a^ \\ & 2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^{(1/2)} - 4*a*b*c^3*d*e \\ & ^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 10*a^3*b*d^6*e*x*(b^2 - 4 \\ & *a*c)^{(1/2)} - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 32*a^3*b*c*d^5*e^2*x + \\ & 6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^{(1/2)} - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c) \\ & ^{(1/2)})*(b^5*d^2 + b^4*d^2*(b^2 - 4*a*c)^{(1/2)} + b^3*c^2*e^2 + 8*a^2*b*c^2* \\ & d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^{(1/2)} + b^2*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} - \\ & 2*b^4*c*d*e - 6*a*b^3*c*d^2 - 4*a*b*c^3*e^2 - 8*a^2*c^3*d*e - 2*a*c^3*e^2 \end{aligned}$$

$$\begin{aligned}
& (b^2 - 4ac)^{1/2} + 10ab^2c^2d^2e - 4ab^2c^2d^2(b^2 - 4ac)^{1/2} \\
& - 2b^3c^2d^2e(b^2 - 4ac)^{1/2} + 6ab^2c^2d^2e(b^2 - 4ac)^{1/2}) / (2 * \\
& (4a^5c^2d^4 - a^4b^2d^4 + 4a^3c^3e^4 + 2a^3b^3d^3e - a^2b^2c^2e^4 - a^2b^4d^2e^2 + 8a^4c^2d^2e^2 - 8a^4b^2c^2d^3e + 2a^2b^3c^2d^3e^3 - 8a^3b^2c^2d^2e^3 + 2a^3b^2c^2d^2e^2)) - (\log(c^4e^7(b^2 - 4ac)^{1/2} - b^5d^4e^3 - 3a^2b^3d^6e + 4b^2c^3d^6e + 4b^4c^2d^3e^4 + b^4d^4e^3(b^2 - 4ac)^{1/2} + 24a^2c^3d^3e^4 - 8a^3c^2d^5e^2 - 6b^3c^2d^2e^5 - 8a^2c^4d^6e - 2a^2c^4e^7x - 2a^3b^2d^7(b^2 - 4ac)^{1/2} - 4a^4d^7x(b^2 - 4ac)^{1/2} + 12a^3b^2c^2d^6e + 17a^2c^2d^4e^3(b^2 - 4ac)^{1/2} + 6b^2c^2d^2e^5(b^2 - 4ac)^{1/2} - 16a^4c^2d^6e^2x + 8a^3c^2d^6e^2(b^2 - 4ac)^{1/2} - 4b^2c^3d^6e^2(b^2 - 4ac)^{1/2} + 18a^2b^2c^3d^2e^5 + 8a^2b^3c^2d^4e^3 + 2a^2b^4d^4e^3x + 4a^3b^2d^6e^2x + 3a^2b^2d^6e^2(b^2 - 4ac)^{1/2} - 6a^2c^3d^2e^5(b^2 - 4ac)^{1/2} - 4b^3c^2d^3e^4(b^2 - 4ac)^{1/2} - 20a^2b^2c^2d^3e^4 - 17a^2b^2c^2d^4e^3 + 2a^2b^2c^2d^5e^2 - 8a^2b^3d^5e^2x + 12a^2c^3d^2e^5x - 34a^3c^2d^4e^3x + 4a^2b^2c^2d^3e^4(b^2 - 4ac)^{1/2} - 18a^2b^2c^2d^5e^2(b^2 - 4ac)^{1/2} + 4a^2b^3d^4e^3x(b^2 - 4ac)^{1/2} - 4a^3c^2d^5e^2x(b^2 - 4ac)^{1/2} - 6a^2b^2c^2d^2e^5x + 4a^2b^2c^2d^3e^4x - 8a^2b^2d^5e^2x(b^2 - 4ac)^{1/2} + 4a^2b^2c^3d^2e^6x + 12a^2c^2d^3e^4x(b^2 - 4ac)^{1/2} + 10a^3b^2d^6e^2x(b^2 - 4ac)^{1/2} - 4a^2c^3d^2e^6x(b^2 - 4ac)^{1/2} + 32a^3b^2c^2d^5e^2x + 6a^2b^2c^2d^2e^5x(b^2 - 4ac)^{1/2} - 8a^2b^2c^2d^3e^4x(b^2 - 4ac)^{1/2}) * (b^4d^2(b^2 - 4ac)^{1/2} - b^5d^2 - b^3c^2e^2 - 8a^2b^2c^2d^2 + 2a^2c^2d^2(b^2 - 4ac)^{1/2} + b^2c^2e^2(b^2 - 4ac)^{1/2} + 2b^4c^2d^2e + 6a^2b^3c^2d^2 + 4a^2b^2c^3e^2 + 8a^2c^3d^2e - 2a^2c^3e^2(b^2 - 4ac)^{1/2} - 10a^2b^2c^2d^2e - 4a^2b^2c^2d^2(b^2 - 4ac)^{1/2} - 2b^3c^2d^2e(b^2 - 4ac)^{1/2} + 6a^2b^2c^2d^2e(b^2 - 4ac)^{1/2})) / (2 * (4a^5c^2d^4 - a^4b^2d^4 + 4a^3c^3e^4 + 2a^3b^3d^3e - a^2b^2c^2e^4 - a^2b^4d^2e^2 + 8a^4c^2d^2e^2 - 8a^4b^2c^2d^3e + 2a^2b^3c^2d^2e^3 - 8a^3b^2c^2d^2e^3 + 2a^3b^2c^2d^2e^2)) - (a^4d^4) / (e * (a^2d^2 + a^2e^3x) * (a^2d^2 + c^2e^2 - b^2d^2e))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)

[Out] Timed out

$$3.72 \quad \int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$$

Optimal. Leaf size=246

$$\frac{(-c(ad^2 - ce^2) + b^2d^2 - 2bcde) \log(ax^2 + bx + c)}{2a(ad^2 - e(bd - ce))^2} + \frac{(-bc(3ad^2 - ce^2) + 4ac^2de + b^3d^2 - 2b^2cde) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2}$$

[Out] d^3/e^2/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^2*(a*d^2-e*(2*b*d-3*c*e))*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))^2+1/2*(b^2*d^2-2*b*c*d*e-c*(a*d^2-c*e^2))*ln(a*x^2+b*x+c)/a/(a*d^2-e*(b*d-c*e))^2+(b^3*d^2-2*b^2*c*d*e+4*a*c^2*d*e-b*c*(3*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.40, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{(-bc(3ad^2 - ce^2) + 4ac^2de - 2b^2cde + b^3d^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \frac{(-c(ad^2 - ce^2) + b^2d^2 - 2bcde) \log(ax^2 + bx + c)}{2a(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[$b^2 - 4ac$, 0] && !NiceSqrtQ[$b^2 - 4ac$]

Rule 1569

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]

Rule 1628

Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx &= \int \frac{x^3}{(d+ex)^2(c+bx+ax^2)} dx \\
&= \int \left(\frac{d^3}{e(-ad^2+e(bd-ce))(d+ex)^2} + \frac{d^2(ad^2-e(2bd-3ce))}{e(ad^2-e(bd-ce))^2(d+ex)} + \frac{cd(bd-2ce)}{(ad^2-e(bd-ce))^2} \right) dx \\
&= \frac{d^3}{e^2(ad^2-e(bd-ce))(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2(ad^2-e(bd-ce))^2} + \frac{\int \frac{cd(bd-2ce)}{(ad^2-e(bd-ce))^2} dx}{(ad^2-e(bd-ce))^2} \\
&= \frac{d^3}{e^2(ad^2-e(bd-ce))(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2(ad^2-e(bd-ce))^2} + \frac{(b^2d^2-2b^2d+2b^2cde+4ac^2de-bc(3ad^2-ce^2))\operatorname{tanh}^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{b^2-4ac}(ad^2-e(bd-ce))^2} \\
&= \frac{d^3}{e^2(ad^2-e(bd-ce))(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2(ad^2-e(bd-ce))^2} + \frac{(b^2d^2-2b^2d+2b^2cde+4ac^2de-bc(3ad^2-ce^2))\operatorname{tanh}^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{b^2-4ac}(ad^2-e(bd-ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 207, normalized size = 0.84

$$\frac{\frac{(c^2e^2-ad^2)+b^2d^2-2bcde}{a} \log(x(ax+b)+c) - \frac{2(bc^2e^2-3ad^2)+4ac^2de+b^3d^2-2b^2cde}{a\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + \frac{2\log(d+ex)(ad^4+d^2e(3ce-2bd))}{e^2} + \frac{2d^3(a^2c^2-b^2d^2)}{e^2}}{2(ad^2+e(ce-bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b + a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [B] time = 36.70, size = 1465, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")

[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x)]

giac [A] time = 0.42, size = 412, normalized size = 1.67

$$\frac{1}{2} \left(\frac{2d^3e^2}{(ad^2e^3 - bde^4 + ce^5)(xe + d)} + \frac{2(b^3d^2e^3 - 3abcd^2e^3 - 2b^2cde^4 + 4ac^2de^4 + bc^2e^5) \arctan \left(-\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d}\right)}{\sqrt{-b^2 + 4ac}} \right)}{(a^3d^4 - 2a^2bd^3e + ab^2d^2e^2 + 2a^2cd^2e^2 - 2abcde^3 + ac^2e^4)\sqrt{-b^2 + 4ac}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3*d^2*e^3 -
3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*
d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)
/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a
^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e
- a*c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e
+ d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^4 -
2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4
) - 2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)*e^(-1)
```

maple [B] time = 0.01, size = 580, normalized size = 2.36

$$\frac{b^3 d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a} + \frac{2b^2 c d e \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a} - \frac{b c^2 e^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a} + \frac{3bc d}{(a d^2 - deb + c e^2)^2 \sqrt{4ac - b^2} a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*d^2+1/2/(a*d^2-b*d*e+c*e^2)^2/
a*ln(a*x^2+b*x+c)*b^2*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d*e
+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*e^2+3/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2-4/(a*d^2-b*
d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d*e-
1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)
))/a*b^3*d^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))/a*b^2*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))/a*b*c^2*e^2+d^4/(a*d^2-b*d*e+c*e^2)^2/e^2*ln
(e*x+d)*a-2*d^3/(a*d^2-b*d*e+c*e^2)^2/e*ln(e*x+d)*b+3*d^2/(a*d^2-b*d*e+c*e^
2)^2*ln(e*x+d)*c+1/e^2*d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 5.11, size = 2037, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x/((d + e*x)^2*(a + b/x + c/x^2)),x)$

[Out] $(\log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 + b^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (\log(a^2*b^2*d^6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^{(1/2)} + 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 - 4*a*c)^{(1/2)} - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e + 4*b*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^{(1/2)} + 4*c^3*d*e^5*(b^2 - 4*a*c)^{(1/2)} + 2*a^3*d^6*x*(b^2 - 4*a*c)^{(1/2)} + 8*a^2*b*c*d^5*e + 8*a*c^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^{(1/2)} - 4*a^2*c*d^5*e*(b^2 - 4*a*c)^{(1/2)} - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2 - 6*a*b^3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2 - 4*a*c)^{(1/2)} - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^{(1/2)} + 2*b^2*c*d^3*e^3*(b^2 - 4*a*c)^{(1/2)} - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^{(1/2)} - 32*a^2*c^2*d^3*e^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^{(1/2)} - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 3*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^{(1/2)} - 6*a^2*b*d^5*e*x*(b^2 - 4*a*c)^{(1/2)} + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x + 23*a^2*b*c*d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^{(1/2)})*(b^4*d^2 - 4*a*c^3*e^2 + b^3*d^2*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} + 8*a*b*c^2*d*e - 3*a*b*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 4*a*c^2*d*e*(b^2 - 4*a*c)^{(1/2)} - 2*b^2*c*d*e*(b^2 - 4*a*c)^{(1/2)})))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e^4 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3 - 8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) - (\log(2*c^4*e^6 + 4*a^3*c*d^6 - a^2*b^2*d^6 + b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^{(1/2)} - 24*a*c^3*d^2*e^4 - 6*b^3*c*d^3*e^3 - 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 - 4*a*c)^{(1/2)} + 10*a^2*c^2*d^4*e^2 + 9*b^2*c^2*d^2*e^4 + 2*a*b^3*d^5*e - 4*b*c^3*d*e^5 + b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^{(1/2)} + 4*c^3*d*e^5*(b^2 - 4*a*c)^{(1/2)} + 2*a^3*d^6*x*(b^2 - 4*a*c)^{(1/2)} - 8*a^2*b*c*d^5*e - 8*a*c^3*d*e^5*x + 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^{(1/2)} - 4*a^2*c*d^5*e*(b^2 - 4*a*c)^{(1/2)} + 20*a*b*c^2*d^3*e^3 - 6*a*b^2*c*d^4*e^2 + 6*a*b^3*d^4*e^2*x - 2*a^2*b^2*d^5*e*x + 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2 - 4*a*c)^{(1/2)} - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^{(1/2)} + 2*b^2*c*d^3*e^3*(b^2 - 4*a*c)^{(1/2)} - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^{(1/2)} + 32*a^2*c^2*d^3*e^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^{(1/2)} - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 3*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^{(1/2)} + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^{(1/2)} - 6*a^2*b*d^5*e*x*(b^2 - 4*a*c)^{(1/2)} - 6*a*b*c^2*d^2*e^4*x - 2*a*b^2*c*d^3*e^3*x - 23*a^2*b*c*d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^{(1/2)})*(b^4*d^2 - 4*a*c^3*e^2 - b^3*d^2*(b^2 - 4*a*c)^{(1/2)} + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*a*b^2*c*d^2 - b*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} + 8*a*b*c^2*d*e + 3*a*b*c*d^2*(b^2 - 4*a*c)^{(1/2)} - 4*a*c^2*d*e*(b^2 - 4*a*c)^{(1/2)} + 2*b^2*c*d*e*(b^2 -$

$$\frac{4ac\sqrt{c}}{(2(4a^4cd^4 - a^3b^2d^4 + 4a^2c^3e^4 - ab^2c^2e^4 - ab^4d^2e^2 + 2a^2b^3d^3e + 8a^3c^2d^2e^2 + 2ab^3cde^3 - 8a^3bcd^3e - 8a^2b^2c^2de^3 + 2a^2b^2cd^2e^2)) + d^3/(e^2(d + ex)(ad^2 + ce^2 - bde))}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)

[Out] Timed out

$$3.73 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx$$

Optimal. Leaf size=194

$$\frac{(-2c(ad^2 - ce^2) + b^2d^2 - 2bcde) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))^2} - \frac{d^2}{e(d+ex)(ad^2 - bde + ce^2)} - \frac{d(bd - 2ce) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2}$$

[Out] $-d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)*\ln(e*x+d)/(a*d^2-e*(b*d-c*e))^2-1/2*d*(b*d-2*c*e)*\ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(b^2*d^2-2*b*c*d*e-2*c*(a*d^2-c*e^2))*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1445, 1628, 634, 618, 206, 628}

$$\frac{(-2c(ad^2 - ce^2) + b^2d^2 - 2bcde) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))^2} - \frac{d^2}{e(d+ex)(ad^2 - bde + ce^2)} - \frac{d(bd - 2ce) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] $-(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*(a*d^2 - c*e^2))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*\operatorname{Log}[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (d*(b*d - 2*c*e)*\operatorname{Log}[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1445

```
Int[((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_) + (e_)*(x
_)^(n_))^(q_), x_Symbol] := Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p
_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)(d+ex)^2} dx &= \int \frac{x^2}{(d+ex)^2(c+bx+ax^2)} dx \\
&= \int \left(\frac{d^2}{(ad^2 - e(bd - ce))(d+ex)^2} + \frac{de(bd - 2ce)}{(ad^2 - e(bd - ce))^2(d+ex)} + \frac{-c(ad^2 - ce^2)}{(ad^2 - e(bd - ce))} \right) dx \\
&= -\frac{d^2}{e(ad^2 - bde + ce^2)(d+ex)} + \frac{d(bd - 2ce)\log(d+ex)}{(ad^2 - e(bd - ce))^2} + \int \frac{-c(ad^2 - ce^2) - ad(bd - 2ce)x}{c+bx+ax^2} dx \\
&= -\frac{d^2}{e(ad^2 - bde + ce^2)(d+ex)} + \frac{d(bd - 2ce)\log(d+ex)}{(ad^2 - e(bd - ce))^2} - \frac{(d(bd - 2ce)) \int \frac{b+2ax}{c+bx+ax^2} dx}{2(ad^2 - e(bd - ce))^2} \\
&= -\frac{d^2}{e(ad^2 - bde + ce^2)(d+ex)} + \frac{d(bd - 2ce)\log(d+ex)}{(ad^2 - e(bd - ce))^2} - \frac{d(bd - 2ce)\log(c+bx+ax^2)}{2(ad^2 - e(bd - ce))} \\
&= -\frac{d^2}{e(ad^2 - bde + ce^2)(d+ex)} - \frac{(b^2d^2 - 2bcde - 2c(ad^2 - ce^2)) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}(ad^2 - e(bd - ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 159, normalized size = 0.82

$$\frac{2(2c(ce^2 - ad^2) + b^2d^2 - 2bcde) \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) - \frac{2d^2(ad^2 + e(ce - bd))}{e(d+ex)} - d(bd - 2ce)\log(x(ax+b)+c) + 2d(bd - 2ce)\log(d+ex)}{2(ad^2 + e(ce - bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2), x]

[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [B] time = 13.25, size = 1120, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d \\ & *e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*\sqrt{b^2 - 4*a*c}*\log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - \sqrt{b^2 - 4*a*c}*(2*a*x + b))/(a*x^2 + b*x + \\ & c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - \\ & 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*\log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - \\ & 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*\log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)* \\ & *d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*\log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*\log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x)] \end{aligned}$$

giac [A] time = 0.35, size = 331, normalized size = 1.71

$$\frac{(b^2 d^2 e^2 - 2 a c d^2 e^2 - 2 b c d e^3 + 2 c^2 e^4) \arctan\left(\frac{\left(2 a d - \frac{2 a d^2}{x e + d} - b e + \frac{2 b d e}{x e + d} - \frac{2 c e^2}{x e + d}\right) e^{-1}}{\sqrt{-b^2 + 4 a c}}\right) e^{-2}}{(a^2 d^4 - 2 a b d^3 e + b^2 d^2 e^2 + 2 a c d^2 e^2 - 2 b c d e^3 + c^2 e^4) \sqrt{-b^2 + 4 a c}} \frac{d^2 e}{(a d^2 e^2 - b d e^3 + c e^4)(x e + d)} \quad (t)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*\arctan((2*a*d - 2*a \\ & *d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^{-1})/\sqrt{- \\ & b^2 + 4*a*c))*e^{-2})/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 \\ & - 2*b*c*d*e^3 + c^2*e^4)*\sqrt{-b^2 + 4*a*c}) - d^2*e/((a*d^2*e^2 - b*d*e^3 \\ & + c*e^4)*(x*e + d)) - 1/2*(b*d^2 - 2*c*d*e)*\log(a - 2*a*d/(x*e + d) + a*d^2 \\ & / (x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2* \\ & d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4) \end{aligned}$$

maple [B] time = 0.01, size = 389, normalized size = 2.01

$$-\frac{2ac d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} + \frac{b^2 d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} - \frac{2bcde \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} + \frac{2c^2 e^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x)

[Out] $-1/2/(a*d^2-b*d*e+c*e^2)^2*\ln(a*x^2+b*x+c)*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2*\ln(a*x^2+b*x+c)*c*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*c*d^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b*c*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*c^2*e^2-d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d^2/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*b-2*d/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*c*e$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 6.09, size = 1585, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)

[Out] $(\log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^{(1/2)} + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x + 2*a*b^2*d^4*(b^2 - 4*a*c)^{(1/2)} - a^2*c*d^4*(b^2 - 4*a*c)^{(1/2)} - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d^3*e^3*x - 2*b*c^2*d^3*e^3*(b^2 - 4*a*c)^{(1/2)} + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^{(1/2)} - b*c^2*e^4*x*(b^2 - 4*a*c)^{(1/2)})$

$$\begin{aligned}
& + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^{(1/2)} + b^2*c*d^2*e^2 \\
& * (b^2 - 4*a*c)^{(1/2)} + b^3*d^2*e^2*x*(b^2 - 4*a*c)^{(1/2)} + 28*a^2*c^2*d^2* \\
& e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^{(1/2)} - 12*a*b*c^2*d*e^3*x - 12*a^2*b* \\
& c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^{(1/2)} + 8*a*c^2*d*e^3*x*(b^2 - 4* \\
& a*c)^{(1/2)} - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^{(1/2)} - 2*b^2*c*d*e^3*x*(b^2 - 4 \\
& *a*c)^{(1/2)} + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^{(1/2)}*(d^2*(b^{3/2} + (b^2*(b^2 \\
& - 4*a*c)^{(1/2}))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^{(1/2})) + d*(b^2*e + \\
& b*e*(b^2 - 4*a*c)^{(1/2}))) + c^2*(e^2*(b^2 - 4*a*c)^{(1/2)} + 4*a*d*e)))/(4*a^ \\
& 3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2 \\
& *d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3* \\
& e + 2*a*b^2*c*d^2*e^2) - (\log(2*a*b^3*d^4 + b*c^3*e^4 + c^3*e^4*(b^2 - 4*a* \\
& c)^{(1/2)} + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4 \\
& *x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3 \\
& *c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)^{(1/2)} + a^2*c*d^4*(b^2 \\
& - 4*a*c)^{(1/2)} - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x + 2*b \\
& *c^2*d*e^3*(b^2 - 4*a*c)^{(1/2)} - 3*a^2*b*d^4*x*(b^2 - 4*a*c)^{(1/2)} + b*c^2* \\
& e^4*x*(b^2 - 4*a*c)^{(1/2)} + 10*a*b*c^2*d^2*e^2 - 14*a*c^2*d^2*e^2*(b^2 - 4* \\
& a*c)^{(1/2)} - b^2*c*d^2*e^2*(b^2 - 4*a*c)^{(1/2)} - b^3*d^2*e^2*x*(b^2 - 4*a*c \\
&)^{(1/2)} + 28*a^2*c^2*d^2*e^2*x + 10*a*b*c*d^3*e*(b^2 - 4*a*c)^{(1/2)} - 12*a* \\
& b*c^2*d*e^3*x - 12*a^2*b*c*d^3*e*x + 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^{(1/2)} - \\
& 8*a*c^2*d*e^3*x*(b^2 - 4*a*c)^{(1/2)} + 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^{(1/2)} + \\
& 2*b^2*c*d*e^3*x*(b^2 - 4*a*c)^{(1/2)} - 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^{(1/2)} \\
&))*(c*(d^2*(2*a*b - a*(b^2 - 4*a*c)^{(1/2})) + d*(b^2*e - b*e*(b^2 - 4*a*c)^{(\\
& 1/2}))) - d^2*(b^{3/2} - (b^2*(b^2 - 4*a*c)^{(1/2}))/2) + c^2*(e^2*(b^2 - 4*a*c) \\
& ^{(1/2)} - 4*a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 \\
& - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c \\
& ^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) + (\log(d + e*x)*(b*d^2 - 2* \\
& c*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a* \\
& c*d^2*e^2) - d^2/(e*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)

[Out] Timed out

$$3.74 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x(d+ex)^2} dx$$

Optimal. Leaf size=183

$$\frac{(ad(bd - 4ce) + bce^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (ad^2 - ce^2) \log(ax^2 + bx + c)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \frac{(ad^2 - ce^2) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2} + \frac{d}{(d + ex)(ad^2 - bde + ce^2)} - \frac{(ad^2 - ce^2)}{(ad^2 - e(bd - ce))}$$

[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^2+1/2*(a*d^2-c*e^2)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2+(b*c*e^2+a*d*(b*d-4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)

Rubi [A] time = 0.24, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{(ad(bd - 4ce) + bce^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (ad^2 - ce^2) \log(ax^2 + bx + c)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \frac{(ad^2 - ce^2) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2} + \frac{d}{(d + ex)(ad^2 - bde + ce^2)} - \frac{(ad^2 - ce^2)}{(ad^2 - e(bd - ce))}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2), x]

[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628


```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (b_)*(x_) +
(c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x(d+ex)^2} dx &= \int \frac{x}{(d+ex)^2(c+bx+ax^2)} dx \\
&= \int \left(\frac{de}{(-ad^2 + e(bd - ce))(d+ex)^2} + \frac{e(-ad^2 + ce^2)}{(ad^2 - e(bd - ce))^2(d+ex)} + \frac{ce(2ad - be)}{(ad^2 - e(bd - ce))^2} \right) dx \\
&= \frac{d}{(ad^2 - bde + ce^2)(d+ex)} - \frac{(ad^2 - ce^2)\log(d+ex)}{(ad^2 - e(bd - ce))^2} + \frac{\int \frac{ce(2ad-be) + a(ad^2 - ce^2)x}{c+bx+ax^2} dx}{(ad^2 - e(bd - ce))^2} \\
&= \frac{d}{(ad^2 - bde + ce^2)(d+ex)} - \frac{(ad^2 - ce^2)\log(d+ex)}{(ad^2 - e(bd - ce))^2} + \frac{(ad^2 - ce^2) \int \frac{b+2ax}{c+bx+ax^2} dx}{2(ad^2 - e(bd - ce))^2} \\
&= \frac{d}{(ad^2 - bde + ce^2)(d+ex)} - \frac{(ad^2 - ce^2)\log(d+ex)}{(ad^2 - e(bd - ce))^2} + \frac{(ad^2 - ce^2)\log(c+bx+ax^2)}{2(ad^2 - e(bd - ce))^2} \\
&= \frac{d}{(ad^2 - bde + ce^2)(d+ex)} + \frac{(bce^2 + ad(bd - 4ce)) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2 - 4ac}}\right)}{\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2} - \frac{(ad^2 - ce^2)\log(c+bx+ax^2)}{(ad^2 - e(bd - ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 148, normalized size = 0.81

$$\frac{-\frac{2(ad(bd-4ce)+bce^2)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + (ad^2 - ce^2)\log(x(ax+b)+c) + \frac{2d(ad^2+e(ce-bd))}{d+ex} + (2ce^2 - 2ad^2)\log(d+ex)}{2(ad^2 + e(ce - bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2), x]

[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2 + 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)]/(2*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [B] time = 14.16, size = 1059, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")

[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]

giac [A] time = 0.37, size = 323, normalized size = 1.77

$$\frac{1}{2} \left(\frac{2 \left(abd^2e - 4acde^2 + bce^3 \right) \arctan \left(\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d} - \frac{2ce^2}{xe+d} \right) e^{(-1)}}{\sqrt{-b^2+4ac}} \right) e^{(-2)}}{\left(a^2d^4 - 2abd^3e + b^2d^2e^2 + 2acd^2e^2 - 2bcde^3 + c^2e^4 \right) \sqrt{-b^2+4ac}} - \frac{\left(ad^2 - ce^2 \right) \log \left(a - \frac{2ad}{xe+d} + \frac{ad^2}{(xe+d)^2} + \frac{1}{x} \right)}{a^2d^4e - 2abd^3e^2 + b^2d^2e^3 + 2acd^2e^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")

[Out] -1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - (a*d^2 - c*e^2)*log(a - 2*a*d/(x*e + d) + a*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*d^4*e - 2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 + c^2*e^5) - 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))e

maple [A] time = 0.01, size = 328, normalized size = 1.79

$$-\frac{ab d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} + \frac{4acde \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} - \frac{bc e^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} - \frac{a d^2 \ln(ex + c)}{(a d^2 - deb + c e^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x)

[Out] 1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)*d^2-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*e^2-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*e^2+d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c*e^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 8.07, size = 1768, normalized size = 9.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)

[Out] d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2*d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 - 4*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*c

$$\begin{aligned}
& c*d*e^3*(b^2 - 4*a*c)^{(3/2)} + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^{(5/2)} - 5*b^2*c* \\
& e^4*x*(b^2 - 4*a*c)^{(3/2)} + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^{(3/2)} + 16*a*b^3*c^ \\
& 2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e - 64* \\
& a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a*b^5*d^2*e^ \\
& 2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3*e*x - 6*a*b \\
& ^2*d^2*e^2*x*(b^2 - 4*a*c)^{(3/2)} - 160*a^2*b^2*c^2*d*e^3*x - 56*a^2*b^3*c*d \\
& ^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(\\
& b^2 - 4*a*c)^{(3/2)} + 96*a^3*b^2*c*d^3*e*x*(b^2*((a*d^2)/2 - (c*e^2)/2) - b \\
& *((a*d^2*(b^2 - 4*a*c)^{(1/2)))/2 + (c*e^2*(b^2 - 4*a*c)^{(1/2)))/2) - 2*a^2*c* \\
& d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^{(1/2)))/(4*a^3*c*d^4 + 4*a*c^3* \\
& e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3 \\
& *d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2* \\
& e^2) - (\log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*c^4*e^4 + 8*b^4*c^2*e^4 \\
& - 56*a^3*b^2*c*d^4 - 56*a*b^2*c^3*e^4 + 4*a^3*b^3*d^4*x - 320*a^3*c^3*d^2* \\
& e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^{(5/2)} - 8*c*d*e^3*(b^2 - 4*a*c)^{(5/2)} - 3*c*e \\
& ^4*x*(b^2 - 4*a*c)^{(5/2)} + 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^{(3/2)} \\
& - 8*b*c^2*e^4*(b^2 - 4*a*c)^{(3/2)} + 12*a^3*d^4*x*(b^2 - 4*a*c)^{(3/2)} - 6*b* \\
& d*e^3*x*(b^2 - 4*a*c)^{(5/2)} - 16*a^4*b*c*d^4*x + 112*a^2*b^2*c^2*d^2*e^2 - \\
& 8*a*b^2*d^3*e*(b^2 - 4*a*c)^{(3/2)} + 8*b^2*c*d*e^3*(b^2 - 4*a*c)^{(3/2)} + 10* \\
& a*d^2*e^2*x*(b^2 - 4*a*c)^{(5/2)} - 5*b^2*c*e^4*x*(b^2 - 4*a*c)^{(3/2)} + 6*b^3 \\
& *d*e^3*x*(b^2 - 4*a*c)^{(3/2)} - 16*a*b^3*c^2*d*e^3 - 8*a*b^4*c*d^2*e^2 + 64* \\
& a^2*b*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e + 64*a^3*b*c^2*d^3*e - 60*a*b^3*c^2*e^ \\
& 4*x + 112*a^2*b*c^3*e^4*x - 4*a*b^5*d^2*e^2*x + 8*a^2*b^4*d^3*e*x - 256*a^3 \\
& *c^3*d*e^3*x + 256*a^4*c^2*d^3*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^{(3/2)} \\
& + 160*a^2*b^2*c^2*d*e^3*x + 56*a^2*b^3*c*d^2*e^2*x - 160*a^3*b*c^2*d^2*e^2* \\
& x - 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(b^2 - 4*a*c)^{(3/2)} - 96*a^3*b^2*c \\
& *d^3*e*x*(b*((a*d^2*(b^2 - 4*a*c)^{(1/2)))/2 + (c*e^2*(b^2 - 4*a*c)^{(1/2)))/2 \\
&) + b^2*((a*d^2)/2 - (c*e^2)/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 - 2*a*c*d*e*(b^ \\
& 2 - 4*a*c)^{(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - \\
& b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^ \\
& 2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (\log(d + e*x)*(a*d^2 - c*e \\
& ^2))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*c*d \\
& ^2*e^2)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)

[Out] Timed out

$$3.75 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^2(d+ex)^2} dx$$

Optimal. Leaf size=189

$$\frac{(2a^2d^2 - 2ae(bd + ce) + b^2e^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))^2} - \frac{e}{(d+ex)(ad^2 - bde + ce^2)} - \frac{e(2ad - be) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2} + \frac{e(2ad - be) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2}$$

[Out] $-e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)*\ln(e*x+d)/(a*d^2-e*(b*d-c*e))^{2-1/2}*e*(2*a*d-b*e)*\ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^{2-2}*a*e*(b*d+c*e)*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/(a*d^2-e*(b*d-c*e))^{2/(-4*a*c+b^2)^{(1/2)}}$

Rubi [A] time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1569, 709, 800, 634, 618, 206, 628}

$$\frac{(2a^2d^2 - 2ae(bd + ce) + b^2e^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac} (ad^2 - e(bd - ce))^2} - \frac{e}{(d+ex)(ad^2 - bde + ce^2)} - \frac{e(2ad - be) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2} + \frac{e(2ad - be) \log(ax^2 + bx + c)}{2(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2), x]`

[Out] $-(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b*d + c*e))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*\operatorname{Log}[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 - (e*(2*a*d - b*e)*\operatorname{Log}[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 709

```
Int[((d_) + (e_)*(x_))^(m_)/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol
] := Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))/((a_) + (b_)*(x_) +
(c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1569

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(mn_) + (c_)*(x_)^(mn2_))^(p_)*((d_)
+ (e_)*(x_)^(n_))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
+ b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^2(d+ex)^2} dx &= \int \frac{1}{(d+ex)^2(c+bx+ax^2)} dx \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} + \frac{\int \frac{ad-be-aex}{(d+ex)(c+bx+ax^2)} dx}{ad^2 - bde + ce^2} \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} + \frac{\int \left(\frac{e^2(2ad-be)}{(ad^2-e(bd-ce))(d+ex)} + \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)}{(ad^2-e(bd-ce))(c+bx+ax^2)} \right) dx}{ad^2 - bde + ce^2} \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} + \frac{e(2ad - be) \log(d+ex)}{(ad^2 - e(bd - ce))^2} + \frac{\int \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)}{c+bx+ax^2} dx}{(ad^2 - e(bd - ce))^2} \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} + \frac{e(2ad - be) \log(d+ex)}{(ad^2 - e(bd - ce))^2} - \frac{(e(2ad - be)) \int \frac{b+2ax}{c+bx+ax^2} dx}{2(ad^2 - e(bd - ce))^2} \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} + \frac{e(2ad - be) \log(d+ex)}{(ad^2 - e(bd - ce))^2} - \frac{e(2ad - be) \log(c+bx+ax^2)}{2(ad^2 - e(bd - ce))} \\
&= -\frac{e}{(ad^2 - bde + ce^2)(d+ex)} - \frac{(2a^2d^2 + b^2e^2 - 2ae(bd + ce)) \tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \dots
\end{aligned}$$

Mathematica [A] time = 0.21, size = 151, normalized size = 0.80

$$\frac{2(2a^2d^2 - 2ae(bd+ce) + b^2e^2) \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) - \frac{2e(ad^2+e(ce-bd))}{d+ex} + e(be - 2ad) \log(x(ax+b) + c) - 2e(be - 2ad) \log(d+ex)}{2(ad^2 + e(ce - bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2), x]

[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)

fricas [B] time = 7.01, size = 1079, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a \\ & *c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - \\ & 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*\sqrt{b^2 - 4*a*c}*\log((2*a^2*x^2 + 2*a \\ & *b*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c}*(2*a*x + b))/(a*x^2 + b*x + c)) + (2 \\ & *(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e \\ & ^2 - (b^3 - 4*a*b*c)*e^3)*x)*\log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)* \\ & d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c \\ &)*e^3)*x)*\log(e*x + d)]/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^ \\ & 4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 \\ & + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^ \\ & 2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c \\ & ^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - \\ & 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d^3 - 2*a*b*d^ \\ & 2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3) \\ & *x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*a*x + b)/(b^2 - 4*a*c) \\ &) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c \\ & c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*\log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a \\ & ^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4 \\ & *a*b*c)*e^3)*x)*\log(e*x + d)]/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b \\ & *c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d \\ & ^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - \\ & 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4 \\ & *a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x] \end{aligned}$$

giac [A] time = 0.35, size = 331, normalized size = 1.75

$$\frac{(2a^2d^2e^2 - 2abde^3 + b^2e^4 - 2ace^4) \arctan\left(\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d} - \frac{2ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}\right) e^{(-2)}}{(a^2d^4 - 2abd^3e + b^2d^2e^2 + 2acd^2e^2 - 2bcde^3 + c^2e^4)\sqrt{-b^2+4ac}} - \frac{(2ade - be^2) \log\left(-a + \frac{2ad}{xe+d} - \frac{2ce^2}{xe+d}\right)}{2(a^2d^4 - 2abd^3e + b^2d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*\arctan(-(2*a*d - 2*a*d \\ & ^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^{(-1)}/\sqrt{-b^ \\ & 2 + 4*a*c})*e^{(-2)}/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - \\ & 2*b*c*d*e^3 + c^2*e^4)*\sqrt{-b^2 + 4*a*c}) - 1/2*(2*a*d*e - b*e^2)*\log(-a + \\ & 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - \\ & c*e^2/(x*e + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2 \\ & *b*c*d*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)) \end{aligned}$$

maple [B] time = 0.01, size = 386, normalized size = 2.04

$$\frac{2a^2d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2}} - \frac{2abde \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2}} - \frac{2ace^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2}} + \frac{b^2e^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x)

[Out] $-1/(a*d^2-b*d*e+c*e^2)^2*a*\ln(a*x^2+b*x+c)*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*\ln(a*x^2+b*x+c)*b*e^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*d*e-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*c*e^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*e^2-e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+2*e/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*a*d-e^2/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*b$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 8.11, size = 1782, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)

[Out] $(\log(c*e^4*(b^2 - 4*a*c)^{(5/2)} - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^2 - 4*a*c)^{(3/2)} - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^{(3/2)} + 60*a*b^3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*a^3*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 10*b*d*e^3*(b^2 - 4*a*c)^{(5/2)} + 4*b*e^4*x*(b^2 - 4*a*c)^{(5/2)} + 16*a^4*b*c*d^4 + 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^{(5/2)} + 7*b^2*c*e^4*(b^2 - 4*a*c)^{(5/2)})$

$$\begin{aligned}
& a^2c^3 \int \frac{1}{(a+c/x^2+b/x)/x^2/(e^x+d)^2} dx \\
& = \frac{1}{2} \left(\frac{2a^2c^3 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 24a^2b^2c^3d^3 + 64a^2b^4c^3d^2e^3 + 32a^2b^5c^3d^2e^3x - 8a^2b^2d^3e^3 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 32a^3d^3e^3x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 96a^3b^2c^3d^3e^3 + 16a^3b^3d^3e^3x + 18a^3b^2d^2e^2 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 160a^2b^2c^2d^2e^3 - 56a^2b^3c^2d^2e^2 + 160a^3b^2c^2d^2e^2 - 136a^2b^2c^2d^2e^4x - 40a^2b^4d^2e^2x - 448a^4c^2d^2e^2x + 48a^2b^2d^2e^2 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 272a^3b^2c^2d^2e^2x - 64a^4b^2c^2d^3e^3x - 24a^2b^2d^2e^3x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 240a^2b^3c^2d^2e^3x + 448a^3b^2c^2d^2e^3x \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + e^2(b^2d - b^2d + b^2d - 4a^2c) \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - e^2(b^3/2 - (b^2(b^2-4ac)^{1/2})/2) + a^2(d^2(b^2-4ac)^{1/2} - 4c^2d^2e) \right) / (4a^3c^2d^4 + 4a^2c^3e^4 - a^2b^2d^4 - b^2c^2e^4 - b^4d^2e^2 + 8a^2c^2d^2e^2 + 2a^2b^3d^3e + 2b^3c^2d^2e^3 - 8a^2b^2c^2d^2e^3 - 8a^2b^2c^2d^3e + 2a^2b^2c^2d^2e^2) - (\log(d+ex))(b^2e^2 - 2a^2d^2e) / (a^2d^4 + c^2e^4 + b^2d^2e^2 - 2a^2b^2d^3e - 2b^2c^2d^2e^3 + 2a^2c^2d^2e^2) - (\log(c^2e^4(b^2-4ac)^{5/2} + 8b^5c^2e^4 + 8b^6e^4x - 4a^3d^4(b^2-4ac)^{3/2} + 4a^3b^3d^4 + 4b^3e^4x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 60a^2b^3c^2e^4 + 112a^2b^2c^3e^4 - 4a^2b^5d^2e^2 + 8a^2b^4d^3e - 256a^3c^3d^2e^3 + 256a^4c^2d^3e + 8a^4b^2d^4x - 32a^3c^3e^4x + 10b^2d^2e^3 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 4b^2e^4x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 16a^4b^2c^2d^4 - 32a^5c^2d^4x - 14a^2d^2e^2 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 7b^2c^2e^4 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 10b^3d^2e^3 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 8a^2d^2e^3x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 24a^2b^4c^2d^2e^3 - 64a^2b^4c^2e^4x - 32a^2b^5d^2e^3x - 8a^2b^2d^3e^3 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 32a^3d^3e^3x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 96a^3b^2c^2d^3e^3 - 16a^3b^3d^3e^3x + 18a^2b^2d^2e^2 \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 160a^2b^2c^2d^2e^3 + 56a^2b^3c^2d^2e^2 - 160a^3b^2c^2d^2e^2 + 136a^2b^2c^2d^2e^4x + 40a^2b^4d^2e^2x + 448a^4c^2d^2e^2x + 48a^2b^2d^2e^2x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) - 272a^3b^2c^2d^2e^2x + 64a^4b^2c^2d^3e^3x - 24a^2b^2d^2e^3x \sqrt{b^2-4ac} \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + 240a^2b^3c^2d^2e^3x - 448a^3b^2c^2d^2e^3x \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + e^2(b^3/2 + (b^2(b^2-4ac)^{1/2})/2) - a^2(e^2(2b^2c + c^2(b^2-4ac)^{1/2}) + e^2(b^2d + b^2d + b^2d - 4a^2c) \operatorname{erf}\left(\frac{b-2ax}{\sqrt{b^2-4ac}}\right) + a^2(d^2(b^2-4ac)^{1/2} + 4c^2d^2e) \right) / (4a^3c^2d^4 + 4a^2c^3e^4 - a^2b^2d^4 - b^2c^2e^4 - b^4d^2e^2 + 8a^2c^2d^2e^2 + 2a^2b^3d^3e + 2b^3c^2d^2e^3 - 8a^2b^2c^2d^2e^3 - 8a^2b^2c^2d^3e + 2a^2b^2c^2d^2e^2) - e / ((d+ex)(a^2d^2 + c^2e^2 - b^2d^2e))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)

[Out] Timed out

$$3.76 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right)x^3(d+ex)^2} dx$$

Optimal. Leaf size=248

$$-\frac{(a^2d^2 - ae(2bd + ce) + b^2e^2) \log(ax^2 + bx + c)}{2c(ad^2 - e(bd - ce))^2} + \frac{(a^2d(bd + 4ce) - abe(2bd + 3ce) + b^3e^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}(ad^2 - e(bd - ce))^2} + d(\dots)$$

[Out] $e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)+\ln(x)/c/d^2-e^2*(3*a*d^2-e*(2*b*d-c*e))*\ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))^2-1/2*(a^2*d^2+b^2*e^2-a*e*(2*b*d+c*e))*\ln(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))^2+(b^3*e^2-a*b*e*(2*b*d+3*c*e)+a^2*d*(b*d+4*c*e))*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/c/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.41, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$-\frac{(a^2d^2 - ae(2bd + ce) + b^2e^2) \log(ax^2 + bx + c)}{2c(ad^2 - e(bd - ce))^2} + \frac{(a^2d(bd + 4ce) - abe(2bd + 3ce) + b^3e^2) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}(ad^2 - e(bd - ce))^2} + d(\dots)$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2), x]

[Out] $e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*e) + a^2*d*(b*d + 4*c*e))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]])/(c*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + \operatorname{Log}[x]/(c*d^2) - (e^2*(3*a*d^2 - e*(2*b*d - c*e))*\operatorname{Log}[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 + b^2*e^2 - a*e*(2*b*d + c*e))*\operatorname{Log}[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d - c*e))^2)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x, x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x_Symbol] \rightarrow \text{Dist}[\frac{2*c*d - b*e}{2*c}, \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 893

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)^{(m_.)} * ((f_.) + (g_.)*(x_.)^{(n_.)} * ((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)})}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(p_.)}}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[p] \&\& ((\text{EqQ}[p, 1] \&\& \text{IntegersQ}[m, n]) \parallel (\text{ILtQ}[m, 0] \&\& \text{ILtQ}[n, 0]))$

Rule 1569

$\text{Int}[(x_.)^{(m_.)} * ((a_.) + (b_.)*(x_.)^{(mn_.)} + (c_.)*(x_.)^{(mn2_.)})^{(p_.)} * ((d_.) + (e_.)*(x_.)^{(n_.)})^{(q_.)}], x_Symbol] \rightarrow \text{Int}[x^{(m - 2*n*p)} * (d + e*x^n)^q * (c + b*x^n + a*x^{(2*n)})^p, x] /;$ $\text{FreeQ}[\{a, b, c, d, e, m, n, q\}, x] \&\& \text{EqQ}[mn, -n] \&\& \text{EqQ}[mn2, 2*mn] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^3 (d + ex)^2} dx &= \int \frac{1}{x(d + ex)^2 (c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{cd^2 x} + \frac{e^3}{d(-ad^2 + e(bd - ce))(d + ex)^2} + \frac{e^3(-3ad^2 + e(2bd - ce))}{d^2(ad^2 - e(bd - ce))^2(d + ex)} + \dots \right) dx \\
&= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 - e(2bd - ce))\log(d + ex)}{d^2(ad^2 - e(bd - ce))^2} + \dots \\
&= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 - e(2bd - ce))\log(d + ex)}{d^2(ad^2 - e(bd - ce))^2} - \dots \\
&= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2 - e(2bd - ce))\log(d + ex)}{d^2(ad^2 - e(bd - ce))^2} - \dots \\
&= \frac{e^2}{d(ad^2 - e(bd - ce))(d + ex)} + \frac{(b^3e^2 - abe(2bd + 3ce) + a^2d(bd + 4ce))\tanh^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{b^2 - 4ac}(ad^2 - e(bd - ce))^2}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 246, normalized size = 0.99

$$\frac{(-a^2d^2 + ae(2bd + ce) - b^2e^2)\log(x(ax + b) + c)}{2c(ad^2 + e(ce - bd))^2} - \frac{(a^2d(bd + 4ce) - abe(2bd + 3ce) + b^3e^2)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}(ad^2 + e(ce - bd))^2} + \frac{1}{d(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2), x]

[Out] $e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 + e*(-(b*d) + c*e))^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.41, size = 391, normalized size = 1.58

$$\frac{(a^2bd^2e^2 - 2ab^2de^3 + 4a^2cde^3 + b^3e^4 - 3abce^4) \arctan\left(\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d} - \frac{2ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}\right) e^{(-2)}}{(a^2cd^4 - 2abcd^3e + b^2cd^2e^2 + 2ac^2d^2e^2 - 2bc^2de^3 + c^3e^4)\sqrt{-b^2+4ac}} - \frac{(a^2d^2 - 2abde + b^2e^2)}{2(a^2cd^4 - 2abcd^3e + b^2cd^2e^2 + 2ac^2d^2e^2 - 2bc^2de^3 + c^3e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")

[Out] $-(a^2b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*\arctan\left(\frac{2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d)}{\sqrt{-b^2 + 4*a*c}}\right)*e^{(-1)}/\sqrt{-b^2 + 4*a*c})*e^{(-2)}/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*\sqrt{-b^2 + 4*a*c}) - 1/2*(a^2*d^2 - 2*a*b*d*e + b^2*e^2 - a*c*e^2)*\log(a - 2*a*d/(x*e + d) + a*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d)) + \log(\text{abs}(-d/(x*e + d) + 1))/(c*d^2)$

maple [B] time = 0.01, size = 589, normalized size = 2.38

$$\frac{a^2b d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2} c} - \frac{4a^2de \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2}} + \frac{2a b^2de \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(a d^2 - deb + c e^2)^2 \sqrt{4ac-b^2} c} + \frac{3ab e^2}{(a d^2 - deb + c e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x)

[Out] $-1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*\ln(a*x^2+b*x+c)*d^2+1/(a*d^2-b*d*e+c*e^2)^2/c*a*\ln(a*x^2+b*x+c)*b*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*a*\ln(a*x^2+b*x+c)*e^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c*\ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*b*d^2-4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a^2*d*e+2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*b*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*e^2+\ln(x)/c/d^2+e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-3*e^2/(a*d^2-b*d*e+c*e^2)^2*\ln(e*x+d)*a+2*e^3/(a*d^2-b*d*e+c*e^2)^2/d*\ln(e*x+d)*b-e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*\ln(e*x+d)*c$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 25.28, size = 3510, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)

[Out]
$$\left(\log\left(\frac{a^4 e^4}{d(a d^2 + c e^2 - b d e)^2}\right) + \frac{a^4 e^5 x}{d^2(a d^2 + c e^2 - b d e)^2}\right) - \frac{((a e^3(3 a^3 b d^4 + b^3 c e^4 - b^4 d e^3 + 5 a b^3 d^2 e^2 - 7 a^2 b^2 d^3 e + 8 a^2 c^2 d e^3 - 3 a b c^2 e^4 + 9 a^3 c d^3 e - a b^2 c d e^3 - 8 a^2 b c d^2 e^2)) / (d^2(a d^2 + c e^2 - b d e)^2) + ((a e(a^3 b d^5 - 4 a c^3 e^5 + b^2 c^2 e^5 - b^4 d^2 e^3 + 3 a b^3 d^3 e^2 - 3 a^2 b^2 d^4 e - 8 a^2 c^2 d^2 e^3 + 4 a^3 c d^4 e - b^3 c d e^4 + 4 a b c^2 d e^4 + 6 a b^2 c d^2 e^3 - 9 a^2 b c d^3 e^2)) / (a d^3 - b d^2 e + c d e^2) + (a e x(3 a^4 d^5 + 2 b^3 c e^5 - 4 b^4 d e^4 + 9 a b^3 d^2 e^3 + 4 a^2 c^2 d e^4 + 19 a^3 c d^3 e^2 - 3 a^2 b^2 d^3 e^2 - 8 a b c^2 e^5 - 5 a^3 b d^4 e + 15 a b^2 c d e^4 - 36 a^2 b c d^2 e^3)) / (a d^3 - b d^2 e + c d e^2) - (a e(b^4 e^2 - 4 a^3 c d^2 + b^3 e^2(b^2 - 4 a c))^{1/2} + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 + a^2 b d^2(b^2 - 4 a c))^{1/2} + 8 a^2 b c d e - 3 a b c e^2(b^2 - 4 a c)^{1/2} - 2 a b^2 d e(b^2 - 4 a c)^{1/2} + 4 a^2 c d e(b^2 - 4 a c)^{1/2}) * (4 a^2 c^2 d^3 e + b^2 c^2 d e^3 + b^3 c d^2 e^2 + 2 a^2 b^2 d^4 x + 2 b^2 c^2 e^4 x + 2 b^4 d^2 e^2 x + a^2 b c d^4 - 4 a c^3 d e^3 - 6 a^3 c d^4 x - 8 a c^3 e^4 x - 2 a b^2 c d^3 e - 4 a b^3 d^3 e x - 2 b^3 c d e^3 x - 3 a b c^2 d^2 e^2 - 6 a^2 c^2 d^2 e^2 x + 8 a b c^2 d e^3 x + 14 a^2 b c d^3 e x - 6 a b^2 c d^2 e^2 x)) / (2 c(4 a c - b^2)(a d^2 + c e^2 - b d e)^2) * (b^4 e^2 - 4 a^3 c d^2 + b^3 e^2(b^2 - 4 a c)^{1/2} + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 + a^2 b d^2(b^2 - 4 a c)^{1/2} + 8 a^2 b c d e - 3 a b c e^2(b^2 - 4 a c)^{1/2} - 2 a b^2 d e(b^2 - 4 a c)^{1/2} + 4 a^2 c d e(b^2 - 4 a c)^{1/2})) / (2 c(4 a c - b^2)(a d^2 + c e^2 - b d e)^2) + (a e^3 x(9 a^4 d^4 + b^4 e^4 + 2 a^2 c^2 e^4 - 6 a^3 c d^2 e^2 + 8 a^2 b^2 d^2 e^2 - 4 a b^2 c e^4 - 4 a b^3 d e^3 - 12 a^3 b d^3 e + 10 a^2 b c d e^3)) / (d^2(a$$

$$\begin{aligned}
& d^2 + c^2 e^2 - b^2 d^2 e^2)) \cdot (b^4 e^2 - 4 a^3 c d^2 + b^3 e^2 (b^2 - 4 a c)^{1/2} \\
& + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 + a^2 b d^2 (b^2 - 4 a c)^{1/2} \\
& + 8 a^2 b c d e - 3 a b c e^2 (b^2 - 4 a c)^{1/2} - 2 a b^2 d e (b^2 - 4 a c)^{1/2} \\
& + 4 a^2 c d e (b^2 - 4 a c)^{1/2})) / (2 c (4 a c - b^2) (a d^2 + c^2 e^2 - b^2 d^2 e^2)) \cdot (b^4 e^2 - 4 a^3 c d^2 + b^3 e^2 (b^2 - 4 a c)^{1/2} \\
& + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 + a^2 b d^2 (b^2 - 4 a c)^{1/2} \\
& + 8 a^2 b c d e - 3 a b c e^2 (b^2 - 4 a c)^{1/2} - 2 a b^2 d e (b^2 - 4 a c)^{1/2} \\
& + 4 a^2 c d e (b^2 - 4 a c)^{1/2})) / (2 (4 a^3 c^4 e^4 + 4 a^3 c^2 d^4 - b^2 c^3 e^4 - a^2 b^2 c d^4 + 2 b^3 c^2 d e^3 \\
& - b^4 c d^2 e^2 + 8 a^2 c^3 d^2 e^2 - 8 a b c^3 d e^3 + 2 a b^3 c d^3 e - 8 a^2 b c^2 d^3 e + 2 a b^2 c^2 d^2 e^2)) + (\log((a^4 e^4) / (d (a d^2 + c^2 e^2 - b^2 d^2 e^2) \\
& + a^4 e^5 x) / (d^2 (a d^2 + c^2 e^2 - b^2 d^2 e^2) - ((a e^3 (3 a^3 b d^4 + b^3 c e^4 - b^4 d e^3 + 5 a b^3 d^2 e^2 - 7 a^2 b^2 d^3 e + 8 a^2 c^2 d e^3 \\
& - 3 a b c^2 e^4 + 9 a^3 c d^3 e - a b^2 c d e^3 - 8 a^2 b c d^2 e^2)) / (d^2 (a d^2 + c^2 e^2 - b^2 d^2 e^2) + ((a e (a^3 b d^5 - 4 a c^3 e^5 + b^2 c^2 e^5 - b^4 d^2 e^3 + 3 a b^3 d^3 e^2 - 3 a^2 b^2 d^4 e - 8 a^2 c^2 d^2 e^3 \\
& + 4 a^3 c d^4 e - b^3 c d e^4 + 4 a b c^2 d e^4 + 6 a b^2 c d^2 e^3 - 9 a^2 b c d^3 e^2)) / (a d^3 - b d^2 e + c d e^2) + (a e x (3 a^4 d^5 + 2 b^3 c e^5 - 4 b^4 d e^4 + 9 a b^3 d^2 e^3 + 4 a^2 c^2 d e^4 + 19 a^3 c d^3 e^2 \\
& - 3 a^2 b^2 d^3 e^2 - 8 a b c^2 e^5 - 5 a^3 b d^4 e + 15 a b^2 c d e^4 - 36 a^2 b c d^2 e^3)) / (a d^3 - b d^2 e + c d e^2) - (a e (b^4 e^2 - 4 a^3 c d^2 - b^3 e^2 (b^2 - 4 a c)^{1/2} \\
& + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 - a^2 b d^2 (b^2 - 4 a c)^{1/2} + 8 a^2 b c d e + 3 a b c e^2 (b^2 - 4 a c)^{1/2} \\
& + 2 a b^2 d e (b^2 - 4 a c)^{1/2} - 4 a^2 c d e (b^2 - 4 a c)^{1/2})) \cdot (4 a^2 c^2 d^3 e + b^2 c^2 d e^3 + b^3 c d^2 e^2 + 2 a^2 b^2 d^4 x + 2 b^2 c^2 e^4 x + 2 b^4 d^2 e^2 x + a^2 b c d^4 - 4 a c^3 d e^3 \\
& - 6 a^3 c d^4 x - 8 a c^3 e^4 x - 2 a b^2 c d^3 e - 4 a b^3 d^3 e x - 2 b^3 c d e^3 x - 3 a b c^2 d^2 e^2 - 6 a^2 c^2 d^2 e^2 x + 8 a b c^2 d e^3 x + 14 a^2 b c d^3 e x - 6 a b^2 c d^2 e^2 x)) / (2 c (4 a c - b^2) (a d^2 + c^2 e^2 - b^2 d^2 e^2)) \cdot (b^4 e^2 - 4 a^3 c d^2 - b^3 e^2 (b^2 - 4 a c)^{1/2} \\
& + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 - a^2 b d^2 (b^2 - 4 a c)^{1/2} + 8 a^2 b c d e + 3 a b c e^2 (b^2 - 4 a c)^{1/2} + 2 a b^2 d e (b^2 - 4 a c)^{1/2} \\
& + 4 a^2 c d e (b^2 - 4 a c)^{1/2})) / (2 c (4 a c - b^2) (a d^2 + c^2 e^2 - b^2 d^2 e^2) + (a e^3 x (9 a^4 d^4 + b^4 e^4 + 2 a^2 c^2 e^4 - 6 a^3 c d^2 e^2 + 8 a^2 b^2 d^2 e^2 - 4 a b^2 c e^4 - 4 a b^3 d e^3 - 12 a^3 b d^3 e + 10 a^2 b c d e^3)) / (d^2 (a d^2 + c^2 e^2 - b^2 d^2 e^2)) \cdot (b^4 e^2 - 4 a^3 c d^2 - b^3 e^2 (b^2 - 4 a c)^{1/2} \\
& + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 - a^2 b d^2 (b^2 - 4 a c)^{1/2} + 8 a^2 b c d e + 3 a b c e^2 (b^2 - 4 a c)^{1/2} + 2 a b^2 d e (b^2 - 4 a c)^{1/2} \\
& - 4 a^2 c d e (b^2 - 4 a c)^{1/2})) / (2 c (4 a c - b^2) (a d^2 + c^2 e^2 - b^2 d^2 e^2)) \cdot (b^4 e^2 - 4 a^3 c d^2 - b^3 e^2 (b^2 - 4 a c)^{1/2} + a^2 b^2 d^2 + 4 a^2 c^2 e^2 - 2 a b^3 d e - 5 a b^2 c e^2 - a^2 b d^2 (b^2 - 4 a c)^{1/2} \\
& + 8 a^2 b c d e + 3 a b c e^2 (b^2 - 4 a c)^{1/2} + 2 a b^2 d e (b^2 - 4 a c)^{1/2} - 4 a^2 c d e (b^2 - 4 a c)^{1/2})) / (2 (4 a^3 c^4 e^4 + 4 a^3 c^2 d^4 - b^2 c^3 e^4 - a^2 b^2 c d^4 + 2 b^3 c^2 d e^3 - b^4 c d^2 e^2 +
\end{aligned}$$

$$8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^3*e - 8*a^2*b*c^2*d^3*e + 2*a*b^2*c^2*d^2*e^2) - (\log(d + e*x)*(c*e^4 + 3*a*d^2*e^2 - 2*b*d*e^3))/(a^2*d^6 + b^2*d^4*e^2 + c^2*d^2*e^4 - 2*a*b*d^5*e + 2*a*c*d^4*e^2 - 2*b*c*d^3*e^3) + \log(x)/(c*d^2) + e^2/(d*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)

[Out] Timed out

$$3.77 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^4 (d+ex)^2} dx$$

Optimal. Leaf size=291

$$\frac{(2a^3cd^2 - a^2(b^2d^2 + 6bcde + 2c^2e^2) + 2ab^2e(bd + 2ce) + b^4(-e^2)) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (ad - be)(abd + 2ace + b^2d)}{c^2\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \frac{(ad - be)(abd + 2ace + b^2d)}{2c^2(ad^2 - e(bd - ce))^2}$$

[Out] $-1/c/d^2/x - e^3/d^2/(a*d^2 - e*(b*d - c*e))/(e*x + d) - (b*d + 2*c*e)*\ln(x)/c^2/d^3 + e^3*(4*a*d^2 - e*(3*b*d - 2*c*e))*\ln(e*x + d)/d^3/(a*d^2 - e*(b*d - c*e))^2 + 1/2*(a*d - b*e)*(a*b*d + 2*a*c*e - b^2*e)*\ln(a*x^2 + b*x + c)/c^2/(a*d^2 - e*(b*d - c*e))^2 + (2*a^3*c*d^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^2))*\arctan\left(\frac{(2*a*x + b)/(-4*a*c + b^2)^{1/2}}{c}\right)/c^2/(a*d^2 - e*(b*d - c*e))^2/(-4*a*c + b^2)^{1/2}$

Rubi [A] time = 0.56, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{(-a^2(b^2d^2 + 6bcde + 2c^2e^2) + 2a^3cd^2 + 2ab^2e(bd + 2ce) + b^4(-e^2)) \tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right) + (ad - be)(abd + 2ace + b^2d)}{c^2\sqrt{b^2 - 4ac} (ad^2 - e(bd - ce))^2} + \frac{(ad - be)(abd + 2ace + b^2d)}{2c^2(ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2), x]

[Out] $-(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^2))*\text{ArcTanh}[(b + 2*a*x)/\text{Sqrt}[b^2 - 4*a*c]]/(c^2*\text{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - ((b*d + 2*c*e)*\text{Log}[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(3*b*d - 2*c*e))*\text{Log}[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*\text{Log}[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e))^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1569

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_), x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^4 (d + ex)^2} dx &= \int \frac{1}{x^2 (d + ex)^2 (c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{cd^2 x^2} + \frac{-bd - 2ce}{c^2 d^3 x} + \frac{e^4}{d^2 (ad^2 - e(bd - ce)) (d + ex)^2} + \frac{e^4 (4ad^2 - e(3bd - ce))}{d^3 (ad^2 - e(bd - ce))} \right) dx \\
&= -\frac{1}{cd^2 x} - \frac{e^3}{d^2 (ad^2 - e(bd - ce)) (d + ex)} - \frac{(bd + 2ce) \log(x)}{c^2 d^3} + \frac{e^3 (4ad^2 - e(3bd - ce))}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cd^2 x} - \frac{e^3}{d^2 (ad^2 - e(bd - ce)) (d + ex)} - \frac{(bd + 2ce) \log(x)}{c^2 d^3} + \frac{e^3 (4ad^2 - e(3bd - ce))}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cd^2 x} - \frac{e^3}{d^2 (ad^2 - e(bd - ce)) (d + ex)} - \frac{(bd + 2ce) \log(x)}{c^2 d^3} + \frac{e^3 (4ad^2 - e(3bd - ce))}{d^3 (ad^2 - e(bd - ce))} \\
&= -\frac{1}{cd^2 x} - \frac{e^3}{d^2 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(2a^3 cd^2 - b^4 e^2 + 2ab^2 e(bd + 2ce) - a^2 (bd + 2ce)^2)}{c^2 \sqrt{b^2 - 4ac}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 287, normalized size = 0.99

$$\frac{(-2a^3 cd^2 + a^2 (b^2 d^2 + 6bcde + 2c^2 e^2) - 2ab^2 e(bd + 2ce) + b^4 e^2) \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + (ad - be) (abd + 2ace + b^2(-e))}{c^2 \sqrt{4ac - b^2} (ad^2 + e(ce - bd))^2} + \frac{e^3 (4ad^2 - e(3bd - ce))}{2c^2 (ad^2 + e(ce - bd))}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2), x]

[Out] $-(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*(-(b*d) + c*e))^2)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.36, size = 487, normalized size = 1.67

$$\frac{(a^2b^2d^2e^2 - 2a^3cd^2e^2 - 2ab^3de^3 + 6a^2bcde^3 + b^4e^4 - 4ab^2ce^4 + 2a^2c^2e^4) \arctan\left(\frac{\left(2ad - \frac{2ad^2}{xe+d} - be + \frac{2bde}{xe+d} - \frac{2ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4ac}}\right)}{(a^2c^2d^4 - 2abc^2d^3e + b^2c^2d^2e^2 + 2ac^3d^2e^2 - 2bc^3de^3 + c^4e^4)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")

[Out] $-(a^2b^2d^2e^2 - 2a^3cd^2e^2 - 2a^2b^3d^2e^3 + 6a^2b^2c^2d^2e^3 + b^4e^4 - 4a^2b^2c^2e^4 + 2a^2c^2e^4) \arctan\left(-\frac{(2ad - 2a^2d^2/(xe + d) - b^2e + 2b^2d^2/(xe + d) - 2c^2e^2/(xe + d))e^{(-1)}}{\sqrt{-b^2 + 4ac}}\right) e^{(-2)} / ((a^2c^2d^4 - 2a^2b^2c^2d^3e + b^2c^2d^2e^2 + 2a^2c^3d^2e^2 - 2b^2c^3d^2e^3 + c^4e^4) \sqrt{-b^2 + 4ac}) + 1/2(a^2b^2d^2 - 2a^2b^2d^2e + 2a^2c^2d^2e + b^3e^2 - 2a^2b^2c^2e^2) \log(-a + 2ad/(xe + d) - a^2d^2/(xe + d)^2 - b^2e/(xe + d) + b^2d^2e/(xe + d)^2 - c^2e^2/(xe + d)^2) / (a^2c^2d^4 - 2a^2b^2c^2d^3e + b^2c^2d^2e^2 + 2a^2c^3d^2e^2 - 2b^2c^3d^2e^3 + c^4e^4) - e^7 / ((a^2d^4e^4 - b^2d^3e^5 + c^2d^2e^6)(xe + d)) - (b^2d^2e + 2c^2e^2)e^{(-1)} \log(\text{abs}(-d/(xe + d) + 1)) / (c^2d^3) + e / (c^2d^3(d/(xe + d) - 1))$

maple [B] time = 0.01, size = 791, normalized size = 2.72

$$\frac{2a^3d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2} c} + \frac{a^2b^2d^2 \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2} c^2} + \frac{6a^2bde \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2} c} + \frac{2a^2e^2}{(ad^2 - deb + ce^2)^2 \sqrt{4ac-b^2} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x)

[Out] $1/2c^2/(a^2d^2-b^2d^2e+c^2e^2)^2 a^2 \ln(ax^2+bx+c) b^2d^2+1/c/(a^2d^2-b^2d^2e+c^2e^2)^2 a^2 \ln(ax^2+bx+c) d^2e-1/c^2/(a^2d^2-b^2d^2e+c^2e^2)^2 a \ln(ax^2+bx+c) b^2d^2e-1/c/(a^2d^2-b^2d^2e+c^2e^2)^2 a \ln(ax^2+bx+c) b^2e^2+1/2c^2/(a^2d^2-b^2d^2e+c^2e^2)^2 \ln(ax^2+bx+c) b^3e^2-2/c/(a^2d^2-b^2d^2e+c^2e^2)^2/(4ac-b^2)^{(1/2)} \arctan((2ax+b)/(4ac-b^2)^{(1/2)}) a^3d^2+1/c^2/(a^2d^2-b^2d^2e+c^2e^2)^2/(4ac-b^2)^{(1/2)} \arctan((2ax+b)/(4ac-b^2)^{(1/2)}) a^2b^2d^2+6/c$

$$\begin{aligned} & (a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)}) \\ & *a^2*b*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a* \\ & c-b^2)^{(1/2)})*a^2*e^2-2/c^2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan(\\ & (2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^3*d*e-4/c/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2 \\ &)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*a*b^2*e^2+1/c^2/(a*d^2-b*d*e+c* \\ & e^2)^2/(4*a*c-b^2)^{(1/2)}*\arctan((2*a*x+b)/(4*a*c-b^2)^{(1/2)})*b^4*e^2-1/c/d^2 \\ & /x-1/c^2/d^2*\ln(x)*b-2/c/d^3*\ln(x)*e-e^3/(a*d^2-b*d*e+c*e^2)/d^2/(e*x+d)+4 \\ & *e^3/(a*d^2-b*d*e+c*e^2)^2/d*\ln(e*x+d)*a-3*e^4/(a*d^2-b*d*e+c*e^2)^2/d^2*\ln \\ & (e*x+d)*b+2*e^5/(a*d^2-b*d*e+c*e^2)^2/d^3*\ln(e*x+d)*c \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details)Is 4*a*c-b^2 positive or negative?

mupad [B] time = 31.16, size = 4948, normalized size = 17.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)

[Out]
$$\begin{aligned} & (\log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2 + \\ & c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) + (x \\ & *(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x + e*x \\ & ^2) - (\log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^ \\ & 4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 + \\ & a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 - \\ & 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7 \\ & - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^ \\ & 3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4)))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)) - (\\ & ((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^5*d^3*e^3 \\ & - 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*e^4 + 8*a^2 \\ & *c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*d^5*e - 7*a* \\ & b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 12*a^2*b^2*c*d \\ & ^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^4*e^2*(b^2 - 4 \\ & *a*c))^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c))^(1/ \\ & 2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c))^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a* \end{aligned}$$

$$\begin{aligned}
& b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^3*d*e*(b^2 - 4*a*c)^{(1/2)} + \\
& 6*a^2*b*c*d*e*(b^2 - 4*a*c)^{(1/2)}*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (2*a*e*x*(a*d - b*e)*(a^3*b*d^5 + 8*a*c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 - a^2*b^2*d^4*e + 16*a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b*c^2*d*e^4 - 8*a*b^2*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e))*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^3*d*e*(b^2 - 4*a*c)^{(1/2)} + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^{(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c^4*e^8 + 4*b^4*c^2*e^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 + 2*a^5*c*d^6*e^2 + a^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 + 18*a^4*c^2*d^4*e^4 - 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6 + 8*a*b^3*c^2*d*e^7 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d^5*e^3 + 10*a^2*b^3*c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2)*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^3*d*e*(b^2 - 4*a*c)^{(1/2)} + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^{(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d - b*e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2)*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^{(1/2)} + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^{(1/2)} - 2*a*b^3*d*e*(b^2 - 4*a*c)^{(1/2)} + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^{(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*e^4 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e^2 - 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*d^2*e^2)) + (log((a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4)))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(b^4*e^2*(b^2 - 4*a*c)^{(1/2)} - b^5*e^2 - a^2*b^3*
\end{aligned}$$

$$\begin{aligned}
& d^2 - 8a^2bc^2e^2 + a^2b^2d^2(b^2 - 4ac)^{1/2} + 2a^2c^2e^2(b^2 - 4ac)^{1/2} + 2ab^4de + 4a^3bcd^2 + 6ab^3c^2e^2 + 8a^3c^2d^2e \\
& - 2a^3cd^2(b^2 - 4ac)^{1/2} - 10a^2b^2cde - 4ab^2c^2e^2(b^2 - 4ac)^{1/2} - 2ab^3d^2e(b^2 - 4ac)^{1/2} + 6a^2bcd^2e(b^2 - 4ac)^{1/2} \\
& (4a^2c^2d^3e + b^2c^2d^2e^3 + b^3cd^2e^2 + 2a^2b^2d^4x + 2b^2c^2e^4x + 2b^4d^2e^2x + a^2bcd^4 - 4ac^3d^2e^3 - 6a^3cd^4x \\
& - 8ac^3e^4x - 2ab^2cd^3e - 4ab^3d^3ex - 2b^3cd^2e^3x - 3abc^2d^2e^2 - 6a^2c^2d^2e^2x + 8abc^2d^2e^3x + 14a^2bcd^3ex \\
& - 6ab^2cd^2e^2x)/(2c^2(4ac - b^2)(ad^2 + ce^2 - bde)^2) - (ae(a^4cd^6 + 8ac^4e^6 - a^3b^2d^6 - 2b^2c^3e^6 + b^5d^3e^3 \\
& - 3ab^4d^4e^2 + 3a^2b^3d^5e + b^3c^2d^2e^5 + b^4cd^2e^4 + 8a^2c^3d^2e^4 - 7a^3c^2d^4e^2 - 4abc^3d^2e^5 - 7a^3bcd^5e \\
& - 7ab^3cd^3e^3 - 6ab^2c^2d^2e^4 + 12a^2b^2cd^3e^3 + 12a^2b^2cd^4e^2))/(cd^2(ad^2 + ce^2 - bde)) + (2ae^2x(ad - b^2e) \\
& (a^3bd^5 + 8ac^3e^5 - 2b^2c^2e^5 + b^4d^2e^3 - ab^3d^3e^2 - a^2b^2d^4e + 16a^2c^2d^2e^3 + 2a^3cd^4e + 2b^3cd^2e^4 - 8abc^2d^2e^4 \\
& - 8ab^2cd^2e^3 + 4a^2bcd^3e^2))/(cd^2(ad^2 + ce^2 - bde)) * (b^4e^2(b^2 - 4ac)^{1/2} - b^5e^2 - a^2b^3d^2 - 8a^2b^2c^2e^2 \\
& + a^2b^2d^2(b^2 - 4ac)^{1/2} + 2a^2c^2e^2(b^2 - 4ac)^{1/2} + 2ab^4de + 4a^3bcd^2 + 6ab^3c^2e^2 + 8a^3c^2d^2e - 2a^3cd^2(b^2 - 4ac)^{1/2} \\
& - 10a^2b^2cde - 4ab^2c^2e^2(b^2 - 4ac)^{1/2} - 2ab^3d^2e(b^2 - 4ac)^{1/2} + 6a^2bcd^2e(b^2 - 4ac)^{1/2}))/ \\
& (2c^2(4ac - b^2)(ad^2 + ce^2 - bde)^2) + (ae^2x(a^6d^8 + 8a^2c^4e^8 + 4b^4c^2e^8 + b^6d^2e^6 - 16ab^2c^3e^8 - 2ab^5d^3e^5 + 2a^5cd^6e^2 \\
& + a^2b^4d^4e^4 + a^4b^2d^6e^2 + 8a^3c^3d^2e^6 + 18a^4c^2d^4e^4 - 2a^5b^2d^7e - 4b^5cd^2e^7 - 26a^2b^2c^2d^2e^6 + 8ab^3c^2d^2e^7 \\
& + 4ab^4cd^2e^6 + 16a^2b^3cd^2e^7 + 6a^4b^2cd^5e^3 + 10a^2b^3cd^3e^5 - 18a^3b^2cd^4e^4))/(c^2d^4(ad^2 + ce^2 - bde)^2) * (b^4e^2(b^2 - 4ac)^{1/2} - b^5e^2 - a^2b^3d^2 - 8a^2b^2c^2e^2 \\
& + a^2b^2d^2(b^2 - 4ac)^{1/2} + 2a^2c^2e^2(b^2 - 4ac)^{1/2} + 2ab^4de + 4a^3bcd^2 + 6ab^3c^2e^2 + 8a^3c^2d^2e - 2a^3cd^2(b^2 - 4ac)^{1/2} \\
& - 10a^2b^2cde - 4ab^2c^2e^2(b^2 - 4ac)^{1/2} - 2ab^3d^2e(b^2 - 4ac)^{1/2} + 6a^2bcd^2e(b^2 - 4ac)^{1/2}))/ \\
& (2c^2(4ac - b^2)(ad^2 + ce^2 - bde)^2) + (4a^5e^4x(ad - b^2e))/(c^2d^2(ad^2 + ce^2 - bde)^2) * (b^4e^2(b^2 - 4ac)^{1/2} - b^5e^2 - a^2b^3d^2 - 8a^2b^2c^2e^2 \\
& + a^2b^2d^2(b^2 - 4ac)^{1/2} + 2a^2c^2e^2(b^2 - 4ac)^{1/2} + 2ab^4de + 4a^3bcd^2 + 6ab^3c^2e^2 + 8a^3c^2d^2e - 2a^3cd^2(b^2 - 4ac)^{1/2} \\
& - 10a^2b^2cde - 4ab^2c^2e^2(b^2 - 4ac)^{1/2} - 2ab^3d^2e(b^2 - 4ac)^{1/2} + 6a^2bcd^2e(b^2 - 4ac)^{1/2}))/ \\
& (2(4ac^5e^4 + 4a^3c^3d^4 - b^2c^4e^4 + 2b^3c^3d^2e^3 - a^2b^2c^2d^4 + 8a^2c^4d^2e^2 - b^4c^2d^2e^2 - 8abc^4d^2e^3 + 2ab^3c^2d^3e - 8a^2b^3cd^3e + 2ab^2c^3d^2e^2)) \\
& - (\log(x)(bd + 2ce))/(c^2d^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)

[Out] Timed out

$$3.78 \quad \int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^5 (d + ex)^2} dx$$

Optimal. Leaf size=372

$$\frac{(a^3 cd^2 - a^2 (b^2 d^2 + 4bcde + c^2 e^2) + ab^2 e(2bd + 3ce) + b^4 (-e^2)) \log(ax^2 + bx + c) + (-a^3 cd(3bd + 4ce) + a^2 b (b^2 d^2 + 4bcde + 5c^2 e^2))}{2c^3 (ad^2 - e(bd - ce))^2}$$

[Out] $-1/2/c/d^2/x^2+(b*d+2*c*e)/c^2/d^3/x+e^4/d^3/(a*d^2-e*(b*d-c*e))/(e*x+d)+(b^2*d^2+2*b*c*d*e-c*(a*d^2-3*c*e^2))*\ln(x)/c^3/d^4-e^4*(5*a*d^2-e*(4*b*d-3*c*e))*\ln(e*x+d)/d^4/(a*d^2-e*(b*d-c*e))^2+1/2*(a^3*c*d^2-b^4*e^2+a*b^2*e*(2*b*d+3*c*e)-a^2*(b^2*d^2+4*b*c*d*e+c^2*e^2))*\ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))^2+(b^5*e^2-a^3*c*d*(3*b*d+4*c*e)-a*b^3*e*(2*b*d+5*c*e)+a^2*b*(b^2*d^2+8*b*c*d*e+5*c^2*e^2))*\operatorname{arctanh}((2*a*x+b)/(-4*a*c+b^2)^{(1/2)})/c^3/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^{(1/2)}$

Rubi [A] time = 0.85, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{(-a^2 (b^2 d^2 + 4bcde + c^2 e^2) + a^3 cd^2 + ab^2 e(2bd + 3ce) + b^4 (-e^2)) \log(ax^2 + bx + c) + (a^2 b (b^2 d^2 + 8bcde + 5c^2 e^2))}{2c^3 (ad^2 - e(bd - ce))^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2), x]

[Out] $-1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d + 5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*\operatorname{ArcTanh}[(b + 2*a*x)/\operatorname{Sqrt}[b^2 - 4*a*c]]/(c^3*\operatorname{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 + 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*\operatorname{Log}[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4*b*d - 3*c*e))*\operatorname{Log}[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2 - b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2))*\operatorname{Log}[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 1569

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^p_)*((d_) + (e_.)*(x_)^(n_.))^q_, x_Symbol] := Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{c}{x^2} + \frac{b}{x}\right) x^5 (d + ex)^2} dx &= \int \frac{1}{x^3 (d + ex)^2 (c + bx + ax^2)} dx \\
&= \int \left(\frac{1}{cd^2 x^3} + \frac{-bd - 2ce}{c^2 d^3 x^2} + \frac{b^2 d^2 + 2bcde - c(ad^2 - 3ce^2)}{c^3 d^4 x} + \frac{e^5}{d^3 (-ad^2 + e(bd - ce))} \right) dx \\
&= -\frac{1}{2cd^2 x^2} + \frac{bd + 2ce}{c^2 d^3 x} + \frac{e^4}{d^3 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(b^2 d^2 + 2bcde - c(ad^2 - 3ce^2))}{c^3 d^4} \\
&= -\frac{1}{2cd^2 x^2} + \frac{bd + 2ce}{c^2 d^3 x} + \frac{e^4}{d^3 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(b^2 d^2 + 2bcde - c(ad^2 - 3ce^2))}{c^3 d^4} \\
&= -\frac{1}{2cd^2 x^2} + \frac{bd + 2ce}{c^2 d^3 x} + \frac{e^4}{d^3 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(b^2 d^2 + 2bcde - c(ad^2 - 3ce^2))}{c^3 d^4} \\
&= -\frac{1}{2cd^2 x^2} + \frac{bd + 2ce}{c^2 d^3 x} + \frac{e^4}{d^3 (ad^2 - e(bd - ce)) (d + ex)} + \frac{(b^5 e^2 - a^3 cd(3bd + 4ce))}{c^3 d^4}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 370, normalized size = 0.99

$$\frac{(-a^3 cd^2 + a^2 (b^2 d^2 + 4bcde + c^2 e^2) - ab^2 e(2bd + 3ce) + b^4 e^2) \log(x(ax + b) + c) + (a^3 cd(3bd + 4ce) - a^2 b (b^2 d^2 + 4bcde + c^2 e^2))}{2c^3 (ad^2 + e(ce - bd))^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2), x]

[Out] $-\frac{1}{2} \frac{1}{(c d^2 x^2)} + \frac{(b d + 2 c e)}{c^2 d^3 x} + \frac{e^4}{d^3 (a d^2 + e(-b d + c e)) (d + e x)} + \frac{((-(b^5 e^2) + a^3 c d (3 b d + 4 c e) + a b^3 e (2 b d + 5 c e) - a^2 b (b^2 d^2 + 8 b c d e + 5 c^2 e^2)) \operatorname{ArcTan}[(b + 2 a x) / \operatorname{Sqrt}[-b^2 + 4 a c]])}{c^3 \operatorname{Sqrt}[-b^2 + 4 a c] (a d^2 + e(-b d + c e))^2} + \frac{((b^2 d^2 + 2 b c d e + c(-a d^2) + 3 c e^2)) \operatorname{Log}[x]}{c^3 d^4} - \frac{(e^4 (5 a d^2 + e(-4 b d + 3 c e)) \operatorname{Log}[d + e x])}{d^4 (a d^2 + e(-b d + c e))^2} - \frac{((-(a^3 c d^2) + b^4 e^2 - a b^2 e (2 b d + 3 c e) + a^2 (b^2 d^2 + 4 b c d e + c^2 e^2)) \operatorname{Log}[c + x (b + a x)])}{(2 c^3 (a d^2 + e(-b d + c e))^2)}$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.49, size = 587, normalized size = 1.58

$$\frac{\left(a^2 b^3 d^2 e^2 - 3 a^3 b c d^2 e^2 - 2 a b^4 d e^3 + 8 a^2 b^2 c d e^3 - 4 a^3 c^2 d e^3 + b^5 e^4 - 5 a b^3 c e^4 + 5 a^2 b c^2 e^4\right) \arctan\left(\frac{\left(2 a d - \frac{2 a d^2}{x e + d} - b e + \frac{2}{x}\right)}{\sqrt{-b^2 + 4 a c}}\right)}{\left(a^2 c^3 d^4 - 2 a b c^3 d^3 e + b^2 c^3 d^2 e^2 + 2 a c^4 d^2 e^2 - 2 b c^4 d e^3 + c^5 e^4\right) \sqrt{-b^2 + 4 a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")

[Out] $(a^2 b^3 d^2 e^2 - 3 a^3 b c d^2 e^2 - 2 a^4 b^2 d e^3 + 8 a^2 b^2 c d e^3 - 4 a^3 c^2 d e^3 + b^5 e^4 - 5 a^2 b^3 c e^4 + 5 a^2 b c^2 e^4) \arctan\left(-\frac{2 a^2 d - 2 a^2 d^2 / (x e + d) - b e + 2 b d e / (x e + d) - 2 c e^2 / (x e + d)}{\sqrt{-b^2 + 4 a c}}\right) e^{-1} / \sqrt{-b^2 + 4 a c} e^{-2} / \left((a^2 c^3 d^4 - 2 a^2 b c^3 d^3 e + b^2 c^3 d^2 e^2 + 2 a^2 c^4 d^2 e^2 - 2 b^2 c^4 d e^3 + c^5 e^4) \sqrt{-b^2 + 4 a c}\right) - 1/2 (a^2 b^2 d^2 - a^3 c d^2 - 2 a^2 b^3 d e + 4 a^2 b^2 c d e + b^4 e^2 - 3 a^2 b^2 c e^2 + a^2 c^2 e^2) \log(-a + 2 a d / (x e + d) - a d^2 / (x e + d)^2 - b e / (x e + d) + b d e / (x e + d)^2 - c e^2 / (x e + d)^2) / (a^2 c^3 d^4 - 2 a^2 b c^3 d^3 e + b^2 c^3 d^2 e^2 + 2 a^2 c^4 d^2 e^2 - 2 b^2 c^4 d e^3 + c^5 e^4) + e^9 / \left((a d^5 e^5 - b d^4 e^6 + c d^3 e^7) (x e + d) + (b^2 d^2 e - a c d^2 e + 2 b^2 c d e^2 + 3 c^2 e^3) e^{-1} \log(\text{abs}(-d / (x e + d) + 1)) / (c^3 d^4) + 1/2 (2 b^2 c d e + 5 c^2 e^2 - 2 (b c d^2 e^2 + 3 c^2 d e^3) e^{-1} / (x e + d)) / (c^3 d^4 (d / (x e + d) - 1)^2)\right)$

maple [B] time = 0.02, size = 993, normalized size = 2.67

$$\frac{3 a^3 b d^2 \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{\left(a d^2 - d e b + c e^2\right)^2 \sqrt{4 a c - b^2} c^2} + \frac{4 a^3 d e \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{\left(a d^2 - d e b + c e^2\right)^2 \sqrt{4 a c - b^2} c} - \frac{a^2 b^3 d^2 \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right)}{\left(a d^2 - d e b + c e^2\right)^2 \sqrt{4 a c - b^2} c^3} - \frac{8 a^2 b^2 a}{\left(a d^2 - d e b + c e^2\right)^2 \sqrt{4 a c - b^2} c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x)

[Out] $-8 / (a d^2 - b d e + c e^2)^2 / c^2 / (4 a^2 c - b^2)^{(1/2)} \arctan\left(\frac{2 a x + b}{(4 a^2 c - b^2)^{(1/2)}}\right) + a^2 b^2 d e - 1 / (a d^2 - b d e + c e^2)^2 / c^3 / (4 a^2 c - b^2)^{(1/2)} \arctan\left(\frac{2 a x + b}{(4 a^2 c - b^2)^{(1/2)}}\right) + a^2 b^3 d^2 e - 5 / (a d^2 - b d e + c e^2)^2 / c / (4 a^2 c - b^2)^{(1/2)} \arctan\left(\frac{2 a x + b}{(4 a^2 c - b^2)^{(1/2)}}\right) + a^2 b^2 e^2 + 5 / (a d^2 - b d e + c e^2)$

$$\begin{aligned} & \frac{2}{c^2} \sqrt{4ac-b^2} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) a^3 b^3 e^{-2} - \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^2} a^2 \ln(ax^2 + bx + c) b^3 d e + \frac{1}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^3} a^3 \ln(ax^2 + bx + c) b^3 d e + \frac{3}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^2} \sqrt{4ac-b^2} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) a^3 b^3 d^2 + \frac{4}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c} (4ac-b^2)^{1/2} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) a^3 b^3 d e - \frac{1}{c^2} \frac{2}{d^2} \ln(x) a + \frac{1}{c^3} \frac{2}{d^2} \ln(x) b^2 + \frac{3}{c} \frac{2}{d^4} \ln(x) e^2 + \frac{1}{c^2} \frac{2}{d^2} x b + \frac{2}{c} \frac{2}{d^3} x e + \frac{e^4}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{d^3} (e x + d) + \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^3} \sqrt{4ac-b^2} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) a^3 b^4 d e + \frac{1}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^2} a^3 \ln(ax^2 + bx + c) d^2 - \frac{1}{2} \frac{2}{c} \frac{2}{d^2} x^2 - \frac{1}{2} \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c} a^2 \ln(ax^2 + bx + c) e^2 - \frac{1}{2} \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^3} \ln(ax^2 + bx + c) b^4 e^2 + \frac{2}{c^2} \frac{2}{d^3} \ln(x) b^3 e - \frac{5}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{d^2} \ln(e x + d) a + \frac{4}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{d^3} \ln(e x + d) b - \frac{3}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{d^4} \ln(e x + d) c - \frac{1}{2} \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^3} a^2 \ln(ax^2 + bx + c) b^2 d^2 + \frac{3}{2} \frac{2}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^2} a \ln(ax^2 + bx + c) b^2 e^2 - \frac{1}{(a^2 d^2 - b^2 d e + c^2 e^2)^2} \frac{2}{c^3} \sqrt{4ac-b^2} \arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) b^5 e^2 \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more details) Is 4*a*c-b^2 positive or negative?

mupad [B] time = 45.61, size = 7144, normalized size = 19.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)

[Out]
$$\begin{aligned} & \left(\frac{(x(2bd + 3ce))}{(2c^2d^2) - 1/(2cd)} + \frac{(x^2(3c^2e^4 - b^2d^2e^2 + a^2bd^3e - b^3cd^3e^3 + 2ac^2d^2e^2))}{(c^2d^3(a^2d^2 + ce^2 - b^2de))} \right) / \left(dx^2 + ex^3 \right) - \frac{(\log(d + ex)(3ce^6 + 5ad^2e^4 - 4b^2de^5))}{(a^2d^8 + b^2d^6e^2 + c^2d^4e^4 - 2abd^7e + 2ac^2d^6e^2 - 2b^2cd^5e^3) + (\log(((27a^2b^3c^6e^{11} - 9a^2b^3c^5e^{11} - ab^8d^5e^6 - a^6b^3d^{10}e - 36a^3c^6d^2e^{10} + 2a^2b^7d^6e^5 - a^3b^6d^7e^4 - a^4b^5d^8e^3 + 2a^5b^4d^9e^2 - 36a^4c^5d^3e^8 + 4a^5c^4d^5e^6 + 3a^6c^3d^7e^4 + a^7b^3cd^{10}e - 39a^2b^3c^4d^2e^9 - 15a^2b^4c^3d^3e^8 + 7a^2b^5c^2d^4e^7 + 53a^3b^2c^4d^3e^8 + 7a^3b^3c^3d^4e^7 - 33a^3b^4c^2d^5e^6 + 20a^4b^2c^3d^5e^6 + 33a^4b^3c^2 \end{aligned}$$

$$\begin{aligned}
& *d^6e^5 - 9a^5b^2c^2d^7e^4 + 6a^4b^3c^4d^8e^3 - a^6b^2c^2d^8e^3 - 4a^6b^2c^2d^9e^2)/(c^4d^6(a^2d^2 + c^2e^2 - b^2d^2e)) + (((a^2e^2(12a^2c^5e^7 - a^3b^3d^7 - 3b^2c^4e^7 + b^6d^4e^3 - 3a^2b^4d^6e + 4a^4c^2d^6e + b^3c^3d^6e + b^5c^2d^3e^4 + 8a^2c^4d^2e^5 - 8a^3c^3d^4e^3 + b^4c^2d^2e^5 + 2a^4b^2c^2d^7 - 4a^2b^3c^4d^6e + 18a^2b^2c^2d^4e^3 - 8a^2b^4c^2d^4e^3 - 10a^3b^2c^2d^6e - 6a^2b^2c^3d^2e^5 - 7a^2b^3c^2d^3e^4 + 12a^2b^2c^3d^3e^4 + 15a^2b^3c^2d^5e^2 - 16a^3b^2c^2d^5e^2)))/(c^2d^3(a^2d^2 + c^2e^2 - b^2d^2e)) + (a^2e^2(4a^2c^2d^3e + b^2c^2d^2e^3 + b^3c^2d^2e^2 + 2a^2b^2d^4e^2x + 2b^2c^2e^4x + 2b^4d^2e^2x + a^2b^2c^2d^4 - 4a^2c^3d^2e^3 - 6a^3c^2d^4e^2x - 8a^2c^3e^4x - 2a^2b^2c^2d^3e - 4a^2b^3d^3e^2x - 2b^3c^2d^2e^3x - 3a^2b^2c^2d^2e^2 - 6a^2c^2d^2e^2x + 8a^2b^2c^2d^2e^3x + 14a^2b^2c^2d^3e^2x - 6a^2b^2c^2d^2e^2x)*(b^6e^2 + b^5e^2(b^2 - 4ac)^(1/2) + a^2b^4d^2 + 4a^4c^2d^2 - 4a^3c^3e^2 - 5a^3b^2c^2d^2 + a^2b^3d^2(b^2 - 4ac)^(1/2) - 2a^2b^5d^2e + 13a^2b^2c^2e^2 - 7a^2b^4c^2e^2 + 12a^2b^3c^2d^2e - 16a^3b^2c^2d^2e - 3a^3b^2c^2d^2e(b^2 - 4ac)^(1/2) - 5a^2b^3c^2e^2(b^2 - 4ac)^(1/2) - 4a^2b^3c^2e^2(b^2 - 4ac)^(1/2) - 2a^2b^4d^2e(b^2 - 4ac)^(1/2) + 8a^2b^2c^2d^2e(b^2 - 4ac)^(1/2)))/(2c^3(4ac - b^2)(a^2d^2 + c^2e^2 - b^2d^2e)) - (a^2e^2x*(2a^4b^2d^7 - 3a^5c^2d^7 + 6b^3c^3e^7 - 2b^6d^3e^4 + 4a^2b^5d^4e^3 - 4a^3b^3d^6e + 24a^2c^4d^6e - 5b^4c^2d^6e - b^5c^2d^2e^5 + 32a^3c^3d^3e^4 - 7a^4c^2d^5e^2 - 24a^2b^3c^4e^7 + 9a^4b^2c^6e - 36a^2b^2c^2d^3e^4 + 14a^2b^2c^3d^6e + 15a^2b^4c^2d^3e^4 + 16a^2b^3c^2d^2e^5 - 48a^2b^2c^3d^2e^5 - 24a^2b^3c^2d^4e^3 + 32a^3b^2c^2d^4e^3 + 4a^3b^2c^2d^5e^2)))/(c^2d^3(a^2d^2 + c^2e^2 - b^2d^2e)) + (a^2e^2(2a^2b^2c^2e^2 - 7a^2b^4c^2e^2 + 12a^2b^3c^2d^2e - 16a^3b^2c^2d^2e - 3a^3b^2c^2d^2e(b^2 - 4ac)^(1/2) - 5a^2b^3c^2e^2(b^2 - 4ac)^(1/2) - 4a^2b^3c^2e^2(b^2 - 4ac)^(1/2) - 2a^2b^4d^2e(b^2 - 4ac)^(1/2) + 8a^2b^2c^2d^2e(b^2 - 4ac)^(1/2)))/(2c^3(4ac - b^2)(a^2d^2 + c^2e^2 - b^2d^2e)) - (x*(18a^3c^6e^11 + 9a^2b^4c^4e^11 + a^2b^8d^4e^7 + a^7b^2d^10e - 36a^2b^2c^5e^11 - 2a^2b^7d^5e^6 + a^3b^6d^6e^5 + a^5b^4d^8e^3 - 2a^6b^3d^9e^2 + 6a^4c^5d^2e^9 - 10a^5c^4d^4e^7 - 12a^6c^3d^6e^5 + 3a^7c^2d^8e^3 + 44a^2b^4c^3d^2e^9 - 2a^2b^5c^2d^3e^8 - 85a^3b^2c^4d^2e^9 - 46a^3b^3c^3d^3e^8 + 45a^3b^4c^2d^4e^7 - 42a^4b^2c^3d^4e^7 - 56a^4b^3c^2d^5e^6 + 19a^5b^2c^2d^6e^5 - 6a^2b^5c^3d^6e^10 + 2a^2b^7c^2d^3e^8 + 42a^3b^2c^5d^6e^10 + 2a^7b^2c^9e^2 - 5a^2b^6c^2d^2e^9 + 6a^2b^3c^4d^6e^10 - 12a^2b^6c^2d^4e^7 + 16a^3b^5c^2d^5e^6 + 88a^4b^2c^4d^3e^8 - 6a^4b^4c^2d^6e^5 + 62a^5b^2c^3d^5e^6 - 2a^6b^2c^2d^7e^4 - 2a^6b^2c^2d^8e^3))/(c^4d^6(a^2d^2 + c^2e^2 - b^2d^2e)) + (b^6e^2 +
\end{aligned}$$

$$\begin{aligned}
& - 4*a*c)^{(1/2)} + 5*a*b^3*c*e^2*(b^2 - 4*a*c)^{(1/2)} + 4*a^3*c^2*d*e*(b^2 - \\
& 4*a*c)^{(1/2)} - 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^{(1/2)} + 2*a*b^4*d*e*(b^2 - 4*a \\
& *c)^{(1/2)} - 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^{(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d \\
& ^2 + c*e^2 - b*d*e)^2) - (a*e*x*(2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^ \\
& 7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 - \\
& 5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 + 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 - \\
& 24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e - 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d* \\
& e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24 \\
& *a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2)))/(c^2*d^3* \\
& (a*d^2 + c*e^2 - b*d*e)))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^4*d \\
& ^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - \\
& 4*a*c)^{(1/2)} - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^ \\
& 3*c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^3*c* \\
& e^2*(b^2 - 4*a*c)^{(1/2)} + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^{(1/2)} - 5*a^2*b*c^2*e \\
& ^2*(b^2 - 4*a*c)^{(1/2)} + 2*a*b^4*d*e*(b^2 - 4*a*c)^{(1/2)} - 8*a^2*b^2*c*d*e* \\
& (b^2 - 4*a*c)^{(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x* \\
& (18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a \\
& ^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2 \\
& *a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6* \\
& e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 - \\
& 85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 - \\
& 42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 - \\
& 6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^ \\
& 9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 + \\
& 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5 \\
& *b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^ \\
& 2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^4*d^2 \\
& + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4*a \\
& *c)^{(1/2)} - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c \\
& *d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^3*c*e^2 \\
& *(b^2 - 4*a*c)^{(1/2)} + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^{(1/2)} - 5*a^2*b*c^2*e^2* \\
& (b^2 - 4*a*c)^{(1/2)} + 2*a*b^4*d*e*(b^2 - 4*a*c)^{(1/2)} - 8*a^2*b^2*c*d*e*(b^ \\
& 2 - 4*a*c)^{(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e \\
& ^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4 \\
& + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e - \\
& 12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^ \\
& 2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c \\
& ^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a* \\
& d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 - b^5*e^2*(b^2 - 4*a*c)^{(1/2)} + a^2*b^4*d \\
& ^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 - a^2*b^3*d^2*(b^2 - 4 \\
& *a*c)^{(1/2)} - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3 \\
& *c*d*e - 16*a^3*b*c^2*d*e + 3*a^3*b*c*d^2*(b^2 - 4*a*c)^{(1/2)} + 5*a*b^3*c*e \\
& ^2*(b^2 - 4*a*c)^{(1/2)} + 4*a^3*c^2*d*e*(b^2 - 4*a*c)^{(1/2)} - 5*a^2*b*c^2*e^ \\
& 2*(b^2 - 4*a*c)^{(1/2)} + 2*a*b^4*d*e*(b^2 - 4*a*c)^{(1/2)} - 8*a^2*b^2*c*d*e*(\\
& b^2 - 4*a*c)^{(1/2)))/(2*(4*a*c^6*e^4 + 4*a^3*c^4*d^4 - b^2*c^5*e^4 + 2*b^3*
\end{aligned}$$

$$c^4*d*e^3 - a^2*b^2*c^3*d^4 + 8*a^2*c^5*d^2*e^2 - b^4*c^3*d^2*e^2 - 8*a*b*c^5*d*e^3 + 2*a*b^3*c^3*d^3*e - 8*a^2*b*c^4*d^3*e + 2*a*b^2*c^4*d^2*e^2) + (\log(x)*(3*c^2*e^2 - d^2*(a*c - b^2) + 2*b*c*d*e))/(c^3*d^4)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)

[Out] Timed out

$$3.79 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} dx$$

Optimal. Leaf size=981

$$\frac{2}{11} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{d + ex} x^5 + \frac{2(ad + be) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} (d + ex)^{7/2} x}{99ae^4} - \frac{2(29a^2d^2 + 8b^2e^2 + ae(19bd - 18ce)) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{693a^2e^4}$$

[Out] $2/3465*(233*a^3*d^3+48*b^3*e^3+a*b*e^2*(67*b*d-157*c*e)+4*a^2*d*e*(18*b*d-37*c*e))*x*(e*x+d)^{(3/2)}*(a+c/x^2+b/x)^{(1/2)}/a^3/e^4-2/693*(29*a^2*d^2+8*b^2*e^2+a*e*(19*b*d-18*c*e))*x*(e*x+d)^{(5/2)}*(a+c/x^2+b/x)^{(1/2)}/a^2/e^4+2/99*(a*d+b*e)*x*(e*x+d)^{(7/2)}*(a+c/x^2+b/x)^{(1/2)}/a/e^4-2/3465*(187*a^4*d^4+64*b^4*e^4+4*a*b^2*e^3*(7*b*d-69*c*e)-4*a^3*d^2*e*(2*b*d+3*c*e)+3*a^2*e^2*(3*b^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/a^4/e^4+2/11*x^5*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}+1/3465*(128*a^5*d^5+128*b^5*e^5-4*a^4*d^3*e*(14*b*d-27*c*e)-8*a*b^3*e^4*(7*b*d+87*c*e)-a^2*b*e^3*(37*b^2*d^2-258*b*c*d*e-771*c^2*e^2)-a^3*d*e^2*(37*b^2*d^2-135*b*c*d*e+156*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2), (-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3465*(a*d^2-e*(b*d-c*e))*(128*a^4*d^4-64*b^4*e^4-4*a*b^2*e^3*(7*b*d-69*c*e)+4*a^3*d^2*e*(2*b*d+3*c*e)-3*a^2*e^2*(3*b^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2), (-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(e*x+d)^(1/2)$

Rubi [A] time = 6.17, antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$\frac{2}{11} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{d + ex} x^5 + \frac{2(ad + be) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} (d + ex)^{7/2} x}{99ae^4} - \frac{2(29a^2d^2 + 8b^2e^2 + ae(19bd - 18ce)) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{693a^2e^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]

[Out]
$$\begin{aligned} & (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*\text{Sqrt}[a + c/x^2 + b/x]*x*\text{Sqrt}[d + e*x]) / (3465*a^4*e^4) + (2*\text{Sqrt}[a + c/x^2 + b/x]*x^5*\text{Sqrt}[d + e*x]) / 11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*\text{Sqrt}[a + c/x^2 + b/x]*x*(d + e*x)^{(3/2)}) / (3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*\text{Sqrt}[a + c/x^2 + b/x]*x*(d + e*x)^{(5/2)}) / (693*a^2*e^4) + (2*(a*d + b*e)*\text{Sqrt}[a + c/x^2 + b/x]*x*(d + e*x)^{(7/2)}) / (99*a*e^4) + (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*\text{Sqrt}[a + c/x^2 + b/x]*x*\text{Sqrt}[d + e*x]*\text{Sqrt}[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*a*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*e)/(2*a*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)]) / (3465*a^5*e^5*\text{Sqrt}[(a*(d + e*x))/(2*a*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)])*(c + b*x + a*x^2) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*\text{Sqrt}[a + c/x^2 + b/x]*x*\text{Sqrt}[(a*(d + e*x))/(2*a*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)])*\text{Sqrt}[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*a*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*e)/(2*a*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)]) / (3465*a^5*e^5*\text{Sqrt}[d + e*x]*(c + b*x + a*x^2)) \end{aligned}$$

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 718

Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]) / (c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2

$- 4ac, 2] + 2cx)/(2\sqrt{b^2 - 4ac, 2})], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d^2 - bde + ae^2, 0] \&\& \text{NeQ}[2cd - b^2e, 0] \&\& \text{EqQ}[m^2, 1/4]$

Rule 843

$\text{Int}[(d + e)x^m((f + g)x + (a + b)x + c)x^2)^p, x_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + ex)^{m+1}(a + bx + cx^2)^p, x], x] + \text{Dist}[(ef - dg)/e, \text{Int}[(d + ex)^m(a + bx + cx^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d^2 - bde + ae^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 918

$\text{Int}[(d + e)x^m \sqrt{(f + g)x} \sqrt{(a + b)x + c} x^2], x_Symbol] \rightarrow \text{Simp}[(2(d + ex)^{m+1} \sqrt{f + gx} \sqrt{a + bx + cx^2})/(e(2m + 5)), x] - \text{Dist}[1/(e(2m + 5)), \text{Int}[(d + ex)^m \text{Simp}[bdf - 3aef + adg + 2(cdf - bef + bdg - aeg)x - (cef - 3cdg + beg)x^2, x]]/(\sqrt{f + gx} \sqrt{a + bx + cx^2}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m\}, x] \&\& \text{NeQ}[ef - dg, 0] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d^2 - bde + ae^2, 0] \&\& \text{IntegerQ}[2m] \&\& !\text{LtQ}[m, -1]$

Rule 1573

$\text{Int}[x^m((a + b)x^{mn} + c)x^{mn2})^p((d + e)x^n)^q, x_Symbol] \rightarrow \text{Dist}[(x^{2n \text{FracPart}[p]}(a + b/x^n + c/x^{2n}))^{\text{FracPart}[p]}]/(c + bx^n + ax^{2n})^{\text{FracPart}[p]}, \text{Int}[x^{m - 2np}(d + ex^n)^q(c + bx^n + ax^{2n})^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{EqQ}[mn, -n] \&\& \text{EqQ}[mn2, 2mn] \&\& !\text{IntegerQ}[p] \&\& !\text{IntegerQ}[q] \&\& \text{PosQ}[n]$

Rule 1653

$\text{Int}[(Pq)(d + e)x^m((a + b)x + c)x^2)^p, x_Symbol] \rightarrow \text{With}\{q = \text{Expon}[Pq, x], f = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(f(d + ex)^{m+q-1}(a + bx + cx^2)^{p+1})/(c^q e^{q-1}(m+q+2p+1)), x] + \text{Dist}[1/(c^q e^q(m+q+2p+1)), \text{Int}[(d + ex)^m(a + bx + cx^2)^p \text{ExpandToSum}[c^q e^q(m+q+2p+1)Pq - cf(m+q+2p+1)(d + ex)^q - f(d + ex)^{q-2}(bd^2e^2(p+1) + ae^2(m+q-1) - cd^2(m+q+2p+1) - e(2cd - b^2e)(m+q+p)x), x], x], x] /; \text{GtQ}[q, 1] \&\& \text{NeQ}[m+q+2p+1, 0] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4ac, 0] \&\& \text{NeQ}[c^2d^2 - bde + ae^2, 0] \&\& !(IGtQ[m, 0] \&\& \text{RationalQ}[a, b, c, d, e] \&\& (\text{IntegerQ}[p] || \text{ILtQ}[p + 1/2, 0]))$

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int x^3 \sqrt{d + ex} \sqrt{c + bx + ax^2} dx}{\sqrt{c + bx + ax^2}} \\
&= \frac{2}{11} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^5 \sqrt{d + ex} - \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{x^3(-3cd-2(bd+ce)x-(ad+be)x^2)}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{11\sqrt{c + bx + ax^2}} \\
&= \frac{2}{11} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^5 \sqrt{d + ex} + \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x(d + ex)^{7/2}}{99ae^4} - \frac{(2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}})}{99ae^4} \\
&= \frac{2}{11} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^5 \sqrt{d + ex} - \frac{2(29a^2d^2 + 8b^2e^2 + ae(19bd - 18ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{693a^2e^4} \\
&= \frac{2}{11} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^5 \sqrt{d + ex} + \frac{2(233a^3d^3 + 48b^3e^3 + abe^2(67bd - 157ce) + 4a^2d^2e)}{3465a^4e^4} \\
&= -\frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d + 3bd^2 + 3e^2d^2))}{3465a^4e^4} \\
&= -\frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d + 3bd^2 + 3e^2d^2))}{3465a^4e^4} \\
&= -\frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d + 3bd^2 + 3e^2d^2))}{3465a^4e^4} \\
&= -\frac{2(187a^4d^4 + 64b^4e^4 + 4ab^2e^3(7bd - 69ce) - 4a^3d^2e(2bd + 3ce) + 3a^2e^2(3b^2d + 3bd^2 + 3e^2d^2))}{3465a^4e^4}
\end{aligned}$$

Mathematica [C] time = 14.20, size = 10904, normalized size = 11.12

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x], x]

[Out] Result too large to show

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex + d} x^4 \sqrt{\frac{ax^2 + bx + c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*x^4*sqrt((a*x^2 + b*x + c)/x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)

maple [B] time = 0.16, size = 11938, normalized size = 12.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2), x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{d + ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)

[Out] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)

[Out] Timed out

$$3.80 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^3 \sqrt{d + ex} dx$$

Optimal. Leaf size=778

$$\frac{4x(d+ex)^{3/2} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} (8a^2d^2 + ae(4bd - 7ce) + 3b^2e^2)}{315a^2e^3} + \frac{2x\sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} (19a^3d^3 - 6a^2cde^2 + 3abe^2)}{315a^3e^3}$$

[Out] $-4/315*(8*a^2*d^2+3*b^2*e^2+a*e*(4*b*d-7*c*e))*x*(e*x+d)^{(3/2)}*(a+c/x^2+b/x)^{(1/2)}/a^2/e^3+2/63*(a*d+b*e)*x*(e*x+d)^{(5/2)}*(a+c/x^2+b/x)^{(1/2)}/a/e^3+2/315*(19*a^3*d^3-6*a^2*c*d*e^2+8*b^3*e^3+3*a*b*e^2*(b*d-9*c*e))*x*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/a^3/e^3+2/9*x^4*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}-2/315*(8*a^4*d^4+8*b^4*e^4-a^3*d^2*e*(4*b*d-9*c*e)-4*a*b^2*e^3*(b*d+9*c*e)-3*a^2*e^2*(b^2*d^2-5*b*c*d*e-7*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^2^(1/2), (-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))^2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^(1/2))/a^4/e^4/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(16*a^3*d^3+6*a^2*c*d*e^2-8*b^3*e^3-3*a*b*e^2*(b*d-9*c*e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^2^(1/2), (-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))^2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^(1/2))*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^4/e^4/(a*x^2+b*x+c)/(e*x+d)^(1/2)$

Rubi [A] time = 2.35, antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(-3a^2e^2(b^2d^2-5bcde-7c^2e^2)-a^3d^2e(4bd-9ce)+8a^4d^3)$$

$$315a^4e^4(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e(\sqrt{b^2-4ac}+...)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x], x]

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)]/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
- 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

$c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 918

$\text{Int}[(d + e*x)^m*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2], x_Symbol] \rightarrow \text{Simp}[(2*(d + e*x)^{m+1}*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(e*(2*m + 5)), x] - \text{Dist}[1/(e*(2*m + 5)), \text{Int}[(d + e*x)^m*\text{Simp}[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x]]/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[2*m] \&\& !\text{LtQ}[m, -1]$

Rule 1573

$\text{Int}[x^m*((a + b*x)^{mn} + c*x^{mn2})^p, x_Symbol] \rightarrow \text{Dist}[(x^{2*n*\text{FracPart}[p]}*(a + b/x^n + c/x^{2*n})^{\text{FracPart}[p]})/(c + b*x^n + a*x^{2*n})^{\text{FracPart}[p]}, \text{Int}[x^{m - 2*n*p}*(d + e*x^n)^q*(c + b*x^n + a*x^{2*n})^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \&\& \text{EqQ}[mn, -n] \&\& \text{EqQ}[mn2, 2*mn] \&\& !\text{IntegerQ}[p] \&\& !\text{IntegerQ}[q] \&\& \text{PosQ}[n]$

Rule 1653

$\text{Int}[(Pq)*(d + e*x)^m*((a + b*x + c*x^2)^p), x_Symbol] \rightarrow \text{With}\{q = \text{Expon}[Pq, x], f = \text{Coeff}[Pq, x, \text{Expon}[Pq, x]]\}, \text{Simp}[(f*(d + e*x)^{m+q-1}*(a + b*x + c*x^2)^{p+1})/(c*e^{q-1}*(m+q+2*p+1)), x] + \text{Dist}[1/(c*e^q*(m+q+2*p+1)), \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p*\text{ExpandToSum}[c*e^q*(m+q+2*p+1)*Pq - c*f*(m+q+2*p+1)*(d + e*x)^q - f*(d + e*x)^{q-2}*(b*d*e*(p+1) + a*e^2*(m+q-1) - c*d^2*(m+q+2*p+1) - e*(2*c*d - b*e)*(m+q+p)*x], x], x] /; \text{GtQ}[q, 1] \&\& \text{NeQ}[m+q+2*p+1, 0] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x\} \&\& \text{PolyQ}[Pq, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !(\text{IGtQ}[m, 0] \&\& \text{RationalQ}[a, b, c, d, e] \&\& (\text{IntegerQ}[p] || \text{ILtQ}[p + 1/2, 0]))$

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^3 \sqrt{d + ex} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int x^2 \sqrt{d + ex} \sqrt{c + bx + ax^2} dx}{\sqrt{c + bx + ax^2}} \\
&= \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} - \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{x^2(-3cd - 2(bd + ce)x - (ad + be)x^2)}{\sqrt{d + ex} \sqrt{c + bx + ax^2}} dx}{9\sqrt{c + bx + ax^2}} \\
&= \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} + \frac{2(ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x(d + ex)^{5/2}}{63ae^3} - \frac{\left(2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right)}{315a^2e^3} \\
&= \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^4 \sqrt{d + ex} - \frac{4(8a^2d^2 + 3b^2e^2 + ae(4bd - 7ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{315a^2e^3} \\
&= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{315a^3e^3} + \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \\
&= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{315a^3e^3} + \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \\
&= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{315a^3e^3} + \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \\
&= \frac{2(19a^3d^3 - 6a^2cde^2 + 8b^3e^3 + 3abe^2(bd - 9ce))\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{315a^3e^3} + \frac{2}{9} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}}
\end{aligned}$$

Mathematica [C] time = 13.74, size = 7531, normalized size = 9.68

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x], x]

[Out] Result too large to show

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}x^3\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*x^3*sqrt((a*x^2 + b*x + c)/x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)

maple [B] time = 0.07, size = 9182, normalized size = 11.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

```
[Out] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2), x)
```

```
[Out] Timed out
```

$$3.81 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^2 \sqrt{d + ex} dx$$

Optimal. Leaf size=636

$$\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2)}{105a^2e^2} \quad 2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a}{x}}$$

[Out] $-2/105*x*(4*a^2*d^2+4*b^2*e^2-a*e*(2*b*d-5*c*e)-3*a*e*(a*d-4*b*e)*x)*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/a^2/e^2+2/7*x*(a*x^2+b*x+c)*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/a+1/105*(8*a^3*d^3+8*b^3*e^3-a^2*d*e*(5*b*d-16*c*e)-a*b*e^2*(5*b*d+29*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)},(-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}/a^3/e^3/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}-2/105*(8*a^2*d^2-4*b^2*e^2-a*e*(b*d-10*c*e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)},(-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/a^3/e^3/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}$

Rubi [A] time = 0.99, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1573, 832, 814, 843, 718, 424, 419}

$$\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2)}{105a^2e^2} \quad 2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a}{x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x], x]

[Out] $(-2*\text{Sqrt}[a + c/x^2 + b/x]*x*\text{Sqrt}[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*\text{Sqrt}[a + c/x^2 + b/x]*x^2*\text{Sqrt}[d + e*x])$

$$\frac{1}{x} \sqrt{d + ex} (c + bx + ax^2) / (7a) + (\sqrt{2} \sqrt{b^2 - 4ac}) (8a^3 d^3 + 8b^3 e^3 - a^2 d e (5bd - 16ce) - a b e^2 (5bd + 29ce)) \sqrt{a + c/x^2 + b/x} \sqrt{d + ex} \sqrt{-((a(c + bx + ax^2))/(b^2 - 4ac))} \text{EllipticE}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4ac} + 2ax)/\sqrt{b^2 - 4ac}}]/\sqrt{2}], (-2\sqrt{b^2 - 4ac}e)/(2ad - (b + \sqrt{b^2 - 4ac})e)] / (105a^3 e^3 \sqrt{(a(d + ex))/(2ad - (b + \sqrt{b^2 - 4ac})e)}) (c + bx + ax^2) - (2\sqrt{2} \sqrt{b^2 - 4ac}) (8a^2 d^2 - 4b^2 e^2 - a e (bd - 10ce)) (a d^2 - e (bd - ce)) \sqrt{a + c/x^2 + b/x} \sqrt{(a(d + ex))/(2ad - (b + \sqrt{b^2 - 4ac})e)} \sqrt{-((a(c + bx + ax^2))/(b^2 - 4ac))} \text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4ac} + 2ax)/\sqrt{b^2 - 4ac}}]/\sqrt{2}], (-2\sqrt{b^2 - 4ac}e)/(2ad - (b + \sqrt{b^2 - 4ac})e)] / (105a^3 e^3 \sqrt{d + ex} (c + bx + ax^2))$$

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[
  (1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
  (Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[
  (2*Rt[b^2 - 4ac, 2]*(d + ex)^m*Sqrt[-((c*(a + bx + cx^2))/(b^2 - 4ac))]) / (c*Sqrt[a + bx + cx^2]*((2*c*(d + ex))/(2*c*d - b*e - e*Rt[b^2 - 4ac, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4ac, 2]*x^2)/(2*c*d - b*e - e*Rt[b^2 - 4ac, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2 - 4ac, 2] + 2*c*x)/(2*Rt[b^2 - 4ac, 2])]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4ac, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[
  ((d + ex)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + bx + cx^2)^p) / (c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + ex)^m*(a + bx + cx^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```

2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))) * x, x], x], x]
/; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) && !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])

```

Rule 832

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1573

```

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] &&
!IntegerQ[q] && PosQ[n]

```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x^2 \sqrt{d + ex} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x\right) \int x \sqrt{d + ex} \sqrt{c + bx + ax^2} dx}{\sqrt{c + bx + ax^2}} \\
&= \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} (c + bx + ax^2)}{7a} + \frac{\left(2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x\right) \int \frac{\left(\frac{1}{2}(-3bd - ce) + \frac{1}{2}(ad + bx + ax^2)\right) \sqrt{d + ex}}{\sqrt{c + bx + ax^2}} dx}{7a\sqrt{c + bx + ax^2}} \\
&= -\frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} (4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x)}{105a^2e^2} + \dots \\
&= -\frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} (4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x)}{105a^2e^2} + \dots \\
&= -\frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} (4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x)}{105a^2e^2} + \dots \\
&= -\frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} (4a^2d^2 + 4b^2e^2 - ae(2bd - 5ce) - 3ae(ad - 4be)x)}{105a^2e^2} + \dots
\end{aligned}$$

Mathematica [C] time = 13.05, size = 5350, normalized size = 8.41

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x], x]

[Out] Result too large to show

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex + d} x^2 \sqrt{\frac{ax^2 + bx + c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x + d)*x^2*sqrt((a*x^2 + b*x + c)/x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)

maple [B] time = 0.05, size = 6302, normalized size = 9.91

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{d + ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)

[Out] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{d + ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2), x)

[Out] Integral(x**2*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)

$$3.82 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} dx$$

Optimal. Leaf size=550

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)(ad^2-e(bd-ce))\sqrt{\frac{a(d+ex)}{2ad-e(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15a^2e^2\sqrt{d+ex}(ax^2+bx+c)}$$

[Out] $2/5*x*(e*x+d)^{(3/2)}*(a+c/x^2+b/x)^{(1/2)}/e-2/15*(2*a*d-b*e)*x*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/a/e-2/15*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)},(-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}/a^2/e^2/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}+2/15*(2*a*d-b*e)*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)},(-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/a^2/e^2/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}$

Rubi [A] time = 0.66, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1573, 734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)(ad^2-e(bd-ce))\sqrt{\frac{a(d+ex)}{2ad-e(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15a^2e^2\sqrt{d+ex}(ax^2+bx+c)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]

[Out] $(-2*(2*a*d-b*e)*Sqrt[a+c/x^2+b/x]*x*Sqrt[d+e*x])/(15*a*e)+(2*Sqrt[a+c/x^2+b/x]*x*(d+e*x)^{(3/2)})/(5*e)-(2*Sqrt[2]*Sqrt[b^2-4*a*c]*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*Sqrt[a+c/x^2+b/x]*x*Sqrt[d+e*x]*Sqrt[-((a*(c+b*x+a*x^2))/(b^2-4*a*c))]*EllipticE[ArcSin[Sqrt[(b+S$

```

qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)))/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))

```

Rule 419

```

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

```

Rule 424

```

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

```

Rule 718

```

Int[((d_) + (e_)*(x_))^(m_)/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Sy
mbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
- 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
b*e, 0] && EqQ[m^2, 1/4]

```

Rule 734

```

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_S
ymbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) &
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 832

```

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c

```

```

_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

```

Rule 843

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1573

```

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p]))*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] &&
!IntegerQ[q] && PosQ[n]

```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \sqrt{d + ex} \sqrt{c + bx + ax^2} dx}{\sqrt{c + bx + ax^2}} \\
&= \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x (d + ex)^{3/2}}{5e} - \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{\sqrt{d+ex}(bd-2ce+(2ad-be)x)}{\sqrt{c+bx+ax^2}} dx}{5e\sqrt{c + bx + ax^2}} \\
&= -\frac{2(2ad - be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{15ae} + \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x (d + ex)^{3/2}}{5e} - \frac{\left(2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right)}{(2ad - be)} \\
&= -\frac{2(2ad - be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{15ae} + \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x (d + ex)^{3/2}}{5e} + \frac{\left(2ad - be\right)}{\left(2\sqrt{2} \sqrt{b^2}\right)} \\
&= -\frac{2(2ad - be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{15ae} + \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x (d + ex)^{3/2}}{5e} - \frac{\left(2\sqrt{2} \sqrt{b^2}\right)}{2\sqrt{2} \sqrt{b^2}} \\
&= -\frac{2(2ad - be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex}}{15ae} + \frac{2\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x (d + ex)^{3/2}}{5e} - \frac{2\sqrt{2} \sqrt{b^2}}{2\sqrt{2} \sqrt{b^2}}
\end{aligned}$$

Mathematica [C] time = 11.77, size = 693, normalized size = 1.26

$$x \sqrt{a + \frac{bx+c}{x^2}} \left(\frac{4e^2(-a^2d^2+ae(bd+3ce)-b^2e^2)}{\sqrt{d+ex}} + \frac{i(d+ex) \sqrt{1-\frac{2(ad^2+e(ce-bd))}{(d+ex)(\sqrt{e^2(b^2-4ac)}+2ad-be)}} \sqrt{\frac{4(ad^2+e(ce-bd))}{(d+ex)(\sqrt{e^2(b^2-4ac)}-2ad+be)}} + 2 \left(a^2d(8ce^2-d\sqrt{e^2(b^2-4ac)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x], x]

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((4*e^2*(-(a^2*d^2) - b^2*e^2 + a*e*(b*d + 3*c*e
)))/Sqrt[d + e*x] + 2*a*e^2*Sqrt[d + e*x]*(b*e + a*(d + 3*e*x)) + (I*(d + e
*x)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*d) + c*e)))/((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)
*e^2])*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sq
rt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2]))) + (b^2*e^2*(b*e - Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*d*
(8*c*e^2 - d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(-2*b^2*d*e - 4*b*c*e^2 + b*d*S
qrt[(b^2 - 4*a*c)*e^2] + 3*c*e*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)
*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d -
b*e + Sqrt[(b^2 - 4*a*c)*e^2])))/Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/((15*a^2*e^3)
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex+d}x\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x*sqrt((a*x^2 + b*x + c)/x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

maple [B] time = 0.05, size = 4361, normalized size = 7.93

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] -1/15*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/((-4*a
*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c
+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3+2*2^(1/2)*(-a*(e*x+d)
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-
4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+
b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+
b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*d^3*e+8*2^(1/2)*(-a*
(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/
2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2
)))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-
4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*a^2*b*d^3*e+8*2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4
*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*
c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2
)^(1/2)*e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2))*a^2*c*d^2*e^2-8*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b
^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2
)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*
e+2*a*d-b*e))^(1/2))*a*b^2*d^2*e^2-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^2)^(1/2)*e-2*a
*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*
e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*e^4-12*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(
1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1
/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^2)^(1/2
)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*
a*d+b*e))^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2))*a^2*c*d^2*e^2+3*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)
)^(1/2),(-((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e)
)^(1/2))*a*b^2*d^2*e^2-6*x^4*a^3*e^4-8*x^3*a^3*d*e^3-8*x^3*a^2*b*e^4-2*x^2*
a^3*d^2*e^2-6*x^2*a^2*c*e^4-2*x^2*a*b^2*e^4-2*a^2*c*d^2*e^2-8*x*a^2*c*d*e^3
-2*x*a*b^2*d*e^3-2*x*a*b*c*e^4-12*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2
*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)))/((-4*a*c+b^2)^(1/2)*e-2*a*
```


$(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{2*a*d-b*e})^{(1/2)})*a*b*c*d*e^{-3-3*2^{(1/2)}}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{2*a*d-b*e})^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)})))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})^{(1/2)}*EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{2*a*d-b*e})^{(1/2)})*(-4*a*c+b^2)^{(1/2)}*a*b*d^2*e^2)/a^2/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/e^3$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)

[Out] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)

[Out] Integral(x*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)

$$3.83 \quad \int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} dx$$

Optimal. Leaf size=955

$$\frac{\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{d + ex} \sqrt{-\frac{a(ax^2 + bx + c)}{b^2 - 4ac}} E \left(\sin^{-1} \left(\frac{\sqrt{\frac{b + 2ax + \sqrt{b^2 - 4ac}}{\sqrt{b^2 - 4ac}}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2 - 4ac} e}{2ad - (b + \sqrt{b^2 - 4ac})e} \right) x}{3ae \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}} (ax^2 + bx + c) \quad 2\sqrt{2}$$

[Out] $2/3*x*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}+1/3*(a*d+b*e)*x*\text{EllipticE}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^{(1/2)}/a/e/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}-2/3*d*(a*d+b*e)*x*\text{EllipticF}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^{(1/2)}/a/e/(a*x^2+b*x+c)/(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/a/e/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}+4/3*(b*d+c*e)*x*\text{EllipticF}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)^{(1/2)}/a/e/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}-c*x*\text{EllipticPi}(2^{(1/2)}*a^{(1/2)}*(e*x+d)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}, 1/2*(e*(-4*a*c+b^2)^{(1/2)}+2*a*d-b*e)/a/d, ((b-2*a*d/e-(-4*a*c+b^2)^{(1/2)})/(b-2*a*d/e+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*2^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/(a*x^2+b*x+c)/a^{(1/2)}$

Rubi [A] time = 3.31, antiderivative size = 955, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1449, 918, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{d + ex} \sqrt{-\frac{a(ax^2 + bx + c)}{b^2 - 4ac}} E \left(\sin^{-1} \left(\frac{\sqrt{\frac{b + 2ax + \sqrt{b^2 - 4ac}}{\sqrt{b^2 - 4ac}}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2 - 4ac} e}{2ad - (b + \sqrt{b^2 - 4ac})e} \right) x}{3ae \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}} (ax^2 + bx + c) \quad 2\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a
*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)))/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)))/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
- (2*a*d)/e)]/(Sqrt[a]*(c + b*x + a*x^2))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/((a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]

Rule 538

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

Rule 718

Int[((d_) + (e_)*(x_)^m)/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]

Rule 843

Int[((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^p, x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 918

Int[((d_) + (e_)*(x_)^m)*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d + e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && !LtQ[m,

-1]

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(f_.) + (g_.)*(x_.)]*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1449

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + b/x^n + c/x^(2*n)))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p]/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex} \, dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{\sqrt{d+ex} \sqrt{c+bx+ax^2}}{x} \, dx}{\sqrt{c+bx+ax^2}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} - \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{-3cd-2(bd+ce)x-(ad+be)x^2}{x\sqrt{d+ex} \sqrt{c+bx+ax^2}} \, dx}{3\sqrt{c+bx+ax^2}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} - \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \left(-\frac{2(bd+ce)}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} - \frac{3cd}{x\sqrt{d+ex} \sqrt{c+bx+ax^2}}\right) \, dx}{3\sqrt{c+bx+ax^2}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{\left(cd\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{1}{x\sqrt{d+ex} \sqrt{c+bx+ax^2}} \, dx}{\sqrt{c+bx+ax^2}} - \frac{(-ad-b)}{\sqrt{c+bx+ax^2}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{\left(cd\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{b - \sqrt{b^2 - 4ac}} + 2ax \sqrt{b + \sqrt{b^2 - 4ac}}\right)}{c + b} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{4\sqrt{2} \sqrt{b^2 - 4ac} (bd + ce) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})}}}{3a\sqrt{d+ex}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} \sqrt{-\frac{a(c)}{2ad - (b + \sqrt{b^2 - 4ac})}}}{3ae \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})}}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} \sqrt{-\frac{a(c)}{2ad - (b + \sqrt{b^2 - 4ac})}}}{3ae \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})}}} \\
&= \frac{2}{3} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} + \frac{\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d+ex} \sqrt{-\frac{a(c)}{2ad - (b + \sqrt{b^2 - 4ac})}}}{3ae \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})}}}
\end{aligned}$$

Mathematica [C] time = 10.53, size = 1258, normalized size = 1.32

$$x\sqrt{a + \frac{c+bx}{x^2}} \left(\frac{4(ad+be)\sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}}{(d+ex)^2} (c+x(b+ax))e^2 + \frac{6i\sqrt{2}ac\sqrt{\frac{-2ce^2+2adx+b(d-ex)e+\sqrt{(b^2-4ac)e^2}}{(2ad-be+\sqrt{(b^2-4ac)e^2})(d+ex)}}}{\sqrt{\frac{2ce^2-2adx+b(ex-d)e+\sqrt{(b^2-4ac)e^2}}{(-2ad+be+\sqrt{(b^2-4ac)e^2})}}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x], x]

[Out] $(2*x*\text{Sqrt}[d + e*x]*\text{Sqrt}[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^{(3/2)}*\text{Sqrt}[a + (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*\text{Sqrt}[(a*d^2 + e*(-(b*d) + c*e)]/(-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2]))*(c + x*(b + a*x)))/(d + e*x)^2 - (I*\text{Sqrt}[2]*(a*d + b*e)*(2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])* \text{Sqrt}[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))* \text{Sqrt}[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])*(d + e*x)))* \text{EllipticE}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])])/ \text{Sqrt}[d + e*x]], -((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])))/ \text{Sqrt}[d + e*x] + (I*\text{Sqrt}[2]*(b*e*(-(b*e) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]) + a*(3*b*d*e - 2*c*e^2 + d*\text{Sqrt}[(b^2 - 4*a*c)*e^2]))*\text{Sqrt}[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))* \text{Sqrt}[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])*(d + e*x))* \text{EllipticF}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])])/ \text{Sqrt}[d + e*x]], -((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])))/ \text{Sqrt}[d + e*x] + ((6*I)*\text{Sqrt}[2]*a*c*e^2*\text{Sqrt}[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])*(d + e*x))* \text{Sqrt}[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + \text{Sqrt}[(b^2 - 4*a*c)*e^2]*(d + e*x)))/((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])*(d + e*x))* \text{EllipticPi}[(d*(2*a*d - b*e - \text{Sqrt}[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) + c*e))], I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])])/ \text{Sqrt}[d + e*x]], -((-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2])))/ \text{Sqrt}[d + e*x])/((6*a*e^2*\text{Sqrt}[(a*d^2 + e*(-(b*d) + c*e)]/(-2*a*d + b*e + \text{Sqrt}[(b^2 - 4*a*c)*e^2]))*(c + x*(b + a*x)))$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)

maple [B] time = 0.05, size = 3023, normalized size = 3.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)

[Out] $\frac{1}{3} \left(\frac{(a^2x^2 + b^2x + c)^{1/2}}{x^2} \right)^{1/2} x (ex+d)^{1/2} / a (2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be}))^{1/2} (e(-2ax + (-4ac + b^2)^{1/2} - b) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (e(b + 2ax + (-4ac + b^2)^{1/2})) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} \text{EllipticF}(2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be}))^{1/2}, (-((-4ac + b^2)^{1/2} e^{-2ad+be}) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (-4ac + b^2)^{1/2} a d^2 e^{-1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} (e(-2ax + (-4ac + b^2)^{1/2} - b) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (e(b + 2ax + (-4ac + b^2)^{1/2})) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} \text{EllipticF}(2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be}))^{1/2}, (-((-4ac + b^2)^{1/2} e^{-2ad+be}) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (-4ac + b^2)^{1/2} b d e^2 2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} (e(-2ax + (-4ac + b^2)^{1/2} - b) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (e(b + 2ax + (-4ac + b^2)^{1/2})) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} \text{EllipticF}(2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be}))^{1/2}, (-((-4ac + b^2)^{1/2} e^{-2ad+be}) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (-4ac + b^2)^{1/2} c e^3 + 3 2^{1/2} (-a(ex+d)) / ((-4ac + b^2)^{1/2} e^{-2ad+be})^{1/2} (e(-2ax + (-4ac + b^2)^{1/2} - b) / ((-4ac + b^2)^{1/2} e^{2ad-be}))^{1/2} (e(b + 2ax + (-4ac + b^2)^{1/2})) /$

$$2)^{(1/2)} * e^{-2*a*d+b*e})^{(1/2)} * \text{EllipticPi}(2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)}, -1/2 * ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}) / d/a, (-((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)}) * b*c*e^3 + 2*x^3*a^2*e^3 + 2*x^2*a^2*d*e^2 + 2*x^2*a*b*e^3 + 2*x*a*b*d*e^2 + 2*x*a*c*e^3 + 2*a*c*d*e^2) / e^2 / (a*e*x^3 + a*d*x^2 + b*e*x^2 + b*d*x + c*e*x + c*d)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d + ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)

[Out] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)

[Out] Timed out

$$3.84 \quad \int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex}}{x} dx$$

Optimal. Leaf size=929

$$\frac{3\sqrt{b^2-4ac} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x \sqrt{d+ex} \sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}} E \left(\sin^{-1} \left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}e}{2ad-(b+\sqrt{b^2-4ac})e} \right)}{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad-(b+\sqrt{b^2-4ac})e}} (ax^2+bx+c)}$$

[Out] $-(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}+3/2*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)))/(-4*a*c+b^2)^{(1/2)})^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}/(a*x^2+b*x+c)*2^{(1/2)}/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}-3*d*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)))/(-4*a*c+b^2)^{(1/2)})^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)})*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}+2*(a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)))/(-4*a*c+b^2)^{(1/2)})^{(1/2)})*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)})*2^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/a/(a*x^2+b*x+c)/(e*x+d)^{(1/2)}-1/2*(b*d+c*e)*x*EllipticPi(2^{(1/2)}*a^{(1/2)}*(e*x+d)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}, 1/2*(e*(-4*a*c+b^2)^{(1/2)}+2*a*d-b*e)/a/d, ((b-2*a*d/e-(-4*a*c+b^2)^{(1/2)})/(b-2*a*d/e+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/d/(a*x^2+b*x+c)*2^{(1/2)}/a^{(1/2)}$

Rubi [A] time = 2.72, antiderivative size = 929, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1573, 916, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{3\sqrt{b^2-4ac} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x \sqrt{d+ex} \sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}} E \left(\sin^{-1} \left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}e}{2ad-(b+\sqrt{b^2-4ac})e} \right)}{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad-(b+\sqrt{b^2-4ac})e}} (ax^2+bx+c)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(Sqrt[2]*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)]/(Sqrt[2]*Sqrt[a]*d*(c + b*x + a*x^2))
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c)
```


), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rule 538

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

Rule 718

Int[((d_) + (e_)*(x_))^(m_)/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m^2, 1/4]

Rule 843

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 916

Int[((d_) + (e_)*(x_))^(m_)*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ

erQ[2*m] && LtQ[m, -1]

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(f_.) + (g_.)*(x_.)]*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^p_)*((d_) + (e_.)*(x_)^(n_.))^q_., x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + b/x^n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{x} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{\sqrt{d+ex} \sqrt{c+bx+ax^2}}{x^2} dx}{\sqrt{c + bx + ax^2}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{bd+ce+2(ad+be)x+3aex^2}{x\sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{2\sqrt{c + bx + ax^2}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \left(\frac{2(ad+be)}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} + \frac{bd+ce}{x\sqrt{d+ex} \sqrt{c+bx+ax^2}}\right) dx}{2\sqrt{c + bx + ax^2}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{\left(3ae\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{x}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{2\sqrt{c + bx + ax^2}} + \frac{(ad + be)}{2\sqrt{c + bx + ax^2}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{\left((bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{b - \sqrt{b^2 - 4ac}} + 2ax \sqrt{b + \sqrt{b^2 - 4ac}}\right)}{2(c + bx + ax^2)} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{2\sqrt{2} \sqrt{b^2 - 4ac} (ad + be) \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}}{a\sqrt{d + ex}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{3\sqrt{b^2 - 4ac} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} \sqrt{-\frac{a(c+bx+ax^2)}{b^2 - 4ac}} E \left(\frac{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}}{c + bx + ax^2} \right)}{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{3\sqrt{b^2 - 4ac} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} \sqrt{-\frac{a(c+bx+ax^2)}{b^2 - 4ac}} E \left(\frac{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}}{c + bx + ax^2} \right)}{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}} \\
&= -\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex} + \frac{3\sqrt{b^2 - 4ac} \sqrt{a + \frac{c}{x^2} + \frac{b}{x}} x \sqrt{d + ex} \sqrt{-\frac{a(c+bx+ax^2)}{b^2 - 4ac}} E \left(\frac{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}}{c + bx + ax^2} \right)}{\sqrt{2} \sqrt{\frac{a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}}}
\end{aligned}$$

Mathematica [C] time = 11.41, size = 1372, normalized size = 1.48

$$x(d+ex)^{3/2} \sqrt{a + \frac{c+bx}{x^2}} \left(12d \sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}} \left(a \left(\frac{d}{d+ex} - 1 \right)^2 + \frac{e \left(-\frac{db}{d+ex} + b + \frac{ce}{d+ex} \right)}{d+ex} \right) - \frac{3i\sqrt{2}d(2ad-be+\sqrt{(b^2-4ac)e^2}) \sqrt{\frac{2ce}{d+ex}}}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]

[Out] -(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]) + (x*(d + e*x)^(3/2)*Sqrt[a + (c + b*x)/x^2]*(12*d*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)) - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))/Sqrt[d + e*x] + (I*Sqrt[2]*(4*a*d^2 - b*d*e - 2*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))/Sqrt[d + e*x] + ((2*I)*Sqrt[2]*e*(b*d + c*e)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2])/(2*(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))/Sqrt[d + e*x])

$$t[d + e*x]))/(4*d*e*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[c + b*x + a*x^2]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d + e*x)))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)

maple [B] time = 0.05, size = 3553, normalized size = 3.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x)

[Out] $\frac{1}{2} * ((a*x^2+b*x+c)/x^2)^{(1/2)} * (e*x+d)^{(1/2)} * (2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * (e * (-2*a*x + (-4*a*c+b^2)^{(1/2)} - b) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)} * (e * (b+2*a*x + (-4*a*c+b^2)^{(1/2)}) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)} * (-4*a*c+b^2)^{(1/2)} * x * a * d^2 * e^{-2*2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * (e * (-2*a*x + (-4*a*c+b^2)^{(1/2)} - b) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)} * (e * (b+2*a*x + (-4*a*c+b^2)^{(1/2)}) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)} * (-4*a*c+b^2)^{(1/2)} * x * b * d * e^{2+4*2^{(1/2)} * (-a*(e*x+d) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * (e * (-2*a*x + (-4*a*c+b^2)^{(1/2)} - b) / ((-4*a*c+b^2)^{(1/2)} * e^{2*a*d-b*e}))^{(1/2)} * (e * (b+2*a*x + (-4*a*c+b^2)^{(1/2)}) / ((-4*a*c+b^2)^{(1/2)} * e^{-2*a*d+b*e}))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (-a*(e*x+d) / ((-4*a$

$$\begin{aligned}
& *c+b^2)^{(1/2)}e^{-2*a*d+b*e})^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a \\
& *c+b^2)^{(1/2)}e^{2*a*d-b*e})^{(1/2)})*x*a^2*d^3-2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b \\
& ^2)^{(1/2)}e^{-2*a*d+b*e})^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2 \\
&)^{(1/2)}e^{2*a*d-b*e})^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)} \\
& e^{-2*a*d+b*e})^{(1/2)}*EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}* \\
& e^{-2*a*d+b*e})^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}* \\
& e^{2*a*d-b*e})^{(1/2)})*x*a*b*d^2*e+6*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}* \\
& e^{-2*a*d+b*e})^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2 \\
& *a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a \\
& *d+b*e})^{(1/2)}*EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b* \\
& e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b* \\
& e}))^{(1/2)})*x*a*c*d*e^2-2*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b* \\
& e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e} \\
&)^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})^{(1/2)} \\
& *EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}, \\
& (-((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}) \\
& *x*b^2*d*e^2-6*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}* \\
& (e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(e \\
& *(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})^{(1/2)}*Ellip \\
& ticE(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c \\
& +b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)})*x*a^2*d^3 \\
& +6*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+ \\
& -4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+ \\
& -4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*EllipticE(2^{(1/2)} \\
& *(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}* \\
& e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)})*x*a*b*d^2*e-6*2^{(1/2)} \\
& *(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2 \\
&)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2 \\
&)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*EllipticE(2^{(1/2)}*(-a*(e*x+ \\
& d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b* \\
& e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)})*x*a*c*d*e^2+2^{(1/2)}*(-a*(e*x+d) \\
& /((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/ \\
& (-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4 \\
& *a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*EllipticPi(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c \\
& +b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}, -1/2*((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/d/a, (\\
& -((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}* \\
& (-4*a*c+b^2)^{(1/2)}*x*b*d*e^2+2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a* \\
& d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d- \\
& b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e} \\
&))^{(1/2)}*EllipticPi(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)} \\
& , -1/2*((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e})/d/a, (-((-4*a*c+b^2)^{(1/2)}e^{-2*a* \\
& d+b*e})/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x*c*e^3- \\
& 2*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+ \\
& -4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}e^{2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+ \\
& -4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}e^{-2*a*d+b*e}))^{(1/2)}*EllipticPi(2^{(1/2)}
\end{aligned}$$

$$\begin{aligned}
 & *(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2), -1/2*((-4*a*c+b^2)^(1/2)* \\
 & e^{-2*a*d+b*e})/d/a, (-((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})/((-4*a*c+b^2)^(1/2)* \\
 & e^{2*a*d-b*e})^(1/2))*x*a*b*d^2*e^{-2*2}^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)* \\
 & e^{-2*a*d+b*e})^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e^{ \\
 & 2*a*d-b*e})^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e^{-2*a \\
 & *d+b*e})^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b \\
 & *e})^(1/2), -1/2*((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})/d/a, (-((-4*a*c+b^2)^(1/2)* \\
 & e^{-2*a*d+b*e})/((-4*a*c+b^2)^(1/2)*e^{2*a*d-b*e})^(1/2))*x*a*c*d*e^{2+2}^(1/2)*(\\
 & -a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(\\
 & 1/2)-b)/((-4*a*c+b^2)^(1/2)*e^{2*a*d-b*e})^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(\\
 & 1/2))/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d) \\
 &)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2), -1/2*((-4*a*c+b^2)^(1/2)*e^{-2*a*d+ \\
 & b*e})/d/a, (-((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})/((-4*a*c+b^2)^(1/2)*e^{2*a*d-b*e \\
 & })^(1/2))*x*b^2*d*e^{2+2}^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e}) \\
 & ^{(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e^{2*a*d-b*e})^(\\
 & 1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2 \\
 &)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})^(1/2), -1 \\
 & /2*((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})/d/a, (-((-4*a*c+b^2)^(1/2)*e^{-2*a*d+b*e})/ \\
 & ((-4*a*c+b^2)^(1/2)*e^{2*a*d-b*e})^(1/2))*x*b*c*e^{3-2*x^3*a^2*d*e^{-2-2*x^2*a^ \\
 & 2*d^2*e^{-2*x^2*a*b*d*e^{-2-2*x*a*b*d^2*e^{-2*x*a*c*d*e^{-2-2*a*c*d^2*e}}/(a*e*x^3+a \\
 & *d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/a/e/d
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x,x)

[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)

[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)

$$3.85 \quad \int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d+ex}}{x^2} dx$$

Optimal. Leaf size=1287

$$\frac{\sqrt{2ad - (b - \sqrt{b^2 - 4ac})e} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x \sqrt{1 - \frac{2a(d+ex)}{2ad - (b - \sqrt{b^2 - 4ac})e}} \sqrt{1 - \frac{2a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})e}} \Pi \left(\frac{2ad - be + \sqrt{b^2 - 4ac}e}{2ad}; \sin^{-1} \right)}{4\sqrt{2} \sqrt{a} cd^2 (ax^2 + bx + c)}$$

[Out] $-1/4*(b*d+c*e)*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/c/d-1/2*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}/x+1/8*(b*d+c*e)*x*\text{EllipticE}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(e*x+d)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}/c/d/(a*x^2+b*x+c)*2^{(1/2)}/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}+3/2*e*x*\text{EllipticF}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/(a*x^2+b*x+c)*2^{(1/2)}/(e*x+d)^{(1/2)}-1/4*(b*d+c*e)*x*\text{EllipticF}(1/2*((b+2*a*x+(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)})^{(1/2)}*2^{(1/2)}, (-2*e*(-4*a*c+b^2)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^{(1/2)}*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/c/(a*x^2+b*x+c)*2^{(1/2)}/(e*x+d)^{(1/2)}-1/2*(a*d+b*e)*x*\text{EllipticPi}(2^{(1/2)}*a^{(1/2)}*(e*x+d)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}, 1/2*(e*(-4*a*c+b^2)^{(1/2)}+2*a*d-b*e)/a/d, ((b-2*a*d/e-(-4*a*c+b^2)^{(1/2)})/(b-2*a*d/e+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/d/(a*x^2+b*x+c)*2^{(1/2)}/a^{(1/2)}+1/8*(b*d+c*e)^2*x*\text{EllipticPi}(2^{(1/2)}*a^{(1/2)}*(e*x+d)^{(1/2)}/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}, 1/2*(e*(-4*a*c+b^2)^{(1/2)}+2*a*d-b*e)/a/d, ((b-2*a*d/e-(-4*a*c+b^2)^{(1/2)})/(b-2*a*d/e+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(a+c/x^2+b/x)^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}*(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^{(1/2)})))^{(1/2)}/c/d^2/(a*x^2+b*x+c)*2^{(1/2)}/a^{(1/2)}$

Rubi [A] time = 5.30, antiderivative size = 1287, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1573, 916, 6742, 718, 419, 939, 934, 169, 538, 537, 843,

424}

$$\frac{\sqrt{2ad - (b - \sqrt{b^2 - 4ac})} e \sqrt{a + \frac{b}{x} + \frac{c}{x^2}} x \sqrt{1 - \frac{2a(d+ex)}{2ad - (b - \sqrt{b^2 - 4ac})} e} \sqrt{1 - \frac{2a(d+ex)}{2ad - (b + \sqrt{b^2 - 4ac})} e} \Pi \left(\frac{2ad - be + \sqrt{b^2 - 4ac} e}{2ad}; \sin^{-1} \right)}{4\sqrt{2} \sqrt{a} cd^2 (ax^2 + bx + c)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]

[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(4*Sqrt[2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]/(2*Sqrt[2]*c*Sqrt[d + e*x]*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c])*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)]/(Sqrt[2]*Sqrt[a]*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c])*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)]/(4*Sqrt[2]*Sqrt[a]*c*d^2*(c + b*x + a*x^2))

Rule 169

Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -

```
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && !SimplerQ[e + f*x, c + d*x] && !SimplerQ[g + h*x, c + d
*x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] := Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
- 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1573

```
Int[(x_)^(m_)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + b/x^n + c/x^(2*n)))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
```

d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] && !IntegerQ[p] &&
!IntegerQ[q] && PosQ[n]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{x^2} dx &= \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{\sqrt{d+ex} \sqrt{c+bx+ax^2}}{x^3} dx}{\sqrt{c + bx + ax^2}} \\
&= -\frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{bd+ce+2(ad+be)x+3aex^2}{x^2 \sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{\left(\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \left(\frac{3ae}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} + \frac{bd+ce}{x^2 \sqrt{d+ex} \sqrt{c+bx+ax^2}}\right) dx}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{\left(3ae\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{1}{\sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{4\sqrt{c + bx + ax^2}} + \frac{\left((ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{1}{x \sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{\left((ad + be)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}\right) \int \frac{1}{x \sqrt{d+ex} \sqrt{c+bx+ax^2}} dx}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{3\sqrt{b^2 - 4ac} e \sqrt{a + \frac{c}{x^2}}}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{3\sqrt{b^2 - 4ac} e \sqrt{a + \frac{c}{x^2}}}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{3\sqrt{b^2 - 4ac} e \sqrt{a + \frac{c}{x^2}}}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{3\sqrt{b^2 - 4ac} e \sqrt{a + \frac{c}{x^2}}}{4\sqrt{c + bx + ax^2}} \\
&= -\frac{(bd + ce)\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{4cd} - \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}} \sqrt{d + ex}}{2x} + \frac{\sqrt{b^2 - 4ac} (bd + ce) \sqrt{a + \frac{c}{x^2}}}{4\sqrt{c + bx + ax^2}}
\end{aligned}$$

Mathematica [C] time = 12.53, size = 811, normalized size = 0.63

$$x\sqrt{a + \frac{c+bx}{x^2}} \left(-\frac{8cd^3}{x^2} - \frac{8ced^2}{x} - \frac{4(bd+ce)d^2}{x} - \frac{i(d+ex)^{3/2} \sqrt{1 - \frac{2(ad^2+e(cc-bd))}{(2ad-be+\sqrt{(b^2-4ac)e^2})(d+ex)}} \sqrt{\frac{4(ad^2+e(cc-bd))}{(-2ad+be+\sqrt{(b^2-4ac)e^2})(d+ex)}} + 2 \right) d(bd+ce)(2ad-be)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]

[Out] (x*Sqrt[a + (c + b*x)/x^2]*((-8*c*d^3)/x^2 - (8*c*d^2*e)/x - (4*d^2*(b*d + c*e))/x - (I*(d + e*x)^(3/2)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e))]/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))*(d + e*x)))*Sqrt[2 + (4*(a*d^2 + e*(-(b*d) + c*e))]/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]))*(d + e*x)))*(d*(b*d + c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2])/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])) - (b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + c*e*(4*a*d^2 + 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2])/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])) + 2*e*(b^2*d^2 - 2*b*c*d*e + c*(-4*a*d^2 + c*e^2))*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2])]/(2*(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2])/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])))/(e*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(16*c*d^2*Sqrt[d + e*x])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] Timed out

$$\begin{aligned}
& x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})^{(1/2)}, -1/2*((-4*a*c+b^2)^{(1/2)}*e^{-2* \\
& a*d+b*e})/d/a, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d \\
& -b*e}))^{(1/2)}*x^2*a*c^2*d*e^3+2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a \\
& *d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d \\
& -b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b* \\
& e})^{(1/2)}*EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, (- \\
& (-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)} \\
&)*(-4*a*c+b^2)^{(1/2)}*x^2*a*b*d^3*e^{-5*2^{(1/2)}}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)} \\
& *e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)} \\
& *e^{+2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)} \\
& *e^{-2*a*d+b*e}))^{(1/2)}*EllipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2* \\
& a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2* \\
& a*d-b*e}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^2*a*c*d^2*e^{-2-2*x^3*a*b*c*d*e^3+2*2^{(1/2)} \\
& (1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c \\
& +b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c \\
& +b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}*EllipticPi(2^{(1/2)}*(-a* \\
& (e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, -1/2*((-4*a*c+b^2)^{(1/2)}*e^{- \\
& 2*a*d+b*e})/d/a, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a \\
& *d-b*e}))^{(1/2)}*x^2*b^2*c*d*e^3-2*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{- \\
& 2*a*d+b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2 \\
& *a*d-b*e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a* \\
& d+b*e}))^{(1/2)}*EllipticE(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e} \\
&))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e} \\
&))^{(1/2)}*x^2*a*c^2*d*e^3-2*2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d \\
& +b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b \\
& *e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e} \\
&))^{(1/2)}*EllipticE(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, (- \\
& (-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)} \\
&)*x^2*a^2*c*d^3*e^{+2*2^{(1/2)}}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)} \\
&)*(-4*a*c+b^2)^{(1/2)}*x^2*a*c^2*d*e^3+2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d \\
& +b*e}))^{(1/2)}*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b \\
& *e}))^{(1/2)}*(e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e} \\
&))^{(1/2)}*EllipticE(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, (- \\
& (-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}*x^ \\
& 2*a*b^2*d^3*e^{+12*2^{(1/2)}}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)} \\
&)*(e*(-2*a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}* \\
& (e*(b+2*a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}*Ell \\
& ipticF(2^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a \\
& *c+b^2)^{(1/2)}*e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}*x^2*a^2 \\
& *c*d^3*e^{-2^{(1/2)}}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}*(e*(-2 \\
& *a*x+(-4*a*c+b^2)^{(1/2)}-b)/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}*(e*(b+2* \\
& a*x+(-4*a*c+b^2)^{(1/2)}))/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}*EllipticF(2 \\
& ^{(1/2)}*(-a*(e*x+d)/((-4*a*c+b^2)^{(1/2)}*e^{-2*a*d+b*e}))^{(1/2)}, (-((-4*a*c+b^2)^{(1/2)} \\
& *e^{-2*a*d+b*e})/((-4*a*c+b^2)^{(1/2)}*e^{+2*a*d-b*e}))^{(1/2)}*x^2*a*b^2*d^3*e \\
& -4*x^2*a^2*c*d^3*e^{-2*x^2*a*b^2*d^3*e^{-2*x^4*a^2*b*d^2*e^2-2*x^4*a^2*c*d*e^3- \\
& 2*x^3*a^2*b*d^3*e^{-6*x^3*a^2*c*d^2*e^2-2*x^3*a*b^2*d^2*e^2-2*x^2*a*c^2*d*e^3}
\end{aligned}$$

```

-6*x*a*c^2*d^2*e^2-2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(
(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1
/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)
*EllipticE(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-(
-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2
*a^2*b*d^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(
-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+
2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticP
i(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-( (-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^
2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*b*c^2*e^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)
^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1
/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*
e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-( (-4*a*c+b^
2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)
^(1/2)*x^2*c^2*e^4-2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1
/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)
*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*E
llipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*
((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-( (-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-
4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*b^3*d^2*e^2-5*2^(1/2)*(-a*(e*x+d)
/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/
((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4
*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/((-4*a*c+
b^2)^(1/2)*e-2*a*d+b*e))^(1/2),(-( (-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+
b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*x^2*a*b*c*d^2*e^2+4*2^(1/2)*(-a*(e*x+d)/((-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*
a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c
+b^2)^(1/2)*e-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)
^(1/2)*e-2*a*d+b*e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-( (-
4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*
a*c+b^2)^(1/2)*x^2*a*c*d^2*e^2+2*2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/((-4*a*c+b^2)^(1/2)*e+2*
a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/((-4*a*c+b^2)^(1/2)*e-2*a*d
+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/((-4*a*c+b^2)^(1/2)*e-2*a*d+b*
e))^(1/2),-1/2*((-4*a*c+b^2)^(1/2)*e-2*a*d+b*e)/d/a,(-( (-4*a*c+b^2)^(1/2)*e-
2*a*d+b*e)/((-4*a*c+b^2)^(1/2)*e+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*
b*c*d*e^3)/x/a/e/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/c/d^2

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+ex} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2,x)

[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)

[Out] Timed out

$$3.86 \quad \int (fx)^m (d + ex^n)^q (a + cx^{2n})^p dx$$

Optimal. Leaf size=29

$$\text{Int}\left((fx)^m (a + cx^{2n})^p (d + ex^n)^q, x\right)$$

[Out] Unintegrable((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Defer[Int] [(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]

Rubi steps

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p dx = \int (fx)^m (d + ex^n)^q (a + cx^{2n})^p dx$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^n)^q (a + cx^{2n})^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]

[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]

fricas [A] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^{2n} + a\right)^p (ex^n + d)^q (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + a)^p (ex^n + d)^q (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int (fx)^m (cx^{2n} + a)^p (ex^n + d)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)^q*(c*x^(2*n)+a)^p,x)

[Out] int((f*x)^m*(e*x^n+d)^q*(c*x^(2*n)+a)^p,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + a)^p (ex^n + d)^q (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int (a + cx^{2n})^p (fx)^m (d + ex^n)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q,x)

[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)

[Out] Timed out

$$3.87 \quad \int (fx)^m (d + ex^n)^3 (a + cx^{2n})^p dx$$

Optimal. Leaf size=358

$$\frac{d^3(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{3d^2ex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m}{2n}, -p; \frac{m}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{m+n+1}$$

[Out] $d^3(f*x)^{(1+m)}*(a+c*x^{(2*n)})^p*\text{hypergeom}([-p, 1/2*(1+m)/n], [1+1/2*(1+m)/n], -c*x^{(2*n)}/a)/f/(1+m)/((1+c*x^{(2*n)}/a)^p)+3*d^2*e*x^{(1+n)}*(f*x)^m*(a+c*x^{(2*n)})^p*\text{hypergeom}([-p, 1/2*(1+m+n)/n], [1/2*(1+m+3*n)/n], -c*x^{(2*n)}/a)/(1+m+n)/((1+c*x^{(2*n)}/a)^p)+3*d*e^2*x^{(1+2*n)}*(f*x)^m*(a+c*x^{(2*n)})^p*\text{hypergeom}([-p, 1/2*(1+m+2*n)/n], [1/2*(1+m+4*n)/n], -c*x^{(2*n)}/a)/(1+m+2*n)/((1+c*x^{(2*n)}/a)^p)+e^3*x^{(1+3*n)}*(f*x)^m*(a+c*x^{(2*n)})^p*\text{hypergeom}([-p, 1/2*(1+m+3*n)/n], [1/2*(1+m+5*n)/n], -c*x^{(2*n)}/a)/(1+m+3*n)/((1+c*x^{(2*n)}/a)^p)$

Rubi [A] time = 0.24, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1561, 365, 364, 20}

$$\frac{3d^2ex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a}\right)}{m+n+1} + \frac{d^3(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m}{2n}, -p; \frac{m}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(f*x)^m*(d + e*x^n)^3*(a + c*x^{(2*n)})^p, x]$

[Out] $(d^3*(f*x)^{(1+m)}*(a + c*x^{(2*n)})^p*\text{Hypergeometric2F1}[(1+m)/(2*n), -p, 1 + (1+m)/(2*n), -((c*x^{(2*n)})/a)])/(f*(1+m)*(1 + (c*x^{(2*n)})/a)^p) + (3*d^2*e*x^{(1+n)}*(f*x)^m*(a + c*x^{(2*n)})^p*\text{Hypergeometric2F1}[(1+m+n)/(2*n), -p, (1+m+3*n)/(2*n), -((c*x^{(2*n)})/a)])/((1+m+n)*(1 + (c*x^{(2*n)})/a)^p) + (3*d*e^2*x^{(1+2*n)}*(f*x)^m*(a + c*x^{(2*n)})^p*\text{Hypergeometric2F1}[(1+m+2*n)/(2*n), -p, (1+m+4*n)/(2*n), -((c*x^{(2*n)})/a)])/((1+m+2*n)*(1 + (c*x^{(2*n)})/a)^p) + (e^3*x^{(1+3*n)}*(f*x)^m*(a + c*x^{(2*n)})^p*\text{Hypergeometric2F1}[(1+m+3*n)/(2*n), -p, (1+m+5*n)/(2*n), -((c*x^{(2*n)})/a)])/((1+m+3*n)*(1 + (c*x^{(2*n)})/a)^p)$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
&& (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^n)^3 (a + cx^{2n})^p dx &= \int \left(d^3 (fx)^m (a + cx^{2n})^p + 3d^2 ex^n (fx)^m (a + cx^{2n})^p + 3de^2 x^{2n} (fx)^m (a + cx^{2n})^p + e^3 x^{3n} (fx)^m (a + cx^{2n})^p \right) dx \\
&= d^3 \int (fx)^m (a + cx^{2n})^p dx + (3d^2 e) \int x^n (fx)^m (a + cx^{2n})^p dx + (3de^2) \int x^{2n} (fx)^m (a + cx^{2n})^p dx + (e^3) \int x^{3n} (fx)^m (a + cx^{2n})^p dx \\
&= (3d^2 ex^{-m} (fx)^m) \int x^{m+n} (a + cx^{2n})^p dx + (3de^2 x^{-m} (fx)^m) \int x^{m+2n} (a + cx^{2n})^p dx + (e^3 x^{-m} (fx)^m) \int x^{m+3n} (a + cx^{2n})^p dx \\
&= \frac{d^3 (fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a} \right)}{f(1+m)} + \left(3d^2 ex^n \right) \frac{d^3 (fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a} \right)}{f(1+m)} \\
&= \frac{d^3 (fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a} \right)}{f(1+m)} + \frac{3d^2 ex^{n+1} (fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a} \right)}{m+n+1} + \frac{e^3 x^{n+1} (fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a} \right)}{m+n+1}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 249, normalized size = 0.70

$$x (fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1 \right)^{-p} \left(\frac{d^3 {}_2F_1 \left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a} \right)}{m+1} + ex^n \left(\frac{3d^2 {}_2F_1 \left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a} \right)}{m+n+1} + \frac{e^3 {}_2F_1 \left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a} \right)}{m+n+1} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]

[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1 + (c*x^(2*n))/a)^p

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3\right)(cx^{2n} + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + a)^p*(f*x)^m, x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming x near 0Simplification assuming f near 0Simplification assuming x near 0Simplification assuming f near 0Unable to divide, perhaps due to rounding error%{-16, [1,0,6,3,1,2,4,4,1]%%}%+%%{-64, [1,0,6,3,1,2,3,4,1]%%}%+%%{-96, [1,0,6,3,1,2,2,4,1]%%}%+%%{-64, [1,0,6,3,1,2,1,4,1]%%}%+%%{-16, [1,0,6,3,1,2,0,4,1]%%}%+%%{-16, [1,0,6,3,0,2,4,4,1]%%}%+%%{-64, [1,0,6,3,0,2,3,4,1]%%}%+%%{-96, [1,0,6,3,0,2,2,4,1]%%}%+%%{-64, [1,0,6,3,0,2,1,4,1]%%}%+%%{-16, [1,0,6,3,0,2,0,4,1]%%}% / %%{16, [0,0,6,4,0,2,4,4,0]%%}%+%%{64, [0,0,6,4,0,2,3,4,0]%%}%+%%{96, [0,0,6,4,0,2,2,4,0]%%}%+%%{64, [0,0,6,4,0,2,1,4,0]%%}%+%%{16, [0,0,6,4,0,2,0,4,0]%%}% Error: Bad Argument Value

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (ex^n + d)^3 (fx)^m (cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(e*x^n+d)^3*(c*x^(2*n)+a)^p,x)`

[Out] `int((f*x)^m*(e*x^n+d)^3*(c*x^(2*n)+a)^p,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)^3 (cx^{2n} + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")`

[Out] `integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + cx^{2n})^p (fx)^m (d + ex^n)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3,x)`

[Out] `int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)`

[Out] Timed out

3.88 $\int (fx)^m (d + ex^n)^2 (a + cx^{2n})^p dx$

Optimal. Leaf size=262

$$\frac{d^2(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{2dex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{m+n+1}$$

[Out] $d^{2*(f*x)^{(1+m)*(a+c*x^{(2*n)})^p}$ hypergeom([-p, 1/2*(1+m)/n], [1+1/2*(1+m)/n], -c*x^{(2*n)}/a)/f/(1+m)/((1+c*x^{(2*n)}/a)^p)+2*d*e*x^{(1+n)*(f*x)^m*(a+c*x^{(2*n)})^phypergeom([-p, 1/2*(1+m+n)/n], [1/2*(1+m+3*n)/n], -c*x^{(2*n)}/a)/(1+m+n)/((1+c*x^{(2*n)}/a)^p)+e^{2*x^{(1+2*n)*(f*x)^m*(a+c*x^{(2*n)})^p}hypergeom([-p, 1/2*(1+m+2*n)/n], [1/2*(1+m+4*n)/n], -c*x^{(2*n)}/a)/(1+m+2*n)/((1+c*x^{(2*n)}/a)^p)

Rubi [A] time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1561, 365, 364, 20}

$$\frac{d^2(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{2dex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{m+n+1}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]

[Out] $(d^{2*(f*x)^{(1+m)*(a+c*x^{(2*n)})^p}$ Hypergeometric2F1[(1+m)/(2*n), -p, 1+(1+m)/(2*n), -((c*x^{(2*n)}/a)])/f*(1+m)*(1+(c*x^{(2*n)}/a)^p)+(2*d*e*x^{(1+n)*(f*x)^m*(a+c*x^{(2*n)})^p}Hypergeometric2F1[(1+m+n)/(2*n), -p, (1+m+3*n)/(2*n), -((c*x^{(2*n)}/a)])/((1+m+n)*(1+(c*x^{(2*n)}/a)^p)+(e^{2*x^{(1+2*n)*(f*x)^m*(a+c*x^{(2*n)})^p}Hypergeometric2F1[(1+m+2*n)/(2*n), -p, (1+m+4*n)/(2*n), -((c*x^{(2*n)}/a)])/((1+m+2*n)*(1+(c*x^{(2*n)}/a)^p)

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a

)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 365

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

Rule 1561

Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && (IGtQ[p, 0] || IGtQ[q, 0])

Rubi steps

$$\begin{aligned}
 \int (fx)^m (d + ex^n)^2 (a + cx^{2n})^p dx &= \int \left(d^2 (fx)^m (a + cx^{2n})^p + 2dex^n (fx)^m (a + cx^{2n})^p + e^2 x^{2n} (fx)^m (a + cx^{2n})^p \right) dx \\
 &= d^2 \int (fx)^m (a + cx^{2n})^p dx + (2de) \int x^n (fx)^m (a + cx^{2n})^p dx + e^2 \int x^{2n} (fx)^m (a + cx^{2n})^p dx \\
 &= (2dex^{-m} (fx)^m) \int x^{m+n} (a + cx^{2n})^p dx + (e^2 x^{-m} (fx)^m) \int x^{m+2n} (a + cx^{2n})^p dx \\
 &= \frac{d^2 (fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(2dex^{-m} (fx)^m\right) \int x^{m+2n} (a + cx^{2n})^p dx \\
 &= \frac{d^2 (fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{2dex^{1+m} (fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)}
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 189, normalized size = 0.72

$$x(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} \left(\frac{d^2 {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{m+1} + ex^n \left(\frac{2d {}_2F_1\left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a}\right)}{m+n+1} + e \int x^{m+2n} (a + cx^{2n})^p dx \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
(1 + m)/(2*n), -((c*x^(2*n))/a)]/(1 + m) + e*x^n*((2*d*Hypergeometric2F1[
(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]/(1 + m + n)
+ (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((
(c*x^(2*n))/a)]/(1 + m + 2*n)))))/(1 + (c*x^(2*n))/a)^p
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2x^{2n} + 2dex^n + d^2\right)(cx^{2n} + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Simplification assuming x near 0Simplificati
on assuming f near 0Simplification assuming x near 0Simplification assuming
f near 0Unable to divide, perhaps due to rounding error%%{4,[0,0,3,2,1,0,
2,3,1]%%}+%%{8,[0,0,3,2,1,0,1,3,1]%%}+%%{4,[0,0,3,2,1,0,0,3,1]%%}+%%{
4,[0,0,3,2,0,0,2,3,1]%%}+%%{8,[0,0,3,2,0,0,1,3,1]%%}+%%{4,[0,0,3,2,0,0,
0,3,1]%%} / %%{-8,[0,0,4,3,0,1,3,3,0]%%}+%%{-24,[0,0,4,3,0,1,2,3,0]%%}
+%%{-24,[0,0,4,3,0,1,1,3,0]%%}+%%{-8,[0,0,4,3,0,1,0,3,0]%%} Error: Bad
Argument Value
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (ex^n + d)^2 (fx)^m (cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^2*(c*x^(2*n)+a)^p,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)^2*(c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)^2 (cx^{2n} + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + c x^{2n})^p (f x)^m (d + e x^n)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

3.89 $\int (fx)^m (d + ex^n) (a + cx^{2n})^p dx$

Optimal. Leaf size=166

$$\frac{d(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n}\right)}{m+n+1}$$

[Out] d*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n], [1+1/2*(1+m)/n], -c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n], [1/2*(1+m+3*n)/n], -c*x^(2*n)/a)/(1+m+n)/((1+c*x^(2*n)/a)^p)

Rubi [A] time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1561, 365, 364, 20}

$$\frac{d(fx)^{m+1} (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n}\right)}{m+n+1}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]

[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/f*(1 + m)*(1 + (c*x^(2*n))/a)^p + (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p)

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
&& (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^n) (a + cx^{2n})^p dx &= \int \left(d(fx)^m (a + cx^{2n})^p + ex^n (fx)^m (a + cx^{2n})^p \right) dx \\
&= d \int (fx)^m (a + cx^{2n})^p dx + e \int x^n (fx)^m (a + cx^{2n})^p dx \\
&= \left(ex^{-m} (fx)^m \right) \int x^{m+n} (a + cx^{2n})^p dx + \left(d (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int (fx)^m dx \\
&= \frac{d(fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a} \right)}{f(1+m)} + \left(ex^{-m} (fx)^m \right) \int (fx)^m dx \\
&= \frac{d(fx)^{1+m} (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} {}_2F_1 \left(\frac{1+m}{2n}, -p; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a} \right)}{f(1+m)} + \frac{ex^{1+n} (fx)^{m+1}}{m+1}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 136, normalized size = 0.82

$$\frac{x(fx)^m (a + cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1 \right)^{-p} \left(d(m+n+1) {}_2F_1 \left(\frac{m+1}{2n}, -p; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a} \right) + e(m+1)x^n {}_2F_1 \left(\frac{m+n+1}{2n}, -p; \frac{m+3n+1}{2n}; -\frac{cx^{2n}}{a} \right) \right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]

[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric2F1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 + m)*(1 + m + n)*(1 + (c*x^(2*n))/a)^p)

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left((ex^n + d)(cx^{2n} + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)(cx^{2n} + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (ex^n + d)(fx)^m (cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)*(c*x^(2*n)+a)^p,x)

[Out] int((f*x)^m*(e*x^n+d)*(c*x^(2*n)+a)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)(cx^{2n} + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + cx^{2n})^p (fx)^m (d + ex^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p, x)
```

```
[Out] Timed out
```

$$3.90 \quad \int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} dx$$

Optimal. Leaf size=194

$$\frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2n}; -p, 1; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+n+1}{2n}\right)}{d^2(m+n+1)}$$

[Out] $x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n, 1, -p, 1+1/2*(1+m)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d/(1+m)/((1+c*x^(2*n)/a)^p) - e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n, 1, -p, 1/2*(1+m+3*n)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d^2/(1+m+n)/((1+c*x^(2*n)/a)^p)$

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2n}; -p, 1; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+n+1}{2n}\right)}{d^2(m+n+1)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]

[Out] $(x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/((d*(1 + m)*(1 + (c*x^(2*n))/a)^p) - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/((d^2*(1 + m + n)*(1 + (c*x^(2*n))/a)^p))$

Rule 510

Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^(n._))^(p._)*((c._) + (d._)*(x._)^(n._))^(q._), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e._)*(x._))^(m._)*((a._) + (b._)*(x._)^(n._))^(p._)*((c._) + (d._)*(x._)^(n._))^(q._), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 1562

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] && !IntegerQ[p] && ILtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx &= (x^{-m}(fx)^m) \int \left(\frac{dx^m (a + cx^{2n})^p}{d^2 - e^2x^{2n}} + \frac{ex^{m+n} (a + cx^{2n})^p}{-d^2 + e^2x^{2n}} \right) dx \\ &= (dx^{-m}(fx)^m) \int \frac{x^m (a + cx^{2n})^p}{d^2 - e^2x^{2n}} dx + (ex^{-m}(fx)^m) \int \frac{x^{m+n} (a + cx^{2n})^p}{-d^2 + e^2x^{2n}} dx \\ &= \left(dx^{-m}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^m \left(1 + \frac{cx^{2n}}{a} \right)^p}{d^2 - e^2x^{2n}} dx + \left(ex^{-m}(fx)^m (a + cx^{2n})^p \right) \int \frac{x^{m+n} \left(1 + \frac{cx^{2n}}{a} \right)^p}{-d^2 + e^2x^{2n}} dx \\ &= \frac{x(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m}{2n}; -p, 1; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2} \right)}{d(1+m)} - \frac{ex^{1+n}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m}{2n}; -p, 1; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2} \right)}{d(1+m)} \end{aligned}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + cx^{2n})^p}{d + ex^n} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]

[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (cx^{2n} + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d),x)

[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + cx^{2n})^p (fx)^m}{d + ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n),x)

[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n), x)

[Out] Timed out

$$3.91 \quad \int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2} dx$$

Optimal. Leaf size=302

$$\frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2n}; -p, 2; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^2(m+1)} + \frac{e^2 x^{2n+1} (fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m}{2n}; -p, 2; \frac{m}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^4(m+2n+1)}$$

[Out] $x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n, 2, -p, 1+1/2*(1+m)/n, e^{2*x^(2*n)}/d^2, -c*x^(2*n)/a)/d^2/(1+m)/((1+c*x^(2*n)/a)^p) - 2*e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n, 2, -p, 1/2*(1+m+3*n)/n, e^{2*x^(2*n)}/d^2, -c*x^(2*n)/a)/d^3/(1+m+n)/((1+c*x^(2*n)/a)^p) + e^{2*x^(1+2*n)}*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+2*n)/n, 2, -p, 1/2*(1+m+4*n)/n, e^{2*x^(2*n)}/d^2, -c*x^(2*n)/a)/d^4/(1+m+2*n)/((1+c*x^(2*n)/a)^p)$

Rubi [A] time = 0.33, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2n}; -p, 2; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^2(m+1)} - \frac{2ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m}{2n}; -p, 2; \frac{m}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^3(m+n+1)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]

[Out] $(x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a), (e^{2*x^(2*n)})/d^2])/((d^2*(1 + m)*(1 + (c*x^(2*n))/a)^p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^{2*x^(2*n)})/d^2])/((d^3*(1 + m + n)*(1 + (c*x^(2*n))/a)^p) + (e^{2*x^(1 + 2*n)}*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e^{2*x^(2*n)})/d^2])/((d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p))$

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)])/((e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] := Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] && !IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} dx &= (x^{-m}(fx)^m) \int \left(\frac{d^2 x^m (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^2} - \frac{2dex^{m+n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} + \frac{e^2 x^{m+2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} \right) dx \\ &= (d^2 x^{-m}(fx)^m) \int \frac{x^m (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^2} dx - (2dex^{-m}(fx)^m) \int \frac{x^{m+n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} dx + (e^2 x^{-m}(fx)^m) \int \frac{x^{m+2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^2} dx \\ &= \left(d^2 x^{-m}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^m \left(1 + \frac{cx^{2n}}{a} \right)^p}{(d^2 - e^2 x^{2n})^2} dx - \left(2dex^{-m}(fx)^m (a + cx^{2n})^p \right) \int \frac{x^{m+n} \left(1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^2} dx + \left(e^2 x^{-m}(fx)^m (a + cx^{2n})^p \right) \int \frac{x^{m+2n} \left(1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^2} dx \\ &= \frac{x(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m}{2n}; -p, 2; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^2(1+m)} - \frac{2ex^{1+n}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m+n}{2n}; -p, 2; 1 + \frac{1+m+n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^2(1+m)} + \frac{e^2 x^{1+2n}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m+2n}{2n}; -p, 2; 1 + \frac{1+m+2n}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^2(1+m)} \end{aligned}$$

Mathematica [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]

[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^{2n} + a)^p (fx)^m}{e^2x^{2n} + 2dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (cx^{2n} + a)^p}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^2,x)

[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + c x^{2n})^p (f x)^m}{(d + e x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2,x)

[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)

[Out] Timed out

$$3.92 \quad \int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx$$

Optimal. Leaf size=412

$$\frac{e^3 x^{3n+1} (fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+3n+1}{2n}; -p, 3; \frac{m+5n+1}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^6(m+3n+1)} + \frac{3e^2 x^{2n+1} (fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+2n+1}{2n}; -p, 3; \frac{m+4n+1}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^5(m+2n+1)}$$

[Out] $x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n, 3, -p, 1+1/2*(1+m)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d^3/(1+m)/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n, 3, -p, 1/2*(1+m+3*n)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d^4/(1+m+n)/((1+c*x^(2*n)/a)^p)+3*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+2*n)/n, 3, -p, 1/2*(1+m+4*n)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d^5/(1+m+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m+3*n)/n, 3, -p, 1/2*(1+m+5*n)/n, e^2*x^(2*n)/d^2, -c*x^(2*n)/a)/d^6/(1+m+3*n)/((1+c*x^(2*n)/a)^p)$

Rubi [A] time = 0.45, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2n}; -p, 3; \frac{m+1}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^3(m+1)} - \frac{3ex^{n+1}(fx)^m (a+cx^{2n})^p \left(\frac{cx^{2n}}{a} + 1\right)^{-p} F_1\left(\frac{m}{2n}; -p, 3; \frac{m}{2n} + 1; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2}\right)}{d^4(m+n+1)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]

[Out] $(x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/d^3*(1 + m)*(1 + (c*x^(2*n))/a)^p - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/d^4*(1 + m + n)*(1 + (c*x^(2*n))/a)^p + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p - (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/d^6*(1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)$

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a,

b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 1562

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^(n_))^(q_)*((a_) + (c_)*(x_)^(n_2))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x], x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] && !IntegerQ[p] && ILtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx &= (x^{-m}(fx)^m) \int \left(\frac{d^3 x^m (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^3} + \frac{3d^2 ex^{m+n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} - \frac{3de^2 x^{m+2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} + \dots \right) dx \\ &= (d^3 x^{-m} (fx)^m) \int \frac{x^m (a + cx^{2n})^p}{(d^2 - e^2 x^{2n})^3} dx + (3d^2 ex^{-m} (fx)^m) \int \frac{x^{m+n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} dx - (3de^2 ex^{1+n} (fx)^m) \int \frac{x^{m+2n} (a + cx^{2n})^p}{(-d^2 + e^2 x^{2n})^3} dx \\ &= \left(d^3 x^{-m} (fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} \right) \int \frac{x^m \left(1 + \frac{cx^{2n}}{a} \right)^p}{(d^2 - e^2 x^{2n})^3} dx + \left(3d^2 ex^{-m} (fx)^m (a + cx^{2n})^p \right) \int \frac{x^{m+n} \left(1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^3} dx - \left(3de^2 ex^{1+n} (fx)^m (a + cx^{2n})^p \right) \int \frac{x^{m+2n} \left(1 + \frac{cx^{2n}}{a} \right)^p}{(-d^2 + e^2 x^{2n})^3} dx \\ &= \frac{x(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m}{2n}; -p, 3; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^3(1+m)} - \frac{3ex^{1+n}(fx)^m (a + cx^{2n})^p \left(1 + \frac{cx^{2n}}{a} \right)^{-p} F_1 \left(\frac{1+m}{2n}; -p, 3; 1 + \frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2 x^{2n}}{d^2} \right)}{d^3(1+m)} \end{aligned}$$

Mathematica [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + cx^{2n})^p}{(d + ex^n)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]

[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(cx^{2n} + a)^p (fx)^m}{e^3x^{3n} + 3de^2x^{2n} + 3d^2ex^n + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (cx^{2n} + a)^p}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^3,x)

[Out] int((f*x)^m*(c*x^(2*n)+a)^p/(e*x^n+d)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + a)^p (fx)^m}{(ex^n + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + c x^{2n})^p (f x)^m}{(d + e x^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3,x)

[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)

[Out] Timed out

$$3.93 \quad \int (b + 2cx) (a + bx + cx^2)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{14} (a + bx + cx^2)^{14}$$

[Out] 1/14*(c*x^2+b*x+a)^14

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {629}

$$\frac{1}{14} (a + bx + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]

[Out] (a + b*x + c*x^2)^14/14

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (a + bx + cx^2)^{13} dx = \frac{1}{14} (a + bx + cx^2)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 12.56

$$\frac{1}{14} x(b+cx) (14a^{13} + 91a^{12}x(b+cx) + 364a^{11}x^2(b+cx)^2 + 1001a^{10}x^3(b+cx)^3 + 2002a^9x^4(b+cx)^4 + 3003a^8x^5(b$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]

[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 1001*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b

+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13)/14

fricas [B] time = 0.46, size = 1446, normalized size = 90.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")

[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + x^26*c^13*a + 26*x^25*c^11*b^3 + 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 + 78*x^24*c^11*b^2*a + 13/2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 + 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a^2 + 429/2*x^22*c^8*b^6 + 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 + 26*x^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 + 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^3*a^2 + 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 + 1716*x^20*c^7*b^6*a + 6435/2*x^20*c^8*b^4*a^2 + 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x^19*c^5*b^9 + 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 + 4290*x^19*c^8*b^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 + 1287*x^18*c^5*b^8*a + 6006*x^18*c^6*b^6*a^2 + 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 + 143*x^18*c^9*a^5 + 26*x^17*c^3*b^11 + 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*a^2 + 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 + 1287*x^17*c^8*b*a^5 + 13/2*x^16*c^2*b^12 + 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 + 12012*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 + 5148*x^16*c^7*b^2*a^5 + 429/2*x^16*c^8*a^6 + x^15*c*b^13 + 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 + 8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 + 12012*x^15*c^6*b^3*a^5 + 1716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 + 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*a^2 + 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 + 18018*x^14*c^5*b^4*a^5 + 6006*x^14*c^6*b^2*a^6 + 1716/7*x^14*c^7*a^7 + x^13*b^13*a + 78*x^13*c*b^11*a^2 + 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 + 18018*x^13*c^4*b^5*a^5 + 12012*x^13*c^5*b^3*a^6 + 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2 + 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 + 12012*x^12*c^3*b^6*a^5 + 15015*x^12*c^4*b^4*a^6 + 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 + 26*x^11*b^11*a^3 + 715*x^11*c*b^9*a^4 + 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b^5*a^6 + 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4 + 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 + 8580*x^10*c^3*b^4*a^7 + 6435/2*x^10*c^4*b^2*a^8 + 143*x^10*c^5*a^9 + 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*a^6 + 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 + 715*x^9*c^4*b*a^9 + 429/2*x^8*b^8*a^6 + 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 + 1430*x^8*c^3*b^2*a^9 + 143/2*x^8*c^4*a^10 + 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 + 1430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 + 715*x^6*c*b^4*a^9 + 429*x^6*c^2*b^2*a^10 + 26*x^6*c^3*a^11 + 143*x^5*b^5*a^9 + 286*x^5*c*b^3*a^10 + 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 + 78*x^4*c*b^2*a^11 + 13/2*x^4*c^2*a^12 + 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 + x^2*c*a^13 + x*b*a^13

giac [B] time = 0.43, size = 216, normalized size = 13.50

$$\frac{1}{14} (cx^2 + bx)^{14} + (cx^2 + bx)^{13} a + \frac{13}{2} (cx^2 + bx)^{12} a^2 + 26 (cx^2 + bx)^{11} a^3 + \frac{143}{2} (cx^2 + bx)^{10} a^4 + 143 (cx^2 + bx)^9 a^5 + \frac{429}{2} (cx^2 + bx)^8 a^6 + 1716 (cx^2 + bx)^7 a^7 + 429 (cx^2 + bx)^6 a^8 + 143 (cx^2 + bx)^5 a^9 + \frac{143}{2} (cx^2 + bx)^4 a^{10} + 26 (cx^2 + bx)^3 a^{11} + \frac{13}{2} (cx^2 + bx)^2 a^{12} + (cx^2 + bx) a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 26*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*a^5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 + b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26*(c*x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13

maple [B] time = 0.00, size = 46548, normalized size = 2909.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x)

[Out] result too large to display

maxima [A] time = 0.43, size = 14, normalized size = 0.88

$$\frac{1}{14} (cx^2 + bx + a)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")

[Out] 1/14*(c*x^2 + b*x + a)^14

mupad [B] time = 3.34, size = 1203, normalized size = 75.19

$$x^{12} \left(\frac{429 a^8 c^6}{2} + 5148 a^7 b^2 c^5 + 15015 a^6 b^4 c^4 + 12012 a^5 b^6 c^3 + \frac{6435 a^4 b^8 c^2}{2} + 286 a^3 b^{10} c + \frac{13 a^2 b^{12}}{2} \right) + x^{16} \left(\frac{429 a^8 c^6}{2} + 5148 a^7 b^2 c^5 + 15015 a^6 b^4 c^4 + 12012 a^5 b^6 c^3 + \frac{6435 a^4 b^8 c^2}{2} + 286 a^3 b^{10} c + \frac{13 a^2 b^{12}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)

[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*((429*a^8*c^6)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 1

$$\begin{aligned}
& 2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^{13}*(a*b^{13} + 7 \\
& 8*a^2*b^{11}*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018 \\
& *a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^{15}*(b^{13}*c + 78*a*b^{11}*c^2 + 1716*a^6 \\
& *b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^ \\
& 5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^{11}*c^3 + 715*a^9*b^4*c + 429*a^{10} \\
& b^2*c^2) + x^{22}*(26*a^3*c^{11} + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a^2*b^ \\
& 2*c^{10}) + x^{10}*((143*a^4*b^{10})/2 + 143*a^9*c^5 + 1287*a^5*b^8*c + 6006*a^6* \\
& b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^{18}*(143*a^5*c^9 + (1 \\
& 43*b^{10}*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (64 \\
& 35*a^4*b^2*c^8)/2) + x^{14}*(b^{14}/14 + (1716*a^7*c^7)/7 + 429*a^2*b^{10}*c^2 + \\
& 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 \\
& + 13*a*b^{12}*c) + x^8*((429*a^6*b^8)/2 + (143*a^{10}*c^4)/2 + 1716*a^7*b^6*c \\
& + (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^{20}*((143*a^4*c^{10})/2 + (429* \\
& b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9) + (c \\
& ^{14}*x^{28})/14 + x^2*(a^{13}*c + (13*a^{12}*b^2)/2) + (13*a^{10}*x^4*(11*b^4 + a^2* \\
& c^2 + 12*a*b^2*c))/2 + (13*c^{10}*x^{24}*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + b \\
& *c^{13}*x^{27} + (c^{12}*x^{26}*(2*a*c + 13*b^2))/2 + a^{13}*b*x + (143*a^7*b*x^7*(12 \\
& *b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + (143*b*c^7*x^{21}*(12*b \\
& ^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + 143*a^5*b*x^9*(b^8 + 5* \\
& a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 143*b*c^5*x^{19}*(b \\
& ^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 13*a^3*b*x \\
& ^{11}*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2* \\
& c^4 + 55*a*b^8*c) + 13*b*c^3*x^{17}*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 + \\
& 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c) + 13*a^9*b*x^5*(11*b^4 + 6* \\
& a^2*c^2 + 22*a*b^2*c) + 13*b*c^9*x^{23}*(11*b^4 + 6*a^2*c^2 + 22*a*b^2*c) + 1 \\
& 3*a^{11}*b*x^3*(a*c + 2*b^2) + 13*b*c^{11}*x^{25}*(a*c + 2*b^2)
\end{aligned}$$

sympy [B] time = 0.35, size = 1326, normalized size = 82.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)

[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12 /2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a* b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10 + 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4* c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1 287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b **2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x* *19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b **7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 + 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c** 4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**

$$\begin{aligned}
& 6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a** \\
& 6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c** \\
& 5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17 \\
& 16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b* \\
& *7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a* \\
& *7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c* \\
& *4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) + \\
& x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85 \\
& 80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1 \\
& 2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a** \\
& 5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) + \\
& x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514 \\
& 8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5 \\
& + 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287* \\
& a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c* \\
& *3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a* \\
& *10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c \\
& + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a \\
& **8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2 \\
& + 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b* \\
& *3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10 \\
& *b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b \\
& **2/2)
\end{aligned}$$

$$3.94 \quad \int x (b + 2cx^2) (a + bx^2 + cx^4)^{13} dx$$

Optimal. Leaf size=18

$$\frac{1}{28} (a + bx^2 + cx^4)^{14}$$

[Out] 1/28*(c*x^4+b*x^2+a)^14

Rubi [A] time = 0.33, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 629}

$$\frac{1}{28} (a + bx^2 + cx^4)^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]

[Out] (a + b*x^2 + c*x^4)^14/28

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (b + 2cx^2) (a + bx^2 + cx^4)^{13} dx &= \frac{1}{2} \text{Subst} \left(\int (b + 2cx) (a + bx + cx^2)^{13} dx, x, x^2 \right) \\ &= \frac{1}{28} (a + bx^2 + cx^4)^{14} \end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{28} x^2 (b + cx^2) \left(14a^{13} + 91a^{12}x^2 (b + cx^2) + 364a^{11}x^4 (b + cx^2)^2 + 1001a^{10}x^6 (b + cx^2)^3 + 2002a^9x^8 (b + cx^2)^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]

[Out] $(x^2*(b + c*x^2)*(14*a^{13} + 91*a^{12}*x^2*(b + c*x^2) + 364*a^{11}*x^4*(b + c*x^2)^2 + 1001*a^{10}*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8*x^{10}*(b + c*x^2)^5 + 3432*a^7*x^{12}*(b + c*x^2)^6 + 3003*a^6*x^{14}*(b + c*x^2)^7 + 2002*a^5*x^{16}*(b + c*x^2)^8 + 1001*a^4*x^{18}*(b + c*x^2)^9 + 364*a^3*x^{20}*(b + c*x^2)^{10} + 91*a^2*x^{22}*(b + c*x^2)^{11} + 14*a*x^{24}*(b + c*x^2)^{12} + x^{26}*(b + c*x^2)^{13})/28$

fricas [B] time = 0.51, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")

[Out] $1/28*x^{56}*c^{14} + 1/2*x^{54}*c^{13}*b + 13/4*x^{52}*c^{12}*b^2 + 1/2*x^{52}*c^{13}*a + 13*x^{50}*c^{11}*b^3 + 13/2*x^{50}*c^{12}*b*a + 143/4*x^{48}*c^{10}*b^4 + 39*x^{48}*c^{11}*b^2*a + 13/4*x^{48}*c^{12}*a^2 + 143/2*x^{46}*c^9*b^5 + 143*x^{46}*c^{10}*b^3*a + 39*x^{46}*c^{11}*b*a^2 + 429/4*x^{44}*c^8*b^6 + 715/2*x^{44}*c^9*b^4*a + 429/2*x^{44}*c^{10}*b^2*a^2 + 13*x^{44}*c^{11}*a^3 + 858/7*x^{42}*c^7*b^7 + 1287/2*x^{42}*c^8*b^5*a + 715*x^{42}*c^9*b^3*a^2 + 143*x^{42}*c^{10}*b*a^3 + 429/4*x^{40}*c^6*b^8 + 858*x^{40}*c^7*b^6*a + 6435/4*x^{40}*c^8*b^4*a^2 + 715*x^{40}*c^9*b^2*a^3 + 143/4*x^{40}*c^{10}*a^4 + 143/2*x^{38}*c^5*b^9 + 858*x^{38}*c^6*b^7*a + 2574*x^{38}*c^7*b^5*a^2 + 2145*x^{38}*c^8*b^3*a^3 + 715/2*x^{38}*c^9*b*a^4 + 143/4*x^{36}*c^4*b^{10} + 1287/2*x^{36}*c^5*b^8*a + 3003*x^{36}*c^6*b^6*a^2 + 4290*x^{36}*c^7*b^4*a^3 + 6435/4*x^{36}*c^8*b^2*a^4 + 143/2*x^{36}*c^9*a^5 + 13*x^{34}*c^3*b^{11} + 715/2*x^{34}*c^4*b^9*a + 2574*x^{34}*c^5*b^7*a^2 + 6006*x^{34}*c^6*b^5*a^3 + 4290*x^{34}*c^7*b^3*a^4 + 1287/2*x^{34}*c^8*b*a^5 + 13/4*x^{32}*c^2*b^{12} + 143*x^{32}*c^3*b^{10}*a + 6435/4*x^{32}*c^4*b^8*a^2 + 6006*x^{32}*c^5*b^6*a^3 + 15015/2*x^{32}*c^6*b^4*a^4 + 2574*x^{32}*c^7*b^2*a^5 + 429/4*x^{32}*c^8*a^6 + 1/2*x^{30}*c*b^{13} + 39*x^{30}*c^2*b^{11}*a + 715*x^{30}*c^3*b^9*a^2 + 4290*x^{30}*c^4*b^7*a^3 + 9009*x^{30}*c^5*b^5*a^4 + 6006*x^{30}*c^6*b^3*a^5 + 858*x^{30}*c^7*b*a^6 + 1/28*x^{28}*b^{14} + 13/2*x^{28}*c*b^{12}*a + 429/2*x^{28}*c^2*b^{10}*a^2 + 2145*x^{28}*c^3*b^8*a^3 + 15015/2*x^{28}*c^4*b^6*a^4 + 9009*x^{28}*c^5*b^4*a^5 + 3003*x^{28}*c^6*b^2*a^6 + 858/7*x^{28}*c^7*a^7 + 1/2*x^{26}*b^{13}*a + 39*x^{26}*c*b^{11}*a^2 + 715*x^{26}*c^2*b^9*a^3 + 4290*x^{26}*c^3*b^7*a^4 + 9009*x^{26}*c^4*b^5*a^5 + 6006*x^{26}*c^5*b^3*a^6 + 858*x^{26}*c^6*b*a^7 + 13/4*x^{24}*b^{12}*a^2 + 143*x^{24}*c*b^{10}*a^3 + 6435/4*x^{24}*c^2*b^8*a^4 + 6006*x^{24}*c^3*b^6*a^5 + 15015/2*x^{24}*c^4*b^4*a^6 + 2574*x^{24}*c^5*b^2*a^7 + 429/4*x^{24}*c^6*a^8 + 13*x^{22}*b^{11}*a^3 + 715/2*x^{22}*c*b^9*a^4 + 2574*x^{22}*c^2*b^7*a^5 + 6006*x^{22}*c^3*b^5*a^6 + 4290*x^{22}*c^4*b^3*a^7 + 1287/2*x^{22}*c^5*b*a^8 + 143/4*x^{20}*b^{10}*a^4 + 1287/2*x^{20}*c*b^8*a^5 + 3003*x^{20}*c^2*b^6*a^6 + 4290*x^{20}*c^3*b^4*a^7 + 6435/4*x^{20}*c^4*b^2*a^8 + 143/2*x^{20}*c^5*a^8$

$9 + 143/2*x^{18}*b^9*a^5 + 858*x^{18}*c*b^7*a^6 + 2574*x^{18}*c^2*b^5*a^7 + 2145*x^{18}*c^3*b^3*a^8 + 715/2*x^{18}*c^4*b*a^9 + 429/4*x^{16}*b^8*a^6 + 858*x^{16}*c*b^6*a^7 + 6435/4*x^{16}*c^2*b^4*a^8 + 715*x^{16}*c^3*b^2*a^9 + 143/4*x^{16}*c^4*a^{10} + 858/7*x^{14}*b^7*a^7 + 1287/2*x^{14}*c*b^5*a^8 + 715*x^{14}*c^2*b^3*a^9 + 143*x^{14}*c^3*b*a^{10} + 429/4*x^{12}*b^6*a^8 + 715/2*x^{12}*c*b^4*a^9 + 429/2*x^{12}*c^2*b^2*a^{10} + 13*x^{12}*c^3*a^{11} + 143/2*x^{10}*b^5*a^9 + 143*x^{10}*c*b^3*a^{10} + 39*x^{10}*c^2*b*a^{11} + 143/4*x^8*b^4*a^{10} + 39*x^8*c*b^2*a^{11} + 13/4*x^8*c^2*a^{12} + 13*x^6*b^3*a^{11} + 13/2*x^6*c*b*a^{12} + 13/4*x^4*b^2*a^{12} + 1/2*x^4*c*a^{13} + 1/2*x^2*b*a^{13}$

giac [B] time = 0.66, size = 246, normalized size = 13.67

$$\frac{1}{28} (cx^4 + bx^2)^{14} + \frac{1}{2} (cx^4 + bx^2)^{13} a + \frac{13}{4} (cx^4 + bx^2)^{12} a^2 + 13 (cx^4 + bx^2)^{11} a^3 + \frac{143}{4} (cx^4 + bx^2)^{10} a^4 + \frac{143}{2} (cx^4 + bx^2)^9 a^5 + \frac{429}{4} (cx^4 + bx^2)^8 a^6 + \frac{858}{7} (cx^4 + bx^2)^7 a^7 + \frac{429}{4} (cx^4 + bx^2)^6 a^8 + \frac{143}{2} (cx^4 + bx^2)^5 a^9 + \frac{143}{4} (cx^4 + bx^2)^4 a^{10} + 13 (cx^4 + bx^2)^3 a^{11} + \frac{13}{4} (cx^4 + bx^2)^2 a^{12} + \frac{1}{2} (cx^4 + bx^2) a^{13} + \frac{1}{2} a^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")

[Out] 1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^12*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*(c*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2)^7*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a^12 + 1/2*(c*x^4 + b*x^2)*a^13

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x)

[Out] result too large to display

maxima [B] time = 0.49, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")

[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^11)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*b^6*c^8 + 42*a*b^4*c^9 + 11*a^2*b^2*c^10)*x^44 + 13/2*(11*b^7*c^7 + 22*a*b^5*c^8 + 6*a^3*b^3*c^9)*x^42 + 13/4*(33*b^8*c^6 + 42*a*b^6*c^7 + 11*a^2*b^4*c^8)*x^40 + 13/2*(11*b^9*c^5 + 22*a*b^7*c^6 + 6*a^3*b^5*c^7)*x^38 + 13/4*(33*b^10*c^4 + 42*a*b^8*c^5 + 11*a^2*b^6*c^6)*x^36 + 13/2*(11*b^11*c^3 + 22*a*b^9*c^4 + 6*a^3*b^7*c^5)*x^34 + 13/4*(33*b^12*c^2 + 42*a*b^10*c^3 + 11*a^2*b^8*c^4)*x^32 + 13/2*(11*b^13*c + 22*a*b^11*c^2 + 6*a^3*b^9*c^3)*x^30 + 13/4*(33*b^14*c + 42*a*b^12*c^2 + 11*a^2*b^10*c^3)*x^28 + 13/2*(11*b^15*c + 22*a*b^13*c^2 + 6*a^3*b^11*c^3)*x^26 + 13/4*(33*b^16*c + 42*a*b^14*c^2 + 11*a^2*b^12*c^3)*x^24 + 13/2*(11*b^17*c + 22*a*b^15*c^2 + 6*a^3*b^13*c^3)*x^22 + 13/4*(33*b^18*c + 42*a*b^16*c^2 + 11*a^2*b^14*c^3)*x^20 + 13/2*(11*b^19*c + 22*a*b^17*c^2 + 6*a^3*b^15*c^3)*x^18 + 13/4*(33*b^20*c + 42*a*b^18*c^2 + 11*a^2*b^16*c^3)*x^16 + 13/2*(11*b^21*c + 22*a*b^19*c^2 + 6*a^3*b^17*c^3)*x^14 + 13/4*(33*b^22*c + 42*a*b^20*c^2 + 11*a^2*b^18*c^3)*x^12 + 13/2*(11*b^23*c + 22*a*b^21*c^2 + 6*a^3*b^19*c^3)*x^10 + 13/4*(33*b^24*c + 42*a*b^22*c^2 + 11*a^2*b^20*c^3)*x^8 + 13/2*(11*b^25*c + 22*a*b^23*c^2 + 6*a^3*b^21*c^3)*x^6 + 13/4*(33*b^26*c + 42*a*b^24*c^2 + 11*a^2*b^22*c^3)*x^4 + 13/2*(11*b^27*c + 22*a*b^25*c^2 + 6*a^3*b^23*c^3)*x^2 + 13/4*(33*b^28*c + 42*a*b^26*c^2 + 11*a^2*b^24*c^3)*x

$$\begin{aligned}
& b^6c^8 + 110ab^4c^9 + 66a^2b^2c^{10} + 4a^3c^{11})x^{44} + 143/14*(12b^7c^7 + 63ab^5c^8 + 70a^2b^3c^9 + 14a^3b^2c^{10})x^{42} + 143/4*(3b^8c^6 + 24ab^6c^7 + 45a^2b^4c^8 + 20a^3b^2c^9 + a^4c^{10})x^{40} + 143/2*(b^9c^5 + 12ab^7c^6 + 36a^2b^5c^7 + 30a^3b^3c^8 + 5a^4b^2c^9) \\
&)x^{38} + 143/4*(b^{10}c^4 + 18ab^8c^5 + 84a^2b^6c^6 + 120a^3b^4c^7 + 45a^4b^2c^8 + 2a^5c^9)x^{36} + 13/2*(2b^{11}c^3 + 55ab^9c^4 + 396a^2b^7c^5 + 924a^3b^5c^6 + 660a^4b^3c^7 + 99a^5b^2c^8)x^{34} + 13/4 \\
& *(b^{12}c^2 + 44ab^{10}c^3 + 495a^2b^8c^4 + 1848a^3b^6c^5 + 2310a^4b^4c^6 + 792a^5b^2c^7 + 33a^6c^8)x^{32} + 1/2*(b^{13}c + 78ab^{11}c^2 + 1430a^2b^9c^3 + 8580a^3b^7c^4 + 18018a^4b^5c^5 + 12012a^5b^3c^6 + 1716a^6b^2c^7) \\
&)x^{30} + 1/28*(b^{14} + 182ab^{12}c + 6006a^2b^{10}c^2 + 60060a^3b^8c^3 + 210210a^4b^6c^4 + 252252a^5b^4c^5 + 84084a^6b^2c^6 + 3432a^7c^7)x^{28} + 1/2*(ab^{13} + 78a^2b^{11}c + 1430a^3b^9c^2 + 8580a^4b^7c^3 + 18018a^5b^5c^4 + 12012a^6b^3c^5 + 1716a^7b^2c^6) \\
&)x^{26} + 13/4*(a^2b^{12} + 44a^3b^{10}c + 495a^4b^8c^2 + 1848a^5b^6c^3 + 2310a^6b^4c^4 + 792a^7b^2c^5 + 33a^8c^6)x^{24} + 13/2*(2a^3b^{11} + 55a^4b^9c + 396a^5b^7c^2 + 924a^6b^5c^3 + 660a^7b^3c^4 + 99a^8b^2c^5) \\
&)x^{22} + 143/4*(a^4b^{10} + 18a^5b^8c + 84a^6b^6c^2 + 120a^7b^4c^3 + 45a^8b^2c^4 + 2a^9c^5)x^{20} + 143/2*(a^5b^9 + 12a^6b^7c + 36a^7b^5c^2 + 30a^8b^3c^3 + 5a^9b^2c^4)x^{18} + 143/4*(3a^6b^8 + 24a^7b^6c + 45a^8b^4c^2 + 20a^9b^2c^3 + a^{10}c^4)x^{16} + 1/2*a^{13}b*x^2 \\
& + 143/14*(12a^7b^7 + 63a^8b^5c + 70a^9b^3c^2 + 14a^{10}b^2c^3)x^{14} + 13/4*(33a^8b^6 + 110a^9b^4c + 66a^{10}b^2c^2 + 4a^{11}c^3)x^{12} + 13/2*(11a^9b^5 + 22a^{10}b^3c + 6a^{11}b^2c^2)x^{10} + 13/4*(11a^{10}b^4 + 12a^{11}b^2c + a^{12}c^2)x^8 + 13/2*(2a^{11}b^3 + a^{12}b^2c)x^6 + 1/4*(13a^{12}b^2 + 2a^{13}c)x^4
\end{aligned}$$

mupad [B] time = 3.23, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^{13}, x)$

[Out] $x^{24}*((13a^2b^{12})/4 + (429a^8c^6)/4 + 143a^3b^{10}c + (6435a^4b^8c^2)/4 + 6006a^5b^6c^3 + (15015a^6b^4c^4)/2 + 2574a^7b^2c^5) + x^{32}*((429a^6c^8)/4 + (13b^{12}c^2)/4 + 143ab^{10}c^3 + (6435a^2b^8c^4)/4 + 6006a^3b^6c^5 + (15015a^4b^4c^6)/2 + 2574a^5b^2c^7) + x^{26}*((ab^{13})/2 + 39a^2b^{11}c + 858a^7b^8c^6 + 715a^3b^9c^2 + 4290a^4b^7c^3 + 9009a^5b^5c^4 + 6006a^6b^3c^5) + x^{30}*((b^{13}c)/2 + 39ab^{11}c^2 + 858a^6b^8c^7 + 715a^2b^9c^3 + 4290a^3b^7c^4 + 9009a^4b^5c^5 + 6006a^5b^3c^6) + x^{12}*((429a^8b^6)/4 + 13a^{11}c^3 + (715a^9b^4c)/2 + (429a^{10}b^2c^2)/2) + x^{44}*(13a^3c^{11} + (429b^6c^8)/4 + (715ab^4c^9)/2 + (429a^2b^2c^{10})/2) + x^{20}*((143a^4b^{10})/4 + (143a^9c^5)/2 + (1287a^5b^8c)/2 + 3003a^6b^6c^2 + 4290a^7b^4c^3 + (6435a^8b^2c$

$$\begin{aligned} &^4)/4) + x^{36}*((143*a^5*c^9)/2 + (143*b^{10}*c^4)/4 + (1287*a*b^8*c^5)/2 + 30 \\ &03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^{28}*(b^{14}/28 + \\ &(858*a^7*c^7)/7 + (429*a^2*b^{10}*c^2)/2 + 2145*a^3*b^8*c^3 + (15015*a^4*b^6 \\ &*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^{12}*c)/2) + x^{16}*((4 \\ &29*a^6*b^8)/4 + (143*a^{10}*c^4)/4 + 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 + 7 \\ &15*a^9*b^2*c^3) + x^{40}*((143*a^4*c^{10})/4 + (429*b^8*c^6)/4 + 858*a*b^6*c^7 \\ &+ (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^{14}*x^{56})/28 + x^4*((a^{13}*c)/ \\ &2 + (13*a^{12}*b^2)/4) + (13*a^{10}*x^8*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (1 \\ &3*c^{10}*x^{48}*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^{13}*b*x^2)/2 + (b*c^{13}*x \\ &^{54})/2 + (c^{12}*x^{52}*(2*a*c + 13*b^2))/4 + (143*a^7*b*x^{14}*(12*b^6 + 14*a^3*c \\ &c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*b*c^7*x^{42}*(12*b^6 + 14*a^3*c \\ &c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*a^5*b*x^{18}*(b^8 + 5*a^4*c^4 + \\ &36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (143*b*c^5*x^{38}*(b^8 + 5 \\ &*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (13*a^3*b*x^2 \\ &2*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 \\ &+ 55*a*b^8*c))/2 + (13*b*c^3*x^{34}*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 \\ &+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/2 + (13*a^9*b*x^{10}*(11*b \\ &^4 + 6*a^2*c^2 + 22*a*b^2*c))/2 + (13*b*c^9*x^{46}*(11*b^4 + 6*a^2*c^2 + 22*a \\ &*b^2*c))/2 + (13*a^{11}*b*x^6*(a*c + 2*b^2))/2 + (13*b*c^{11}*x^{50}*(a*c + 2*b^2 \\ &))/2 \end{aligned}$$

sympy [B] time = 0.34, size = 1384, normalized size = 76.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)

[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**12/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a**2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**10/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c**7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**

$$\begin{aligned}
& 5 + 15015*a^{**4}*b^{**6}*c^{**4}/2 + 2145*a^{**3}*b^{**8}*c^{**3} + 429*a^{**2}*b^{**10}*c^{**2}/2 + \\
& 13*a*b^{**12}*c/2 + b^{**14}/28) + x^{**26}*(858*a^{**7}*b*c^{**6} + 6006*a^{**6}*b^{**3}*c^{**5} + \\
& 9009*a^{**5}*b^{**5}*c^{**4} + 4290*a^{**4}*b^{**7}*c^{**3} + 715*a^{**3}*b^{**9}*c^{**2} + 39*a^{**2}*b^{**11}*c + a*b^{**13}/2) + x^{**24}*(429*a^{**8}*c^{**6}/4 + 2574*a^{**7}*b^{**2}*c^{**5} + 15015* \\
& a^{**6}*b^{**4}*c^{**4}/2 + 6006*a^{**5}*b^{**6}*c^{**3} + 6435*a^{**4}*b^{**8}*c^{**2}/4 + 143*a^{**3}*b^{**10}*c + 13*a^{**2}*b^{**12}/4) + x^{**22}*(1287*a^{**8}*b*c^{**5}/2 + 4290*a^{**7}*b^{**3}*c^{**4} \\
& + 6006*a^{**6}*b^{**5}*c^{**3} + 2574*a^{**5}*b^{**7}*c^{**2} + 715*a^{**4}*b^{**9}*c/2 + 13*a^{**3}* \\
& b^{**11}) + x^{**20}*(143*a^{**9}*c^{**5}/2 + 6435*a^{**8}*b^{**2}*c^{**4}/4 + 4290*a^{**7}*b^{**4}*c^{**3} + 3003*a^{**6}*b^{**6}*c^{**2} + 1287*a^{**5}*b^{**8}*c/2 + 143*a^{**4}*b^{**10}/4) + x^{**18}*(\\
& 715*a^{**9}*b*c^{**4}/2 + 2145*a^{**8}*b^{**3}*c^{**3} + 2574*a^{**7}*b^{**5}*c^{**2} + 858*a^{**6}*b* \\
& *7*c + 143*a^{**5}*b^{**9}/2) + x^{**16}*(143*a^{**10}*c^{**4}/4 + 715*a^{**9}*b^{**2}*c^{**3} + 64 \\
& 35*a^{**8}*b^{**4}*c^{**2}/4 + 858*a^{**7}*b^{**6}*c + 429*a^{**6}*b^{**8}/4) + x^{**14}*(143*a^{**10} \\
& *b*c^{**3} + 715*a^{**9}*b^{**3}*c^{**2} + 1287*a^{**8}*b^{**5}*c/2 + 858*a^{**7}*b^{**7}/7) + x^{**12}*(13*a^{**11}*c^{**3} + 429*a^{**10}*b^{**2}*c^{**2}/2 + 715*a^{**9}*b^{**4}*c/2 + 429*a^{**8}*b^{**6} \\
& /4) + x^{**10}*(39*a^{**11}*b*c^{**2} + 143*a^{**10}*b^{**3}*c + 143*a^{**9}*b^{**5}/2) + x^{**8}*(\\
& 13*a^{**12}*c^{**2}/4 + 39*a^{**11}*b^{**2}*c + 143*a^{**10}*b^{**4}/4) + x^{**6}*(13*a^{**12}*b*c \\
& /2 + 13*a^{**11}*b^{**3}) + x^{**4}*(a^{**13}*c/2 + 13*a^{**12}*b^{**2}/4)
\end{aligned}$$

$$3.95 \quad \int x^2 (b + 2cx^3) (a + bx^3 + cx^6)^{13} dx$$

Optimal. Leaf size=18

$$\frac{1}{42} (a + bx^3 + cx^6)^{14}$$

[Out] 1/42*(c*x^6+b*x^3+a)^14

Rubi [A] time = 0.30, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 629}

$$\frac{1}{42} (a + bx^3 + cx^6)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]

[Out] (a + b*x^3 + c*x^6)^14/42

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (b + 2cx^3) (a + bx^3 + cx^6)^{13} dx &= \frac{1}{3} \text{Subst} \left(\int (b + 2cx) (a + bx + cx^2)^{13} dx, x, x^3 \right) \\ &= \frac{1}{42} (a + bx^3 + cx^6)^{14} \end{aligned}$$

Mathematica [B] time = 0.18, size = 233, normalized size = 12.94

$$\frac{1}{42} x^3 (b + cx^3) \left(14a^{13} + 91a^{12}x^3 (b + cx^3) + 364a^{11}x^6 (b + cx^3)^2 + 1001a^{10}x^9 (b + cx^3)^3 + 2002a^9x^{12} (b + cx^3)^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]

[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^12 + x^39*(b + c*x^3)^13)/42

fricas [B] time = 0.64, size = 1454, normalized size = 80.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")

[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 1/3*x^78*c^13*a + 26/3*x^75*c^11*b^3 + 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 + 26*x^72*c^11*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 + 286/3*x^69*c^10*b^3*a + 26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 + 715/3*x^66*c^9*b^4*a + 143*x^66*c^10*b^2*a^2 + 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 + 429*x^63*c^8*b^5*a + 1430/3*x^63*c^9*b^3*a^2 + 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 + 572*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 + 1430/3*x^60*c^9*b^2*a^3 + 143/6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 + 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b^5*a^2 + 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10 + 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 + 2860*x^54*c^7*b^4*a^3 + 2145/2*x^54*c^8*b^2*a^4 + 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 + 715/3*x^51*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 + 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*b^3*a^4 + 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 + 286/3*x^48*c^3*b^10*a + 2145/2*x^48*c^4*b^8*a^2 + 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 + 1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 + 26*x^45*c^2*b^11*a + 1430/3*x^45*c^3*b^9*a^2 + 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^5*a^4 + 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 + 13/3*x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 + 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c^4*b^6*a^4 + 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 + 572/7*x^42*c^7*a^7 + 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 + 1430/3*x^39*c^2*b^9*a^3 + 2860*x^39*c^3*b^7*a^4 + 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 + 572*x^39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 + 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2*b^8*a^4 + 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 + 1716*x^36*c^5*b^2*a^7 + 143/2*x^36*c^6*a^8 + 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 + 1716*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 + 2860*x^33*c^4*b^3*a^7 + 429*x^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 + 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^6*a^6 + 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 + 143/3*x^30*c^5*a^

$9 + 143/3*x^{27}*b^9*a^5 + 572*x^{27}*c*b^7*a^6 + 1716*x^{27}*c^2*b^5*a^7 + 1430*x^{27}*c^3*b^3*a^8 + 715/3*x^{27}*c^4*b*a^9 + 143/2*x^{24}*b^8*a^6 + 572*x^{24}*c*b^6*a^7 + 2145/2*x^{24}*c^2*b^4*a^8 + 1430/3*x^{24}*c^3*b^2*a^9 + 143/6*x^{24}*c^4*a^{10} + 572/7*x^{21}*b^7*a^7 + 429*x^{21}*c*b^5*a^8 + 1430/3*x^{21}*c^2*b^3*a^9 + 286/3*x^{21}*c^3*b*a^{10} + 143/2*x^{18}*b^6*a^8 + 715/3*x^{18}*c*b^4*a^9 + 143*x^{18}*c^2*b^2*a^{10} + 26/3*x^{18}*c^3*a^{11} + 143/3*x^{15}*b^5*a^9 + 286/3*x^{15}*c*b^3*a^{10} + 26*x^{15}*c^2*b*a^{11} + 143/6*x^{12}*b^4*a^{10} + 26*x^{12}*c*b^2*a^{11} + 13/6*x^{12}*c^2*a^{12} + 26/3*x^9*b^3*a^{11} + 13/3*x^9*c*b*a^{12} + 13/6*x^6*b^2*a^{11} + 1/3*x^6*c*a^{13} + 1/3*x^3*b*a^{13}$

giac [B] time = 0.61, size = 246, normalized size = 13.67

$$\frac{1}{42} (cx^6 + bx^3)^{14} + \frac{1}{3} (cx^6 + bx^3)^{13} a + \frac{13}{6} (cx^6 + bx^3)^{12} a^2 + \frac{26}{3} (cx^6 + bx^3)^{11} a^3 + \frac{143}{6} (cx^6 + bx^3)^{10} a^4 + \frac{143}{3} (cx^6 + bx^3)^9 a^5 + \frac{143}{2} (cx^6 + bx^3)^8 a^6 + \frac{572}{7} (cx^6 + bx^3)^7 a^7 + \frac{143}{2} (cx^6 + bx^3)^6 a^8 + \frac{143}{3} (cx^6 + bx^3)^5 a^9 + \frac{143}{6} (cx^6 + bx^3)^4 a^{10} + \frac{26}{3} (cx^6 + bx^3)^3 a^{11} + \frac{13}{6} (cx^6 + bx^3)^2 a^{12} + \frac{1}{3} (cx^6 + bx^3) a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")

[Out] 1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^12*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/3*(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x^3)^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13

maple [B] time = 0.00, size = 46552, normalized size = 2586.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x)

[Out] result too large to display

maxima [B] time = 0.55, size = 1240, normalized size = 68.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")

[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^12)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*

$$\begin{aligned}
& b^6c^8 + 110ab^4c^9 + 66a^2b^2c^{10} + 4a^3c^{11})x^{66} + 143/21*(12b^7c^7 + 63ab^5c^8 + 70a^2b^3c^9 + 14a^3b^2c^{10})x^{63} + 143/6*(3b^8c^6 + 24ab^6c^7 + 45a^2b^4c^8 + 20a^3b^2c^9 + a^4c^{10})x^{60} + 143/3*(b^9c^5 + 12ab^7c^6 + 36a^2b^5c^7 + 30a^3b^3c^8 + 5a^4b^2c^9)x^{57} + 143/6*(b^{10}c^4 + 18ab^8c^5 + 84a^2b^6c^6 + 120a^3b^4c^7 + 45a^4b^2c^8 + 2a^5c^9)x^{54} + 13/3*(2b^{11}c^3 + 55ab^9c^4 + 396a^2b^7c^5 + 924a^3b^5c^6 + 660a^4b^3c^7 + 99a^5b^2c^8)x^{51} + 13/6*(b^{12}c^2 + 44ab^{10}c^3 + 495a^2b^8c^4 + 1848a^3b^6c^5 + 2310a^4b^4c^6 + 792a^5b^2c^7 + 33a^6c^8)x^{48} + 1/3*(b^{13}c + 78ab^{11}c^2 + 1430a^2b^9c^3 + 8580a^3b^7c^4 + 18018a^4b^5c^5 + 12012a^5b^3c^6 + 1716a^6b^2c^7)x^{45} + 1/42*(b^{14} + 182ab^{12}c + 6006a^2b^{10}c^2 + 60060a^3b^8c^3 + 210210a^4b^6c^4 + 252252a^5b^4c^5 + 84084a^6b^2c^6 + 3432a^7c^7)x^{42} + 1/3*(a^2b^{13} + 78a^2b^{11}c + 1430a^3b^9c^2 + 8580a^4b^7c^3 + 18018a^5b^5c^4 + 12012a^6b^3c^5 + 1716a^7b^2c^6)x^{39} + 13/6*(a^2b^{12} + 44a^3b^{10}c + 495a^4b^8c^2 + 1848a^5b^6c^3 + 2310a^6b^4c^4 + 792a^7b^2c^5 + 33a^8c^6)x^{36} + 13/3*(2a^3b^{11} + 55a^4b^9c + 396a^5b^7c^2 + 924a^6b^5c^3 + 660a^7b^3c^4 + 99a^8b^2c^5)x^{33} + 143/6*(a^4b^{10} + 18a^5b^8c + 84a^6b^6c^2 + 120a^7b^4c^3 + 45a^8b^2c^4 + 2a^9c^5)x^{30} + 143/3*(a^5b^9 + 12a^6b^7c + 36a^7b^5c^2 + 30a^8b^3c^3 + 5a^9b^2c^4)x^{27} + 143/6*(3a^6b^8 + 24a^7b^6c + 45a^8b^4c^2 + 20a^9b^2c^3 + a^{10}c^4)x^{24} + 143/21*(12a^7b^7 + 63a^8b^5c + 70a^9b^3c^2 + 14a^{10}b^2c^3)x^{21} + 13/6*(33a^8b^6 + 110a^9b^4c + 66a^{10}b^2c^2 + 4a^{11}c^3)x^{18} + 1/3a^{13}bx^3 + 13/3*(11a^9b^5 + 22a^{10}b^3c + 6a^{11}b^2c^2)x^{15} + 13/6*(11a^{10}b^4 + 12a^{11}b^2c + a^{12}c^2)x^{12} + 13/3*(2a^{11}b^3 + a^{12}b^2c)x^9 + 1/6*(13a^{12}b^2 + 2a^{13}c)x^6
\end{aligned}$$

mupad [B] time = 3.18, size = 1210, normalized size = 67.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^{13}, x)$

[Out] $x^{36}*((13a^2b^{12})/6 + (143a^8c^6)/2 + (286a^3b^{10}c)/3 + (2145a^4b^8c^2)/2 + 4004a^5b^6c^3 + 5005a^6b^4c^4 + 1716a^7b^2c^5) + x^{48}*((143a^6c^8)/2 + (13b^{12}c^2)/6 + (286ab^{10}c^3)/3 + (2145a^2b^8c^4)/2 + 4004a^3b^6c^5 + 5005a^4b^4c^6 + 1716a^5b^2c^7) + x^{39}*((ab^{13})/3 + 26a^2b^{11}c + 572a^7b^6c^6 + (1430a^3b^9c^2)/3 + 2860a^4b^7c^3 + 6006a^5b^5c^4 + 4004a^6b^3c^5) + x^{45}*((b^{13}c)/3 + 26ab^{11}c^2 + 572a^6b^6c^7 + (1430a^2b^9c^3)/3 + 2860a^3b^7c^4 + 6006a^4b^5c^5 + 4004a^5b^3c^6) + x^{18}*((143a^8b^6)/2 + (26a^{11}c^3)/3 + (715a^9b^4c)/3 + 143a^{10}b^2c^2) + x^{66}*((26a^3c^{11})/3 + (143b^6c^8)/2 + (715ab^4c^9)/3 + 143a^2b^2c^{10}) + x^{30}*((143a^4b^{10})/6 + (143a^9c^5)/3 + 429a^5b^8c + 2002a^6b^6c^2 + 2860a^7b^4c^3 + (2145a^8b^$

$$\begin{aligned}
& 2*c^4)/2) + x^{54}*((143*a^5*c^9)/3 + (143*b^{10}*c^4)/6 + 429*a*b^8*c^5 + 2002 \\
& *a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^{42}*(b^{14}/42 + (\\
& 572*a^7*c^7)/7 + 143*a^2*b^{10}*c^2 + 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 + 6 \\
& 006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^{12}*c)/3) + x^{24}*((143*a^6*b^8) \\
& /2 + (143*a^{10}*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 + (1430*a^9*b^ \\
& 2*c^3)/3) + x^{60}*((143*a^4*c^{10})/6 + (143*b^8*c^6)/2 + 572*a*b^6*c^7 + (214 \\
& 5*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^{14}*x^{84})/42 + x^6*((a^{13}*c)/3 \\
& + (13*a^{12}*b^2)/6) + (13*a^{10}*x^{12}*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (1 \\
& 3*c^{10}*x^{72}*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^{13}*b*x^3)/3 + (b*c^{13}*x \\
& ^{81})/3 + (c^{12}*x^{78}*(2*a*c + 13*b^2))/6 + (143*a^7*b*x^{21}*(12*b^6 + 14*a^3* \\
& c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*b*c^7*x^{63}*(12*b^6 + 14*a^3*c \\
& ^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*a^5*b*x^{27}*(b^8 + 5*a^4*c^4 + \\
& 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (143*b*c^5*x^{57}*(b^8 + 5 \\
& *a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (13*a^3*b*x^3 \\
& 3*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^ \\
& 4 + 55*a*b^8*c))/3 + (13*b*c^3*x^{51}*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 \\
& + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/3 + (13*a^9*b*x^{15}*(11*b \\
& ^4 + 6*a^2*c^2 + 22*a*b^2*c))/3 + (13*b*c^9*x^{69}*(11*b^4 + 6*a^2*c^2 + 22*a \\
& *b^2*c))/3 + (13*a^{11}*b*x^9*(a*c + 2*b^2))/3 + (13*b*c^{11}*x^{75}*(a*c + 2*b^2 \\
&))/3
\end{aligned}$$

sympy [B] time = 0.34, size = 1394, normalized size = 77.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)

[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c**12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 + 1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b**4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a**4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b

$$\begin{aligned}
& **4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2 \\
& + 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5 \\
& + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a \\
& **2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5 \\
& 005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3 \\
& *b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c** \\
& 4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3 \\
& *b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4 \\
& *c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(\\
& 715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b* \\
& *7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 + \\
& 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a* \\
& *10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) + \\
& x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a** \\
& 8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3) \\
& + x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a \\
& **12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
\end{aligned}$$

$$3.96 \quad \int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^{13} dx$$

Optimal. Leaf size=23

$$\frac{(a + bx^n + cx^{2n})^{14}}{14n}$$

[Out] 1/14*(a+b*x^n+c*x^(2*n))^14/n

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 629}

$$\frac{(a + bx^n + cx^{2n})^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]

[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^{13} dx &= \frac{\text{Subst}\left(\int (b + 2cx) (a + bx + cx^2)^{13} dx, x, x^n\right)}{n} \\ &= \frac{(a + bx^n + cx^{2n})^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.07, size = 22, normalized size = 0.96

$$\frac{(a + x^n (b + cx^n))^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*(b + 2*c*xⁿ)*(a + b*xⁿ + c*x^(2*n))¹³,x]

[Out] (a + xⁿ*(b + c*xⁿ))¹⁴/(14*n)

fricas [B] time = 0.74, size = 1297, normalized size = 56.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(a+b*xⁿ+c*x^(2*n))¹³,x, algorithm="fricas")

[Out] 1/14*(c¹⁴*x^(28*n) + 14*b*c¹³*x^(27*n) + 14*a¹³*b*xⁿ + 7*(13*b²*c¹² + 2*a*c¹³)*x^(26*n) + 182*(2*b³*c¹¹ + a*b*c¹²)*x^(25*n) + 91*(11*b⁴*c¹⁰ + 12*a*b²*c¹¹ + a²*c¹²)*x^(24*n) + 182*(11*b⁵*c⁹ + 22*a*b³*c¹⁰ + 6*a²*b*c¹¹)*x^(23*n) + 91*(33*b⁶*c⁸ + 110*a*b⁴*c⁹ + 66*a²*b²*c¹⁰ + 4*a³*c¹¹)*x^(22*n) + 286*(12*b⁷*c⁷ + 63*a*b⁵*c⁸ + 70*a²*b³*c⁹ + 14*a³*b*c¹⁰)*x^(21*n) + 1001*(3*b⁸*c⁶ + 24*a*b⁶*c⁷ + 45*a²*b⁴*c⁸ + 20*a³*b²*c⁹ + a⁴*c¹⁰)*x^(20*n) + 2002*(b⁹*c⁵ + 12*a*b⁷*c⁶ + 36*a²*b⁵*c⁷ + 30*a³*b³*c⁸ + 5*a⁴*b*c⁹)*x^(19*n) + 1001*(b¹⁰*c⁴ + 18*a*b⁸*c⁵ + 84*a²*b⁶*c⁶ + 120*a³*b⁴*c⁷ + 45*a⁴*b²*c⁸ + 2*a⁵*c⁹)*x^(18*n) + 182*(2*b¹¹*c³ + 55*a*b⁹*c⁴ + 396*a²*b⁷*c⁵ + 924*a³*b⁵*c⁶ + 660*a⁴*b³*c⁷ + 99*a⁵*b*c⁸)*x^(17*n) + 91*(b¹²*c² + 44*a*b¹⁰*c³ + 495*a²*b⁸*c⁴ + 1848*a³*b⁶*c⁵ + 2310*a⁴*b⁴*c⁶ + 792*a⁵*b²*c⁷ + 33*a⁶*c⁸)*x^(16*n) + 14*(b¹³*c + 78*a*b¹¹*c² + 1430*a²*b⁹*c³ + 8580*a³*b⁷*c⁴ + 18018*a⁴*b⁵*c⁵ + 12012*a⁵*b³*c⁶ + 1716*a⁶*b*c⁷)*x^(15*n) + (b¹⁴ + 182*a*b¹²*c + 6006*a²*b¹⁰*c² + 60060*a³*b⁸*c³ + 210210*a⁴*b⁶*c⁴ + 252252*a⁵*b⁴*c⁵ + 84084*a⁶*b²*c⁶ + 3432*a⁷*c⁷)*x^(14*n) + 14*(a*b¹³ + 78*a²*b¹¹*c + 1430*a³*b⁹*c² + 8580*a⁴*b⁷*c³ + 18018*a⁵*b⁵*c⁴ + 12012*a⁶*b³*c⁵ + 1716*a⁷*b*c⁶)*x^(13*n) + 91*(a²*b¹² + 44*a³*b¹⁰*c + 495*a⁴*b⁸*c² + 1848*a⁵*b⁶*c³ + 2310*a⁶*b⁴*c⁴ + 792*a⁷*b²*c⁵ + 33*a⁸*c⁶)*x^(12*n) + 182*(2*a³*b¹¹ + 55*a⁴*b⁹*c + 396*a⁵*b⁷*c² + 924*a⁶*b⁵*c³ + 660*a⁷*b³*c⁴ + 99*a⁸*b*c⁵)*x^(11*n) + 1001*(a⁴*b¹⁰ + 18*a⁵*b⁸*c + 84*a⁶*b⁶*c² + 120*a⁷*b⁴*c³ + 45*a⁸*b²*c⁴ + 2*a⁹*c⁵)*x^(10*n) + 2002*(a⁵*b⁹ + 12*a⁶*b⁷*c + 36*a⁷*b⁵*c² + 30*a⁸*b³*c³ + 5*a⁹*b*c⁴)*x^(9*n) + 1001*(3*a⁶*b⁸ + 24*a⁷*b⁶*c + 45*a⁸*b⁴*c² + 20*a⁹*b²*c³ + a¹⁰*c⁴)*x^(8*n) + 286*(12*a⁷*b⁷*c² + 63*a⁸*b⁵*c + 70*a⁹*b³*c² + 14*a¹⁰*b*c³)*x^(7*n) + 91*(33*a⁸*b⁶*c² + 182*a⁹*b⁵*c + 924*a¹⁰*b⁴*c² + 660*a¹¹*b³*c² + 99*a¹²*b²*c² + 99*a¹³*b*c² + 99*a¹⁴*c²)*x^(6*n) + 182*(2*a³*b¹¹ + 55*a⁴*b⁹*c + 396*a⁵*b⁷*c² + 924*a⁶*b⁵*c³ + 660*a⁷*b³*c⁴ + 99*a⁸*b*c⁵)*x^(5*n) + 1001*(a⁴*b¹⁰ + 18*a⁵*b⁸*c + 84*a⁶*b⁶*c² + 120*a⁷*b⁴*c³ + 45*a⁸*b²*c⁴ + 2*a⁹*c⁵)*x^(4*n) + 2002*(a⁵*b⁹ + 12*a⁶*b⁷*c + 36*a⁷*b⁵*c² + 30*a⁸*b³*c³ + 5*a⁹*b*c⁴)*x^(3*n) + 1001*(3*a⁶*b⁸ + 24*a⁷*b⁶*c + 45*a⁸*b⁴*c² + 20*a⁹*b²*c³ + a¹⁰*c⁴)*x^(2*n) + 286*(12*a⁷*b⁷*c² + 63*a⁸*b⁵*c + 70*a⁹*b³*c² + 14*a¹⁰*b*c³)*x^(1*n) + 91*(33*a⁸*b⁶*c² + 182*a⁹*b⁵*c + 924*a¹⁰*b⁴*c² + 660*a¹¹*b³*c² + 99*a¹²*b²*c² + 99*a¹³*b*c² + 99*a¹⁴*c²)*x^(0*n)

$$\frac{6 + 110*a^9*b^4*c + 66*a^{10}*b^2*c^2 + 4*a^{11}*c^3}{n} * x^{(6*n)} + 182*(11*a^9*b^5 + 22*a^{10}*b^3*c + 6*a^{11}*b*c^2) * x^{(5*n)} + 91*(11*a^{10}*b^4 + 12*a^{11}*b^2*c + a^{12}*c^2) * x^{(4*n)} + 182*(2*a^{11}*b^3 + a^{12}*b*c) * x^{(3*n)} + 7*(13*a^{12}*b^2 + 2*a^{13}*c) * x^{(2*n)}$$

giac [B] time = 1.00, size = 1693, normalized size = 73.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 84084*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 4004*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n) + 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(11*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) + 1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^

$$5*c^2*x^{(9*n)} + 60060*a^8*b^3*c^3*x^{(9*n)} + 10010*a^9*b*c^4*x^{(9*n)} + 3003*a^6*b^8*x^{(8*n)} + 24024*a^7*b^6*c*x^{(8*n)} + 45045*a^8*b^4*c^2*x^{(8*n)} + 20020*a^9*b^2*c^3*x^{(8*n)} + 1001*a^10*c^4*x^{(8*n)} + 3432*a^7*b^7*x^{(7*n)} + 18018*a^8*b^5*c*x^{(7*n)} + 20020*a^9*b^3*c^2*x^{(7*n)} + 4004*a^10*b*c^3*x^{(7*n)} + 3003*a^8*b^6*x^{(6*n)} + 10010*a^9*b^4*c*x^{(6*n)} + 6006*a^10*b^2*c^2*x^{(6*n)} + 364*a^11*c^3*x^{(6*n)} + 2002*a^9*b^5*x^{(5*n)} + 4004*a^10*b^3*c*x^{(5*n)} + 1092*a^11*b*c^2*x^{(5*n)} + 1001*a^10*b^4*x^{(4*n)} + 1092*a^11*b^2*c*x^{(4*n)} + 91*a^12*c^2*x^{(4*n)} + 364*a^11*b^3*x^{(3*n)} + 182*a^12*b*c*x^{(3*n)} + 91*a^12*b^2*x^{(2*n)} + 14*a^13*c*x^{(2*n)} + 14*a^13*b*x^{(n)}/n$$

maple [B] time = 0.06, size = 2042, normalized size = 88.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(n-1)}*(b+2*c*x^n)*(b*x^n+c*x^{(2*n)}+a)^{13}, x)$

[Out] $26*b^{11}*c^3/n*(x^n)^{17}+1716/7/n*(x^n)^{14}*a^7*c^7+1716/7*b^7*a^7/n*(x^n)^{7+1}+43*b^9*c^5/n*(x^n)^{19}+26*b^3*c^{11}/n*(x^n)^{25}+a*b^{13}/n*(x^n)^{13}+143*a^5*b^9/n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^{21}+143*b^5*c^9/n*(x^n)^{23}+143/2*a^{10}/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+143*b^5*a^9/n*(x^n)^5+26*b^{11}*a^3/n*(x^n)^{11}+b^{13}*c/n*(x^n)^{15}+13/2*a^{12}/n*(x^n)^4*c^2+143/2*a^{10}/n*(x^n)^4*b^4+26*a^{11}/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+143*a^9/n*(x^n)^{10}*c^5+143/2*a^4/n*(x^n)^{10}*b^{10}+429/2*c^8/n*(x^n)^{22}*b^6+c^{13}/n*(x^n)^{26}*a+13/2*c^{12}/n*(x^n)^{26}*b^2+429/2*c^8/n*(x^n)^{16}*a^6+13/2*c^2/n*(x^n)^{16}*b^{12}+143*c^9/n*(x^n)^{18}*a^5+143/2*c^4/n*(x^n)^{18}*b^{10}+143/2*c^{10}/n*(x^n)^{20}*a^4+429/2*c^6/n*(x^n)^{20}*b^8+26*c^{11}/n*(x^n)^{22}*a^3+429/2*a^8/n*(x^n)^{12}*c^6+13/2*a^2/n*(x^n)^{12}*b^{12}+26*a^{11}*b^3/n*(x^n)^3+13/2*c^{12}/n*(x^n)^{24}*a^2+143/2*c^{10}/n*(x^n)^{24}*b^4+a^{13}/n*(x^n)^2*c+13/2*a^{12}/n*(x^n)^2*b^2+b*a^{13}/n*x^n+b*c^{13}/n*(x^n)^{27}+1287*b^5*c^8/n*(x^n)^{21}*a+78*b*c^{11}/n*(x^n)^{23}*a^2+286*b^3*c^{10}/n*(x^n)^{23}*a+286*b*a^{10}/n*(x^n)^7*c^3+1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)^7*c+715*b*c^9/n*(x^n)^{19}*a^4+4290*b^3*c^8/n*(x^n)^{19}*a^3+5148*b^5*c^7/n*(x^n)^{19}*a^2+1716*b^7*c^6/n*(x^n)^{19}*a+5148*a^7/n*(x^n)^{12}*b^2*c^5+15015*a^6/n*(x^n)^{12}*b^4*c^4+12012*a^5/n*(x^n)^{12}*b^6*c^3+6435/2*a^4/n*(x^n)^{12}*b^8*c^2+1/14*c^{14}/n*(x^n)^{28}+715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^{14}*b^{14}+6435/2*a^8/n*(x^n)^{10}*b^2*c^4+8580*a^7/n*(x^n)^{10}*b^4*c^3+6006*a^6/n*(x^n)^{10}*b^6*c^2+1287*a^5/n*(x^n)^{10}*b^8*c+1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b^4*c^2+1716*a^7/n*(x^n)^8*b^6*c+78*b*a^{11}/n*(x^n)^5*c^2+286*b^3*a^{10}/n*(x^n)^5*c+1287*b*a^8/n*(x^n)^{11}*c^5+1716*a^7*b/n*(x^n)^{13}*c^6+12012*a^6*b^3/n*(x^n)^{13}*c^5+18018*a^5*b^5/n*(x^n)^{13}*c^4+8580*a^4*b^7/n*(x^n)^{13}*c^3+1430*a^3*b^9/n*(x^n)^{13}*c^2+78*a^2*b^{11}/n*(x^n)^{13}*c+715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c+286*b*c^{10}/n*(x^n)^{21}*a^3+1430*b^3*c^9/n*(x^n)^{21}*a^2+8580*b^3*a^7/n*(x^n)^{11}*c^4+12012*b^5*a^6/n*(x^n)^{11}*c^3+5148*b^7*a^5/n*(x^n)^{11}*c^2+715*b^9*a^4/n*(x^n)^{11}*c+1716*b*c^7/n*(x^n)^{15}*a^6+12012*b^3*c^6/n*(x^n)^{15}*a^5+1$

$$\begin{aligned}
& 8018*b^5*c^5/n*(x^n)^{15*a^4+8580*b^7*c^4/n*(x^n)^{15*a^3+1430*b^9*c^3/n*(x^n)} \\
&)^{15*a^2+78*b^{11}*c^2/n*(x^n)^{15*a+13*b*c^{12}/n*(x^n)^{25*a+1430*c^9/n*(x^n)^2} \\
& 0*a^3*b^2+6435/2*c^8/n*(x^n)^{20*a^2*b^4+1716*c^7/n*(x^n)^{20*a*b^6+429*c^{10}/} \\
& n*(x^n)^{22*a^2*b^2+715*c^9/n*(x^n)^{22*a*b^4+13*a^{12}*b/n*(x^n)^3*c+78*c^{11}/n} \\
& *(x^n)^{24*a*b^2+78*a^{11}/n*(x^n)^4*b^2*c+429*a^{10}/n*(x^n)^6*b^2*c^2+1287*b*c \\
& ^8/n*(x^n)^{17*a^5+8580*b^3*c^7/n*(x^n)^{17*a^4+12012*b^5*c^6/n*(x^n)^{17*a^3+} \\
& 5148*b^7*c^5/n*(x^n)^{17*a^2+715*b^9*c^4/n*(x^n)^{17*a+6006/n*(x^n)^{14*a^6*b^} \\
& 2*c^6+18018/n*(x^n)^{14*a^5*b^4*c^5+15015/n*(x^n)^{14*a^4*b^6*c^4+4290/n*(x^n)} \\
&)^{14*a^3*b^8*c^3+429/n*(x^n)^{14*a^2*b^{10}*c^2+13/n*(x^n)^{14*a*b^{12}*c+286*a^3} \\
& /n*(x^n)^{12*b^{10}*c+5148*c^7/n*(x^n)^{16*a^5*b^2+15015*c^6/n*(x^n)^{16*a^4*b^4} \\
& +12012*c^5/n*(x^n)^{16*a^3*b^6+6435/2*c^4/n*(x^n)^{16*a^2*b^8+286*c^3/n*(x^n)} \\
& ^{16*a*b^{10}+6435/2*c^8/n*(x^n)^{18*a^4*b^2+8580*c^7/n*(x^n)^{18*a^3*b^4+6006*c} \\
& ^6/n*(x^n)^{18*a^2*b^6+1287*c^5/n*(x^n)^{18*a*b^8}
\end{aligned}$$

maxima [B] time = 0.86, size = 2041, normalized size = 88.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima")

[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^13*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n + 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*a*b^5*c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n + 8580*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*a^5*c^9*x^(18*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n + 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286*a*b^10*c^3*x^(16*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5*x^(16*n)/n + 15015*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n + 12012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)/n + 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n + 4290*a^3*b^8*c^3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n + 18018*a^5*b^4*c^5*x^(14*n)/n

$$\begin{aligned}
& + 6006*a^6*b^2*c^6*x^{(14*n)}/n + 1716/7*a^7*c^7*x^{(14*n)}/n + a*b^{13}*x^{(13*n)} \\
& /n + 78*a^2*b^{11}*c*x^{(13*n)}/n + 1430*a^3*b^9*c^2*x^{(13*n)}/n + 8580*a^4*b^7* \\
& c^3*x^{(13*n)}/n + 18018*a^5*b^5*c^4*x^{(13*n)}/n + 12012*a^6*b^3*c^5*x^{(13*n)}/ \\
& n + 1716*a^7*b*c^6*x^{(13*n)}/n + 13/2*a^2*b^{12}*x^{(12*n)}/n + 286*a^3*b^{10}*c*x \\
& ^{(12*n)}/n + 6435/2*a^4*b^8*c^2*x^{(12*n)}/n + 12012*a^5*b^6*c^3*x^{(12*n)}/n + \\
& 15015*a^6*b^4*c^4*x^{(12*n)}/n + 5148*a^7*b^2*c^5*x^{(12*n)}/n + 429/2*a^8*c^6* \\
& x^{(12*n)}/n + 26*a^3*b^{11}*x^{(11*n)}/n + 715*a^4*b^9*c*x^{(11*n)}/n + 5148*a^5*b \\
& ^7*c^2*x^{(11*n)}/n + 12012*a^6*b^5*c^3*x^{(11*n)}/n + 8580*a^7*b^3*c^4*x^{(11*n)} \\
&)/n + 1287*a^8*b*c^5*x^{(11*n)}/n + 143/2*a^4*b^{10}*x^{(10*n)}/n + 1287*a^5*b^8* \\
& c*x^{(10*n)}/n + 6006*a^6*b^6*c^2*x^{(10*n)}/n + 8580*a^7*b^4*c^3*x^{(10*n)}/n + \\
& 6435/2*a^8*b^2*c^4*x^{(10*n)}/n + 143*a^9*c^5*x^{(10*n)}/n + 143*a^5*b^9*x^{(9*n)} \\
&)/n + 1716*a^6*b^7*c*x^{(9*n)}/n + 5148*a^7*b^5*c^2*x^{(9*n)}/n + 4290*a^8*b^3* \\
& c^3*x^{(9*n)}/n + 715*a^9*b*c^4*x^{(9*n)}/n + 429/2*a^6*b^8*x^{(8*n)}/n + 1716*a^ \\
& 7*b^6*c*x^{(8*n)}/n + 6435/2*a^8*b^4*c^2*x^{(8*n)}/n + 1430*a^9*b^2*c^3*x^{(8*n)} \\
& /n + 143/2*a^{10}*c^4*x^{(8*n)}/n + 1716/7*a^7*b^7*x^{(7*n)}/n + 1287*a^8*b^5*c*x \\
& ^{(7*n)}/n + 1430*a^9*b^3*c^2*x^{(7*n)}/n + 286*a^{10}*b*c^3*x^{(7*n)}/n + 429/2*a^ \\
& 8*b^6*x^{(6*n)}/n + 715*a^9*b^4*c*x^{(6*n)}/n + 429*a^{10}*b^2*c^2*x^{(6*n)}/n + 26 \\
& *a^{11}*c^3*x^{(6*n)}/n + 143*a^9*b^5*x^{(5*n)}/n + 286*a^{10}*b^3*c*x^{(5*n)}/n + 78 \\
& *a^{11}*b*c^2*x^{(5*n)}/n + 143/2*a^{10}*b^4*x^{(4*n)}/n + 78*a^{11}*b^2*c*x^{(4*n)}/n \\
& + 13/2*a^{12}*c^2*x^{(4*n)}/n + 26*a^{11}*b^3*x^{(3*n)}/n + 13*a^{12}*b*c*x^{(3*n)}/n + \\
& 13/2*a^{12}*b^2*x^{(2*n)}/n + a^{13}*c*x^{(2*n)}/n + a^{13}*b*x^n/n
\end{aligned}$$

mupad [B] time = 5.78, size = 1395, normalized size = 60.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(n-1)}*(b+2*c*x^n)*(a+b*x^n+c*x^{(2*n)})^{13},x)$

[Out] $x^{(n-1)}*((x^{(11*n+1)}*((13*a^2*b^{12})/2+(429*a^8*c^6)/2+286*a^3*b^{10}*c+(6435*a^4*b^8*c^2)/2+12012*a^5*b^6*c^3+15015*a^6*b^4*c^4+5148*a^7*b^2*c^5))/n+(x^{(15*n+1)}*((429*a^6*c^8)/2+(13*b^{12}*c^2)/2+286*a*b^{10}*c^3+(6435*a^2*b^8*c^4)/2+12012*a^3*b^6*c^5+15015*a^4*b^4*c^6+5148*a^5*b^2*c^7))/n+(x^{(12*n+1)}*(a*b^{13}+78*a^2*b^{11}*c+1716*a^7*b*c^6+1430*a^3*b^9*c^2+8580*a^4*b^7*c^3+18018*a^5*b^5*c^4+12012*a^6*b^3*c^5))/n+(x^{(14*n+1)}*(b^{13}*c+78*a*b^{11}*c^2+1716*a^6*b*c^7+1430*a^2*b^9*c^3+8580*a^3*b^7*c^4+18018*a^4*b^5*c^5+12012*a^5*b^3*c^6))/n+(x^{(5*n+1)}*((429*a^8*b^6)/2+26*a^{11}*c^3+715*a^9*b^4*c+429*a^{10}*b^2*c^2))/n+(x^{(21*n+1)}*(26*a^3*c^{11}+(429*b^6*c^8)/2+715*a*b^4*c^9+429*a^2*b^2*c^{10}))/n+(x^{(9*n+1)}*((143*a^4*b^{10})/2+143*a^9*c^5+1287*a^5*b^8*c+6006*a^6*b^6*c^2+8580*a^7*b^4*c^3+(6435*a^8*b^2*c^4)/2))/n+(x^{(17*n+1)}*(143*a^5*c^9+(143*b^{10}*c^4)/2+1287*a*b^8*c^5+6006*a^2*b^6*c^6+8580*a^3*b^4*c^7+(6435*a^4*b^2*c^8)/2))/n+(x^{(13*n+1)}*(b^{14}/14+(1716*a^7*c^7)/7+429*a^2*b^{10}*c^2+4290*a^3*b^8*c^3+15015*a^4*b^6*c^4+18018*a^5*b^4*c^5+6006*a^6*b^2*c^6+13*a*b^{12}*c))/n+(x^{(7*n+1)}*($

$$\begin{aligned}
& (429*a^6*b^8)/2 + (143*a^{10}*c^4)/2 + 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2 \\
& + 1430*a^9*b^2*c^3)/n + (x^{(19*n + 1)}*((143*a^4*c^{10})/2 + (429*b^8*c^6)/2 \\
& + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9))/n + (c^{14}*x^{(2 \\
& 7*n + 1)})/(14*n) + (a^{12}*x^{(n + 1)}*(a*c + (13*b^2)/2))/n + (a^{10}*x^{(3*n + 1 \\
&)}*((143*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (c^{10}*x^{(23*n + 1)}*((143 \\
& *b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (b*c^{13}*x^{(26*n + 1)})/n + (c^{12 \\
& }*x^{(25*n + 1)}*(a*c + (13*b^2)/2))/n + (a^{13}*b*x)/n + (143*a^7*b*x^{(6*n + 1 \\
&)}*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (143*b*c^7*x^{(\\
& 20*n + 1)}*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (14 \\
& 3*a^5*b*x^{(8*n + 1)}*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12 \\
& *a*b^6*c))/n + (143*b*c^5*x^{(18*n + 1)}*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + \\
& 30*a^3*b^2*c^3 + 12*a*b^6*c))/n + (13*a^3*b*x^{(10*n + 1)}*(2*b^{10} + 99*a^5*c \\
& ^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + \\
& (13*b*c^3*x^{(16*n + 1)}*(2*b^{10} + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^ \\
& 4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + (13*a^9*b*x^{(4*n + 1)}*(11*b^4 + \\
& 6*a^2*c^2 + 22*a*b^2*c))/n + (13*b*c^9*x^{(22*n + 1)}*(11*b^4 + 6*a^2*c^2 + 2 \\
& 2*a*b^2*c))/n + (13*a^{11}*b*x^{(2*n + 1)}*(a*c + 2*b^2))/n + (13*b*c^{11}*x^{(24* \\
& n + 1)}*(a*c + 2*b^2))/n
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)

[Out] Timed out

$$3.97 \quad \int (b + 2cx) (-a + bx + cx^2)^{13} dx$$

Optimal. Leaf size=18

$$\frac{1}{14} (a - bx - cx^2)^{14}$$

[Out] 1/14*(-c*x^2-b*x+a)^14

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {629}

$$\frac{1}{14} (a - bx - cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]

[Out] (a - b*x - c*x^2)^14/14

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (-a + bx + cx^2)^{13} dx = \frac{1}{14} (a - bx - cx^2)^{14}$$

Mathematica [B] time = 0.18, size = 201, normalized size = 11.17

$$\frac{1}{14} x(b+cx) (-14a^{13} + 91a^{12}x(b+cx) - 364a^{11}x^2(b+cx)^2 + 1001a^{10}x^3(b+cx)^3 - 2002a^9x^4(b+cx)^4 + 3003a^8x^5(b+cx)^5 - 3432a^7x^6(b+cx)^6 + 3003a^6x^7(b+cx)^7 - 2002a^5x^8(b+cx)^8 + 1001a^4x^9(b+cx)^9 - 14a^3x^{10}(b+cx)^{10} + 14a^2x^{11}(b+cx)^{11} - 14ax^{12}(b+cx)^{12} + x^{13}(b+cx)^{13})$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]

[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 14*a^3*x^10*(b + c*x)^10 + 14*a^2*x^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13)

$$+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^{10}*(b + c*x)^{10} + 91*a^2*x^{11}*(b + c*x)^{11} - 14*a*x^{12}*(b + c*x)^{12} + x^{13}*(b + c*x)^{13})/14$$

fricas [B] time = 0.50, size = 1450, normalized size = 80.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")

[Out] $1/14*x^{28}*c^{14} + x^{27}*c^{13}*b + 13/2*x^{26}*c^{12}*b^2 - x^{26}*c^{13}*a + 26*x^{25}*c^{11}*b^3 - 13*x^{25}*c^{12}*b*a + 143/2*x^{24}*c^{10}*b^4 - 78*x^{24}*c^{11}*b^2*a + 13/2*x^{24}*c^{12}*a^2 + 143*x^{23}*c^9*b^5 - 286*x^{23}*c^{10}*b^3*a + 78*x^{23}*c^{11}*b*a^2 + 429/2*x^{22}*c^8*b^6 - 715*x^{22}*c^9*b^4*a + 429*x^{22}*c^{10}*b^2*a^2 - 26*x^{22}*c^{11}*a^3 + 1716/7*x^{21}*c^7*b^7 - 1287*x^{21}*c^8*b^5*a + 1430*x^{21}*c^9*b^3*a^2 - 286*x^{21}*c^{10}*b*a^3 + 429/2*x^{20}*c^6*b^8 - 1716*x^{20}*c^7*b^6*a + 6435/2*x^{20}*c^8*b^4*a^2 - 1430*x^{20}*c^9*b^2*a^3 + 143/2*x^{20}*c^{10}*a^4 + 143*x^{19}*c^5*b^9 - 1716*x^{19}*c^6*b^7*a + 5148*x^{19}*c^7*b^5*a^2 - 4290*x^{19}*c^8*b^3*a^3 + 715*x^{19}*c^9*b*a^4 + 143/2*x^{18}*c^4*b^{10} - 1287*x^{18}*c^5*b^8*a + 6006*x^{18}*c^6*b^6*a^2 - 8580*x^{18}*c^7*b^4*a^3 + 6435/2*x^{18}*c^8*b^2*a^4 - 143*x^{18}*c^9*a^5 + 26*x^{17}*c^3*b^{11} - 715*x^{17}*c^4*b^9*a + 5148*x^{17}*c^5*b^7*a^2 - 12012*x^{17}*c^6*b^5*a^3 + 8580*x^{17}*c^7*b^3*a^4 - 1287*x^{17}*c^8*b*a^5 + 13/2*x^{16}*c^2*b^{12} - 286*x^{16}*c^3*b^{10}*a + 6435/2*x^{16}*c^4*b^8*a^2 - 12012*x^{16}*c^5*b^6*a^3 + 15015*x^{16}*c^6*b^4*a^4 - 5148*x^{16}*c^7*b^2*a^5 + 429/2*x^{16}*c^8*a^6 + x^{15}*c*b^{13} - 78*x^{15}*c^2*b^{11}*a + 1430*x^{15}*c^3*b^9*a^2 - 8580*x^{15}*c^4*b^7*a^3 + 18018*x^{15}*c^5*b^5*a^4 - 12012*x^{15}*c^6*b^3*a^5 + 1716*x^{15}*c^7*b*a^6 + 1/14*x^{14}*b^{14} - 13*x^{14}*c*b^{12}*a + 429*x^{14}*c^2*b^{10}*a^2 - 4290*x^{14}*c^3*b^8*a^3 + 15015*x^{14}*c^4*b^6*a^4 - 18018*x^{14}*c^5*b^4*a^5 + 6006*x^{14}*c^6*b^2*a^6 - 1716/7*x^{14}*c^7*a^7 - x^{13}*b^{13}*a + 78*x^{13}*c*b^{11}*a^2 - 1430*x^{13}*c^2*b^9*a^3 + 8580*x^{13}*c^3*b^7*a^4 - 18018*x^{13}*c^4*b^5*a^5 + 12012*x^{13}*c^5*b^3*a^6 - 1716*x^{13}*c^6*b*a^7 + 13/2*x^{12}*b^{12}*a^2 - 286*x^{12}*c*b^{10}*a^3 + 6435/2*x^{12}*c^2*b^8*a^4 - 12012*x^{12}*c^3*b^6*a^5 + 15015*x^{12}*c^4*b^4*a^6 - 5148*x^{12}*c^5*b^2*a^7 + 429/2*x^{12}*c^6*a^8 - 26*x^{11}*b^{11}*a^3 + 715*x^{11}*c*b^9*a^4 - 5148*x^{11}*c^2*b^7*a^5 + 12012*x^{11}*c^3*b^5*a^6 - 8580*x^{11}*c^4*b^3*a^7 + 1287*x^{11}*c^5*b*a^8 + 143/2*x^{10}*b^{10}*a^4 - 1287*x^{10}*c*b^8*a^5 + 6006*x^{10}*c^2*b^6*a^6 - 8580*x^{10}*c^3*b^4*a^7 + 6435/2*x^{10}*c^4*b^2*a^8 - 143*x^{10}*c^5*a^9 - 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*a^6 - 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 - 715*x^9*c^4*b*a^9 + 429/2*x^8*b^8*a^6 - 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 - 1430*x^8*c^3*b^2*a^9 + 143/2*x^8*c^4*a^{10} - 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 - 1430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^{10} + 429/2*x^6*b^6*a^8 - 715*x^6*c*b^4*a^9 + 429*x^6*c^2*b^2*a^{10} - 26*x^6*c^3*a^{11} - 143*x^5*b^5*a^9 + 286*x^5*c*b^3*a^{10} - 78*x^5*c^2*b*a^{11} + 143/2*x^4*b^4*a^{10} - 78*x^4*c*b^2*a^{11} + 13/2*x^4*c^2*a^{12} - 26*x^3*b^3*a^{11} + 13*x^3*c*b*a^{12} + 13/2*x^2*b^2*a^{12} - x^2*c*a^{13} - x*b*a^{13}$

giac [B] time = 0.47, size = 218, normalized size = 12.11

$$\frac{1}{14} (cx^2 + bx)^{14} - (cx^2 + bx)^{13} a + \frac{13}{2} (cx^2 + bx)^{12} a^2 - 26 (cx^2 + bx)^{11} a^3 + \frac{143}{2} (cx^2 + bx)^{10} a^4 - 143 (cx^2 + bx)^9 a^5 + \frac{429}{2} (cx^2 + bx)^8 a^6 - 1716/7 (cx^2 + bx)^7 a^7 + 429/2 (cx^2 + bx)^6 a^8 - 143 (cx^2 + bx)^5 a^9 + 143/2 (cx^2 + bx)^4 a^{10} - 26 (cx^2 + bx)^3 a^{11} + 13/2 (cx^2 + bx)^2 a^{12} - (cx^2 + bx) a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 26*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*a^5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 + b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26*(c*x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13

maple [B] time = 0.00, size = 47685, normalized size = 2649.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x)

[Out] result too large to display

maxima [A] time = 0.44, size = 16, normalized size = 0.89

$$\frac{1}{14} (cx^2 + bx - a)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")

[Out] 1/14*(c*x^2 + b*x - a)^14

mupad [B] time = 1.38, size = 1208, normalized size = 67.11

$$x^{12} \left(\frac{429 a^8 c^6}{2} - 5148 a^7 b^2 c^5 + 15015 a^6 b^4 c^4 - 12012 a^5 b^6 c^3 + \frac{6435 a^4 b^8 c^2}{2} - 286 a^3 b^{10} c + \frac{13 a^2 b^{12}}{2} \right) + x^{16} \left(\frac{429 a^8 c^6}{2} - 5148 a^7 b^2 c^5 + 15015 a^6 b^4 c^4 - 12012 a^5 b^6 c^3 + \frac{6435 a^4 b^8 c^2}{2} - 286 a^3 b^{10} c + \frac{13 a^2 b^{12}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)

[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*((429*a^8*c^6)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5)

$$\begin{aligned}
& 2012a^3b^6c^5 + 15015a^4b^4c^6 - 5148a^5b^2c^7) - x^{13}(ab^{13} - 7 \\
& 8a^2b^{11}c + 1716a^7b^6c^6 + 1430a^3b^9c^2 - 8580a^4b^7c^3 + 18018 \\
& a^5b^5c^4 - 12012a^6b^3c^5) + x^{15}(b^{13}c - 78a^2b^{11}c^2 + 1716a^6 \\
& b^6c^7 + 1430a^2b^9c^3 - 8580a^3b^7c^4 + 18018a^4b^5c^5 - 12012a^5 \\
& b^3c^6) + x^6((429a^8b^6)/2 - 26a^{11}c^3 - 715a^9b^4c + 429a^{10} \\
& b^2c^2) - x^{22}(26a^3c^{11} - (429b^6c^8)/2 + 715a^2b^4c^9 - 429a^2b^ \\
& 2c^{10}) + x^{10}((143a^4b^{10})/2 - 143a^9c^5 - 1287a^5b^8c + 6006a^6 \\
& b^6c^2 - 8580a^7b^4c^3 + (6435a^8b^2c^4)/2) - x^{18}(143a^5c^9 - (1 \\
& 43b^{10}c^4)/2 + 1287a^2b^8c^5 - 6006a^2b^6c^6 + 8580a^3b^4c^7 - (64 \\
& 35a^4b^2c^8)/2) + x^{14}(b^{14}/14 - (1716a^7c^7)/7 + 429a^2b^{10}c^2 - \\
& 4290a^3b^8c^3 + 15015a^4b^6c^4 - 18018a^5b^4c^5 + 6006a^6b^2c^6 \\
& - 13ab^{12}c) + x^8((429a^6b^8)/2 + (143a^{10}c^4)/2 - 1716a^7b^6c \\
& + (6435a^8b^4c^2)/2 - 1430a^9b^2c^3) + x^{20}((143a^4c^{10})/2 + (429 \\
& b^8c^6)/2 - 1716a^2b^6c^7 + (6435a^2b^4c^8)/2 - 1430a^3b^2c^9) + (c \\
& ^{14}x^{28})/14 - x^2(a^{13}c - (13a^{12}b^2)/2) + (13a^{10}x^4(11b^4 + a^2c^2 \\
& - 12ab^2c))/2 + (13c^{10}x^{24}(11b^4 + a^2c^2 - 12ab^2c))/2 + b \\
& c^{13}x^{27} - (c^{12}x^{26}(2ac - 13b^2))/2 - a^{13}bx - (143a^7b^7x^{12} \\
& b^6 - 14a^3c^3 + 70a^2b^2c^2 - 63ab^4c))/7 + (143b^7c^7x^{21}(12b \\
& ^6 - 14a^3c^3 + 70a^2b^2c^2 - 63ab^4c))/7 - 143a^5b^7x^9(b^8 + 5 \\
& a^4c^4 + 36a^2b^4c^2 - 30a^3b^2c^3 - 12ab^6c) + 143b^7c^5x^{19}(b \\
& ^8 + 5a^4c^4 + 36a^2b^4c^2 - 30a^3b^2c^3 - 12ab^6c) - 13a^3b^7x \\
& ^{11}(2b^{10} - 99a^5c^5 + 396a^2b^6c^2 - 924a^3b^4c^3 + 660a^4b^2c^4 \\
& - 55ab^8c) + 13b^7c^3x^{17}(2b^{10} - 99a^5c^5 + 396a^2b^6c^2 - \\
& 924a^3b^4c^3 + 660a^4b^2c^4 - 55ab^8c) - 13a^9b^7x^5(11b^4 + 6 \\
& a^2c^2 - 22ab^2c) + 13b^7c^9x^{23}(11b^4 + 6a^2c^2 - 22ab^2c) + 1 \\
& 3a^{11}b^7x^3(ac - 2b^2) - 13b^7c^{11}x^{25}(ac - 2b^2)
\end{aligned}$$

sympy [B] time = 0.36, size = 1326, normalized size = 73.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)

[Out] $-a^{13}bx + b^{13}x^{27} + c^{14}x^{28}/14 + x^{26}(-a^{13} + 13b^2c^{12}/2) + x^{25}(-13ab^2c^{12} + 26b^3c^{11}) + x^{24}(13a^2c^{12}/2 - 78a^2b^2c^{11} + 143b^4c^{10}/2) + x^{23}(78a^2b^2c^{11} - 286a^2b^3c^{10} + 143b^5c^9) + x^{22}(-26a^3c^{11} + 429a^2b^2c^{10} - 715a^2b^4c^9 + 429b^6c^8/2) + x^{21}(-286a^3b^2c^{10} + 1430a^2b^3c^9 - 1287a^2b^5c^8 + 1716b^7c^7/7) + x^{20}(143a^4c^{10}/2 - 1430a^3b^2c^9 + 6435a^2b^4c^8/2 - 1716a^2b^6c^7 + 429b^8c^6/2) + x^{19}(715a^4b^2c^9 - 4290a^3b^3c^8 + 5148a^2b^5c^7 - 1716a^2b^7c^6 + 143b^9c^5) + x^{18}(-143a^5c^9 + 6435a^4b^2c^8/2 - 8580a^3b^4c^7 + 6006a^2b^6c^6 - 1287a^2b^8c^5 + 143b^{10}c^4/2) + x^{17}(-1287a^5b^2c^8 + 8580a^4b^3c^7 - 12012a^3b^5c^6$

$$\begin{aligned}
& **5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(\\
& 429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b \\
& **6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x \\
& *15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580* \\
& a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(\\
& -1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4 \\
& *b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b** \\
& 14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5* \\
& c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1 \\
& 3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 - \\
& 12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b \\
& **12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c \\
& **3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143* \\
& a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c \\
& *2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a \\
& **8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x \\
& **8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716* \\
& a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c \\
& *2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1 \\
& 0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 + \\
& 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2* \\
& c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13* \\
& c + 13*a**12*b**2/2)
\end{aligned}$$

$$3.98 \quad \int x (b + 2cx^2) (-a + bx^2 + cx^4)^{13} dx$$

Optimal. Leaf size=20

$$\frac{1}{28} (a - bx^2 - cx^4)^{14}$$

[Out] 1/28*(-c*x^4-b*x^2+a)^14

Rubi [A] time = 0.32, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1247, 629}

$$\frac{1}{28} (a - bx^2 - cx^4)^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]

[Out] (a - b*x^2 - c*x^4)^14/28

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (b + 2cx^2) (-a + bx^2 + cx^4)^{13} dx &= \frac{1}{2} \text{Subst} \left(\int (b + 2cx) (-a + bx + cx^2)^{13} dx, x, x^2 \right) \\ &= \frac{1}{28} (a - bx^2 - cx^4)^{14} \end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{28} x^2 (b + cx^2) \left(-14a^{13} + 91a^{12}x^2 (b + cx^2) - 364a^{11}x^4 (b + cx^2)^2 + 1001a^{10}x^6 (b + cx^2)^3 - 2002a^9x^8 (b + cx^2)^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]

[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^12 + x^26*(b + c*x^2)^13)/28

fricas [B] time = 0.50, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")

[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 - 1/2*x^52*c^13*a + 13*x^50*c^11*b^3 - 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 - 39*x^48*c^11*b^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 - 143*x^46*c^10*b^3*a + 39*x^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 - 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^10*b^2*a^2 - 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 - 1287/2*x^42*c^8*b^5*a + 715*x^42*c^9*b^3*a^2 - 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 - 858*x^40*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 - 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^10*a^4 + 143/2*x^38*c^5*b^9 - 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 - 2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 - 1287/2*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 - 4290*x^36*c^7*b^4*a^3 + 6435/4*x^36*c^8*b^2*a^4 - 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 - 715/2*x^34*c^4*b^9*a + 2574*x^34*c^5*b^7*a^2 - 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4 - 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 - 143*x^32*c^3*b^10*a + 6435/4*x^32*c^4*b^8*a^2 - 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 - 2574*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 - 39*x^30*c^2*b^11*a + 715*x^30*c^3*b^9*a^2 - 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 - 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 - 13/2*x^28*c*b^12*a + 429/2*x^28*c^2*b^10*a^2 - 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4*b^6*a^4 - 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 - 858/7*x^28*c^7*a^7 - 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 - 715*x^26*c^2*b^9*a^3 + 4290*x^26*c^3*b^7*a^4 - 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 - 858*x^26*c^6*b*a^7 + 13/4*x^24*b^12*a^2 - 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^4 - 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 - 2574*x^24*c^5*b^2*a^7 + 429/4*x^24*c^6*a^8 - 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 - 2574*x^22*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 - 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22*c^5*b*a^8 + 143/4*x^20*b^10*a^4 - 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^6*a^6 - 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 - 143/2*x^20*c^5*a^8

$$9 - 143/2*x^{18}*b^9*a^5 + 858*x^{18}*c*b^7*a^6 - 2574*x^{18}*c^2*b^5*a^7 + 2145*x^{18}*c^3*b^3*a^8 - 715/2*x^{18}*c^4*b*a^9 + 429/4*x^{16}*b^8*a^6 - 858*x^{16}*c*b^6*a^7 + 6435/4*x^{16}*c^2*b^4*a^8 - 715*x^{16}*c^3*b^2*a^9 + 143/4*x^{16}*c^4*a^{10} - 858/7*x^{14}*b^7*a^7 + 1287/2*x^{14}*c*b^5*a^8 - 715*x^{14}*c^2*b^3*a^9 + 143*x^{14}*c^3*b*a^{10} + 429/4*x^{12}*b^6*a^8 - 715/2*x^{12}*c*b^4*a^9 + 429/2*x^{12}*c^2*b^2*a^{10} - 13*x^{12}*c^3*a^{11} - 143/2*x^{10}*b^5*a^9 + 143*x^{10}*c*b^3*a^{10} - 39*x^{10}*c^2*b*a^{11} + 143/4*x^8*b^4*a^{10} - 39*x^8*c*b^2*a^{11} + 13/4*x^8*c^2*a^{12} - 13*x^6*b^3*a^{11} + 13/2*x^6*c*b*a^{12} + 13/4*x^4*b^2*a^{12} - 1/2*x^4*c*a^{13} - 1/2*x^2*b*a^{13}$$

giac [B] time = 0.57, size = 246, normalized size = 12.30

$$\frac{1}{28} (cx^4 + bx^2)^{14} - \frac{1}{2} (cx^4 + bx^2)^{13} a + \frac{13}{4} (cx^4 + bx^2)^{12} a^2 - 13 (cx^4 + bx^2)^{11} a^3 + \frac{143}{4} (cx^4 + bx^2)^{10} a^4 - \frac{143}{2} (cx^4 + bx^2)^9 a^5 + \frac{429}{4} (cx^4 + bx^2)^8 a^6 - \frac{858}{7} (cx^4 + bx^2)^7 a^7 + \frac{429}{4} (cx^4 + bx^2)^6 a^8 - \frac{143}{2} (cx^4 + bx^2)^5 a^9 + \frac{143}{4} (cx^4 + bx^2)^4 a^{10} - 13 (cx^4 + bx^2)^3 a^{11} + \frac{13}{4} (cx^4 + bx^2)^2 a^{12} - \frac{1}{2} (cx^4 + bx^2) a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")

[Out] 1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^12*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*(c*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2)^7*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a^12 - 1/2*(c*x^4 + b*x^2)*a^13

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x)

[Out] result too large to display

maxima [B] time = 0.52, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")

[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^11)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*b^6*c^8 - 42*a*b^4*c^9 + 13*a^2*b^2*c^10)*x^44 + 13/2*(11*b^7*c^7 - 22*a*b^5*c^8 + 6*a^2*b^3*c^9)*x^42 + 13/4*(33*b^8*c^6 - 42*a*b^6*c^7 + 13*a^2*b^4*c^8)*x^40 + 13/2*(11*b^9*c^5 - 22*a*b^7*c^6 + 6*a^2*b^5*c^7)*x^38 + 13/4*(33*b^10*c^4 - 42*a*b^8*c^5 + 13*a^2*b^6*c^6)*x^36 + 13/2*(11*b^11*c^3 - 22*a*b^9*c^4 + 6*a^2*b^7*c^5)*x^34 + 13/4*(33*b^12*c^2 - 42*a*b^10*c^3 + 13*a^2*b^8*c^4)*x^32 + 13/2*(11*b^13*c - 22*a*b^11*c^2 + 6*a^2*b^9*c^3)*x^30 + 13/4*(33*b^14*c - 42*a*b^12*c^2 + 13*a^2*b^10*c^3)*x^28 + 13/2*(11*b^15*c - 22*a*b^13*c^2 + 6*a^2*b^11*c^3)*x^26 + 13/4*(33*b^16*c - 42*a*b^14*c^2 + 13*a^2*b^12*c^3)*x^24 + 13/2*(11*b^17*c - 22*a*b^15*c^2 + 6*a^2*b^13*c^3)*x^22 + 13/4*(33*b^18*c - 42*a*b^16*c^2 + 13*a^2*b^14*c^3)*x^20 + 13/2*(11*b^19*c - 22*a*b^17*c^2 + 6*a^2*b^15*c^3)*x^18 + 13/4*(33*b^20*c - 42*a*b^18*c^2 + 13*a^2*b^16*c^3)*x^16 + 13/2*(11*b^21*c - 22*a*b^19*c^2 + 6*a^2*b^17*c^3)*x^14 + 13/4*(33*b^22*c - 42*a*b^20*c^2 + 13*a^2*b^18*c^3)*x^12 + 13/2*(11*b^23*c - 22*a*b^21*c^2 + 6*a^2*b^19*c^3)*x^10 + 13/4*(33*b^24*c - 42*a*b^22*c^2 + 13*a^2*b^20*c^3)*x^8 + 13/2*(11*b^25*c - 22*a*b^23*c^2 + 6*a^2*b^21*c^3)*x^6 + 13/4*(33*b^26*c - 42*a*b^24*c^2 + 13*a^2*b^22*c^3)*x^4 + 13/2*(11*b^27*c - 22*a*b^25*c^2 + 6*a^2*b^23*c^3)*x^2 + 13/4*(33*b^28*c - 42*a*b^26*c^2 + 13*a^2*b^24*c^3)*x

$$\begin{aligned}
& b^6c^8 - 110ab^4c^9 + 66a^2b^2c^{10} - 4a^3c^{11})x^{44} + 143/14*(12b^7c^7 - 63ab^5c^8 + 70a^2b^3c^9 - 14a^3b^2c^{10})x^{42} + 143/4*(3b^8c^6 - 24ab^6c^7 + 45a^2b^4c^8 - 20a^3b^2c^9 + a^4c^{10})x^{40} + 143/2*(b^9c^5 - 12ab^7c^6 + 36a^2b^5c^7 - 30a^3b^3c^8 + 5a^4b^2c^9)x^{38} + 143/4*(b^{10}c^4 - 18ab^8c^5 + 84a^2b^6c^6 - 120a^3b^4c^7 + 45a^4b^2c^8 - 2a^5c^9)x^{36} + 13/2*(2b^{11}c^3 - 55ab^9c^4 + 396a^2b^7c^5 - 924a^3b^5c^6 + 660a^4b^3c^7 - 99a^5b^2c^8)x^{34} + 13/4*(b^{12}c^2 - 44ab^{10}c^3 + 495a^2b^8c^4 - 1848a^3b^6c^5 + 2310a^4b^4c^6 - 792a^5b^2c^7 + 33a^6c^8)x^{32} + 1/2*(b^{13}c - 78ab^{11}c^2 + 1430a^2b^9c^3 - 8580a^3b^7c^4 + 18018a^4b^5c^5 - 12012a^5b^3c^6 + 1716a^6b^2c^7)x^{30} + 1/28*(b^{14} - 182ab^{12}c + 6006a^2b^{10}c^2 - 60060a^3b^8c^3 + 210210a^4b^6c^4 - 252252a^5b^4c^5 + 84084a^6b^2c^6 - 3432a^7c^7)x^{28} - 1/2*(a^2b^{13} - 78a^2b^{11}c + 1430a^3b^9c^2 - 8580a^4b^7c^3 + 18018a^5b^5c^4 - 12012a^6b^3c^5 + 1716a^7b^2c^6)x^{26} + 13/4*(a^2b^{12} - 44a^3b^{10}c + 495a^4b^8c^2 - 1848a^5b^6c^3 + 2310a^6b^4c^4 - 792a^7b^2c^5 + 33a^8c^6)x^{24} - 13/2*(2a^3b^{11} - 55a^4b^9c + 396a^5b^7c^2 - 924a^6b^5c^3 + 660a^7b^3c^4 - 99a^8b^2c^5)x^{22} + 143/4*(a^4b^{10} - 18a^5b^8c + 84a^6b^6c^2 - 120a^7b^4c^3 + 45a^8b^2c^4 - 2a^9c^5)x^{20} - 143/2*(a^5b^9 - 12a^6b^7c + 36a^7b^5c^2 - 30a^8b^3c^3 + 5a^9b^2c^4)x^{18} + 143/4*(3a^6b^8 - 24a^7b^6c + 45a^8b^4c^2 - 20a^9b^2c^3 + a^{10}c^4)x^{16} - 1/2*a^{13}b*x^2 - 143/14*(12a^7b^7 - 63a^8b^5c + 70a^9b^3c^2 - 14a^{10}b^2c^3)x^{14} + 13/4*(33a^8b^6 - 110a^9b^4c + 66a^{10}b^2c^2 - 4a^{11}c^3)x^{12} - 13/2*(11a^9b^5 - 22a^{10}b^3c + 6a^{11}b^2c^2)x^{10} + 13/4*(11a^{10}b^4 - 12a^{11}b^2c + a^{12}c^2)x^8 - 13/2*(2a^{11}b^3 - a^{12}b^2c)x^6 + 1/4*(13a^{12}b^2 - 2a^{13}c)x^4
\end{aligned}$$

mupad [B] time = 3.25, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^{13}, x)$

[Out] $x^{24}*((13a^2b^{12})/4 + (429a^8c^6)/4 - 143a^3b^{10}c + (6435a^4b^8c^2)/4 - 6006a^5b^6c^3 + (15015a^6b^4c^4)/2 - 2574a^7b^2c^5) + x^{32}*((429a^6c^8)/4 + (13b^{12}c^2)/4 - 143ab^{10}c^3 + (6435a^2b^8c^4)/4 - 6006a^3b^6c^5 + (15015a^4b^4c^6)/2 - 2574a^5b^2c^7) - x^{26}*((ab^{13})/2 - 39a^2b^{11}c + 858a^7b^2c^6 + 715a^3b^9c^2 - 4290a^4b^7c^3 + 9009a^5b^5c^4 - 6006a^6b^3c^5) + x^{30}*((b^{13}c)/2 - 39ab^{11}c^2 + 858a^6b^2c^7 + 715a^2b^9c^3 - 4290a^3b^7c^4 + 9009a^4b^5c^5 - 6006a^5b^3c^6) + x^{12}*((429a^8b^6)/4 - 13a^{11}c^3 - (715a^9b^4c)/2 + (429a^{10}b^2c^2)/2) - x^{44}*(13a^3c^{11} - (429b^6c^8)/4 + (715a^2b^4c^9)/2 - (429a^2b^2c^{10})/2) + x^{20}*((143a^4b^{10})/4 - (143a^9c^5)/2 - (1287a^5b^8c)/2 + 3003a^6b^6c^2 - 4290a^7b^4c^3 + (6435a^8b^2c^4)/4 - 6006a^9b^2c^5) + x^{24}*((13a^2b^{12})/4 + (429a^8c^6)/4 - 143a^3b^{10}c + (6435a^4b^8c^2)/4 - 6006a^5b^6c^3 + (15015a^6b^4c^4)/2 - 2574a^7b^2c^5) + x^{32}*((429a^6c^8)/4 + (13b^{12}c^2)/4 - 143ab^{10}c^3 + (6435a^2b^8c^4)/4 - 6006a^3b^6c^5 + (15015a^4b^4c^6)/2 - 2574a^5b^2c^7) - x^{26}*((ab^{13})/2 - 39a^2b^{11}c + 858a^7b^2c^6 + 715a^3b^9c^2 - 4290a^4b^7c^3 + 9009a^5b^5c^4 - 6006a^6b^3c^5) + x^{30}*((b^{13}c)/2 - 39ab^{11}c^2 + 858a^6b^2c^7 + 715a^2b^9c^3 - 4290a^3b^7c^4 + 9009a^4b^5c^5 - 6006a^5b^3c^6) + x^{12}*((429a^8b^6)/4 - 13a^{11}c^3 - (715a^9b^4c)/2 + (429a^{10}b^2c^2)/2) - x^{44}*(13a^3c^{11} - (429b^6c^8)/4 + (715a^2b^4c^9)/2 - (429a^2b^2c^{10})/2) + x^{20}*((143a^4b^{10})/4 - (143a^9c^5)/2 - (1287a^5b^8c)/2 + 3003a^6b^6c^2 - 4290a^7b^4c^3 + (6435a^8b^2c^4)/4 - 6006a^9b^2c^5)$

$$\begin{aligned} &^4)/4) - x^{36}*((143*a^5*c^9)/2 - (143*b^{10}*c^4)/4 + (1287*a*b^8*c^5)/2 - 30 \\ &03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^{28}*(b^{14}/28 - \\ &(858*a^7*c^7)/7 + (429*a^2*b^{10}*c^2)/2 - 2145*a^3*b^8*c^3 + (15015*a^4*b^6 \\ &*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^{12}*c)/2) + x^{16}*((4 \\ &29*a^6*b^8)/4 + (143*a^{10}*c^4)/4 - 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 - 7 \\ &15*a^9*b^2*c^3) + x^{40}*((143*a^4*c^{10})/4 + (429*b^8*c^6)/4 - 858*a*b^6*c^7 \\ &+ (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^{14}*x^{56})/28 - x^4*((a^{13}*c)/ \\ &2 - (13*a^{12}*b^2)/4) + (13*a^{10}*x^8*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 + (1 \\ &3*c^{10}*x^{48}*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^{13}*b*x^2)/2 + (b*c^{13}*x \\ &^{54})/2 - (c^{12}*x^{52}*(2*a*c - 13*b^2))/4 - (143*a^7*b*x^{14}*(12*b^6 - 14*a^3*c \\ &^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 + (143*b*c^7*x^{42}*(12*b^6 - 14*a^3*c \\ &^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 - (143*a^5*b*x^{18}*(b^8 + 5*a^4*c^4 + \\ &36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 + (143*b*c^5*x^{38}*(b^8 + 5 \\ &*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 - (13*a^3*b*x^2 \\ &2*(2*b^{10} - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 \\ &- 55*a*b^8*c))/2 + (13*b*c^3*x^{34}*(2*b^{10} - 99*a^5*c^5 + 396*a^2*b^6*c^2 \\ &- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/2 - (13*a^9*b*x^{10}*(11*b \\ &^4 + 6*a^2*c^2 - 22*a*b^2*c))/2 + (13*b*c^9*x^{46}*(11*b^4 + 6*a^2*c^2 - 22*a \\ &*b^2*c))/2 + (13*a^{11}*b*x^6*(a*c - 2*b^2))/2 - (13*b*c^{11}*x^{50}*(a*c - 2*b^2 \\ &))/2 \end{aligned}$$

sympy [B] time = 0.36, size = 1384, normalized size = 69.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)

[Out] $-a^{13}b*x^{13}/2 + b*c^{13}x^{13}/2 + c^{14}x^{56}/28 + x^{52}*(-a*c^{13}/2 + 13$
 $*b^2*c^{12}/4) + x^{50}*(-13*a*b*c^{12}/2 + 13*b^3*c^{11}) + x^{48}*(13*a^2*c$
 $**12/4 - 39*a*b^2*c^{11} + 143*b^4*c^{10}/4) + x^{46}*(39*a^2*b*c^{11} - 143$
 $*a*b^3*c^{10} + 143*b^5*c^9/2) + x^{44}*(-13*a^3*c^{11} + 429*a^2*b^2*c$
 $*10/2 - 715*a*b^4*c^9/2 + 429*b^6*c^8/4) + x^{42}*(-143*a^3*b*c^{10} + 7$
 $15*a^2*b^3*c^9 - 1287*a*b^5*c^8/2 + 858*b^7*c^7/7) + x^{40}*(143*a^4$
 $*c^{10}/4 - 715*a^3*b^2*c^9 + 6435*a^2*b^4*c^8/4 - 858*a*b^6*c^7 + 4$
 $29*b^8*c^6/4) + x^{38}*(715*a^4*b*c^9/2 - 2145*a^3*b^3*c^8 + 2574*a^2$
 $*b^5*c^7 - 858*a*b^7*c^6 + 143*b^9*c^5/2) + x^{36}*(-143*a^5*c^9/2$
 $+ 6435*a^4*b^2*c^8/4 - 4290*a^3*b^4*c^7 + 3003*a^2*b^6*c^6 - 1287*$
 $a*b^8*c^5/2 + 143*b^{10}*c^4/4) + x^{34}*(-1287*a^5*b*c^8/2 + 4290*a^4$
 $b^3*c^7 - 6006*a^3*b^5*c^6 + 2574*a^2*b^7*c^5 - 715*a*b^9*c^4/2 +$
 $13*b^{11}*c^3) + x^{32}*(429*a^6*c^8/4 - 2574*a^5*b^2*c^7 + 15015*a^4$
 $*b^4*c^6/2 - 6006*a^3*b^6*c^5 + 6435*a^2*b^8*c^4/4 - 143*a*b^{10}*c$
 $*3 + 13*b^{12}*c^2/4) + x^{30}*(858*a^6*b*c^7 - 6006*a^5*b^3*c^6 + 9009$
 $*a^4*b^5*c^5 - 4290*a^3*b^7*c^4 + 715*a^2*b^9*c^3 - 39*a*b^{11}*c$
 $2 + b^{13}*c/2) + x^{28}*(-858*a^7*c^7/7 + 3003*a^6*b^2*c^6 - 9009*a^5*$

$$\begin{aligned}
& b^{**4}c^{**5} + 15015a^{**4}b^{**6}c^{**4}/2 - 2145a^{**3}b^{**8}c^{**3} + 429a^{**2}b^{**10}c^{**2}/2 - 13a^{**12}c/2 + b^{**14}/28) + x^{**26}*(-858a^{**7}b^{**6}c^{**6} + 6006a^{**6}b^{**3}c^{**5} - 9009a^{**5}b^{**5}c^{**4} + 4290a^{**4}b^{**7}c^{**3} - 715a^{**3}b^{**9}c^{**2} + 39a^{**2}b^{**11}c - a^{**13}/2) + x^{**24}*(429a^{**8}c^{**6}/4 - 2574a^{**7}b^{**2}c^{**5} + 15015a^{**6}b^{**4}c^{**4}/2 - 6006a^{**5}b^{**6}c^{**3} + 6435a^{**4}b^{**8}c^{**2}/4 - 143a^{**3}b^{**10}c + 13a^{**2}b^{**12}/4) + x^{**22}*(1287a^{**8}b^{**5}c^{**5}/2 - 4290a^{**7}b^{**3}c^{**4} + 6006a^{**6}b^{**5}c^{**3} - 2574a^{**5}b^{**7}c^{**2} + 715a^{**4}b^{**9}c/2 - 13a^{**3}b^{**11}) + x^{**20}*(-143a^{**9}c^{**5}/2 + 6435a^{**8}b^{**2}c^{**4}/4 - 4290a^{**7}b^{**4}c^{**3} + 3003a^{**6}b^{**6}c^{**2} - 1287a^{**5}b^{**8}c/2 + 143a^{**4}b^{**10}/4) + x^{**18}*(-715a^{**9}b^{**4}c^{**4}/2 + 2145a^{**8}b^{**3}c^{**3} - 2574a^{**7}b^{**5}c^{**2} + 858a^{**6}b^{**7}c - 143a^{**5}b^{**9}/2) + x^{**16}*(143a^{**10}c^{**4}/4 - 715a^{**9}b^{**2}c^{**3} + 6435a^{**8}b^{**4}c^{**2}/4 - 858a^{**7}b^{**6}c + 429a^{**6}b^{**8}/4) + x^{**14}*(143a^{**10}b^{**3}c^{**3} - 715a^{**9}b^{**3}c^{**2} + 1287a^{**8}b^{**5}c/2 - 858a^{**7}b^{**7}/7) + x^{**12}*(-13a^{**11}c^{**3} + 429a^{**10}b^{**2}c^{**2}/2 - 715a^{**9}b^{**4}c/2 + 429a^{**8}b^{**6}/4) + x^{**10}*(-39a^{**11}b^{**2}c^{**2} + 143a^{**10}b^{**3}c - 143a^{**9}b^{**5}/2) + x^{**8}*(13a^{**12}c^{**2}/4 - 39a^{**11}b^{**2}c + 143a^{**10}b^{**4}/4) + x^{**6}*(13a^{**12}b^{**2}c/2 - 13a^{**11}b^{**3}) + x^{**4}*(-a^{**13}c/2 + 13a^{**12}b^{**2}/4)
\end{aligned}$$

$$3.99 \quad \int x^2 (b + 2cx^3) (-a + bx^3 + cx^6)^{13} dx$$

Optimal. Leaf size=20

$$\frac{1}{42} (a - bx^3 - cx^6)^{14}$$

[Out] 1/42*(-c*x^6-b*x^3+a)^14

Rubi [A] time = 0.31, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 629}

$$\frac{1}{42} (a - bx^3 - cx^6)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]

[Out] (a - b*x^3 - c*x^6)^14/42

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (b + 2cx^3) (-a + bx^3 + cx^6)^{13} dx &= \frac{1}{3} \text{Subst} \left(\int (b + 2cx) (-a + bx + cx^2)^{13} dx, x, x^3 \right) \\ &= \frac{1}{42} (a - bx^3 - cx^6)^{14} \end{aligned}$$

Mathematica [B] time = 0.17, size = 233, normalized size = 11.65

$$\frac{1}{42} x^3 (b + cx^3) \left(-14a^{13} + 91a^{12}x^3 (b + cx^3) - 364a^{11}x^6 (b + cx^3)^2 + 1001a^{10}x^9 (b + cx^3)^3 - 2002a^9x^{12} (b + cx^3)^4 + 1001a^8x^{15} (b + cx^3)^5 - 364a^7x^{18} (b + cx^3)^6 + 91a^6x^{21} (b + cx^3)^7 - 14a^5x^{24} (b + cx^3)^8 + a^4x^{27} (b + cx^3)^9 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]

[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^12 + x^39*(b + c*x^3)^13)/42

fricas [B] time = 0.54, size = 1454, normalized size = 72.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")

[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 - 1/3*x^78*c^13*a + 26/3*x^75*c^11*b^3 - 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 - 26*x^72*c^11*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 - 286/3*x^69*c^10*b^3*a + 26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 - 715/3*x^66*c^9*b^4*a + 143*x^66*c^10*b^2*a^2 - 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 - 429*x^63*c^8*b^5*a + 1430/3*x^63*c^9*b^3*a^2 - 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 - 572*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 - 1430/3*x^60*c^9*b^2*a^3 + 143/6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 - 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b^5*a^2 - 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10 - 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 - 2860*x^54*c^7*b^4*a^3 + 2145/2*x^54*c^8*b^2*a^4 - 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 - 715/3*x^51*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 - 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*b^3*a^4 - 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 - 286/3*x^48*c^3*b^10*a + 2145/2*x^48*c^4*b^8*a^2 - 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 - 1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 - 26*x^45*c^2*b^11*a + 1430/3*x^45*c^3*b^9*a^2 - 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^5*a^4 - 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 - 13/3*x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 - 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c^4*b^6*a^4 - 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 - 572/7*x^42*c^7*a^7 - 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 - 1430/3*x^39*c^2*b^9*a^3 + 2860*x^39*c^3*b^7*a^4 - 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 - 572*x^39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 - 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2*b^8*a^4 - 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 - 1716*x^36*c^5*b^2*a^7 + 143/2*x^36*c^6*a^8 - 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 - 1716*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 - 2860*x^33*c^4*b^3*a^7 + 429*x^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 - 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^6*a^6 - 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 - 143/3*x^30*c^5*a^8

$$9 - 143/3*x^{27}*b^9*a^5 + 572*x^{27}*c*b^7*a^6 - 1716*x^{27}*c^2*b^5*a^7 + 1430*x^{27}*c^3*b^3*a^8 - 715/3*x^{27}*c^4*b*a^9 + 143/2*x^{24}*b^8*a^6 - 572*x^{24}*c*b^6*a^7 + 2145/2*x^{24}*c^2*b^4*a^8 - 1430/3*x^{24}*c^3*b^2*a^9 + 143/6*x^{24}*c^4*a^{10} - 572/7*x^{21}*b^7*a^7 + 429*x^{21}*c*b^5*a^8 - 1430/3*x^{21}*c^2*b^3*a^9 + 286/3*x^{21}*c^3*b*a^{10} + 143/2*x^{18}*b^6*a^8 - 715/3*x^{18}*c*b^4*a^9 + 143*x^{18}*c^2*b^2*a^{10} - 26/3*x^{18}*c^3*a^{11} - 143/3*x^{15}*b^5*a^9 + 286/3*x^{15}*c*b^3*a^{10} - 26*x^{15}*c^2*b*a^{11} + 143/6*x^{12}*b^4*a^{10} - 26*x^{12}*c*b^2*a^{11} + 13/6*x^{12}*c^2*a^{12} - 26/3*x^9*b^3*a^{11} + 13/3*x^9*c*b*a^{12} + 13/6*x^6*b^2*a^{11} 2 - 1/3*x^6*c*a^{13} - 1/3*x^3*b*a^{13}$$

giac [B] time = 0.68, size = 246, normalized size = 12.30

$$\frac{1}{42} (cx^6 + bx^3)^{14} - \frac{1}{3} (cx^6 + bx^3)^{13} a + \frac{13}{6} (cx^6 + bx^3)^{12} a^2 - \frac{26}{3} (cx^6 + bx^3)^{11} a^3 + \frac{143}{6} (cx^6 + bx^3)^{10} a^4 - \frac{143}{3} (cx^6 + bx^3)^9 a^5 + \frac{143}{2} (cx^6 + bx^3)^8 a^6 - 572/7 (cx^6 + bx^3)^7 a^7 + 143/2 (cx^6 + bx^3)^6 a^8 - 143/3 (cx^6 + bx^3)^5 a^9 + 143/6 (cx^6 + bx^3)^4 a^{10} - 26/3 (cx^6 + bx^3)^3 a^{11} + 13/6 (cx^6 + bx^3)^2 a^{12} - 1/3 (cx^6 + bx^3) a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")

[Out] 1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^12*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/3*(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x^3)^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13

maple [B] time = 0.00, size = 47688, normalized size = 2384.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x)

[Out] result too large to display

maxima [B] time = 0.50, size = 1242, normalized size = 62.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")

[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*b^6*c^8 - 42*a*b^4*c^9 + 13*a^2*b^2*c^10)*x^66 + 13/6*(33*b^7*c^7 - 42*a*b^5*c^8 + 13*a^2*b^3*c^9)*x^63 + 13/6*(33*b^8*c^6 - 42*a*b^6*c^7 + 13*a^2*b^4*c^8)*x^60 + 13/6*(33*b^9*c^5 - 42*a*b^7*c^6 + 13*a^2*b^5*c^7)*x^57 + 13/6*(33*b^10*c^4 - 42*a*b^8*c^5 + 13*a^2*b^6*c^6)*x^54 + 13/6*(33*b^11*c^3 - 42*a*b^9*c^4 + 13*a^2*b^7*c^5)*x^51 + 13/6*(33*b^12*c^2 - 42*a*b^10*c^3 + 13*a^2*b^8*c^4)*x^48 + 13/6*(33*b^13*c - 42*a*b^11*c^2 + 13*a^2*b^9*c^3)*x^45 + 13/6*(33*b^14*c - 42*a*b^12*c^2 + 13*a^2*b^10*c^3)*x^42 + 13/6*(33*b^15*c - 42*a*b^13*c^2 + 13*a^2*b^11*c^3)*x^39 + 13/6*(33*b^16*c - 42*a*b^14*c^2 + 13*a^2*b^12*c^3)*x^36 + 13/6*(33*b^17*c - 42*a*b^15*c^2 + 13*a^2*b^13*c^3)*x^33 + 13/6*(33*b^18*c - 42*a*b^16*c^2 + 13*a^2*b^14*c^3)*x^30 + 13/6*(33*b^19*c - 42*a*b^17*c^2 + 13*a^2*b^15*c^3)*x^27 + 13/6*(33*b^20*c - 42*a*b^18*c^2 + 13*a^2*b^16*c^3)*x^24 + 13/6*(33*b^21*c - 42*a*b^19*c^2 + 13*a^2*b^17*c^3)*x^21 + 13/6*(33*b^22*c - 42*a*b^20*c^2 + 13*a^2*b^18*c^3)*x^18 + 13/6*(33*b^23*c - 42*a*b^21*c^2 + 13*a^2*b^19*c^3)*x^15 + 13/6*(33*b^24*c - 42*a*b^22*c^2 + 13*a^2*b^20*c^3)*x^12 + 13/6*(33*b^25*c - 42*a*b^23*c^2 + 13*a^2*b^21*c^3)*x^9 + 13/6*(33*b^26*c - 42*a*b^24*c^2 + 13*a^2*b^22*c^3)*x^6 + 13/6*(33*b^27*c - 42*a*b^25*c^2 + 13*a^2*b^23*c^3)*x^3 + 13/6*(33*b^28*c - 42*a*b^26*c^2 + 13*a^2*b^24*c^3)*x

$$\begin{aligned}
& b^6c^8 - 110ab^4c^9 + 66a^2b^2c^{10} - 4a^3c^{11})x^{66} + 143/21*(12b^7c^7 - 63ab^5c^8 + 70a^2b^3c^9 - 14a^3b^2c^{10})x^{63} + 143/6*(3b^8c^6 - 24ab^6c^7 + 45a^2b^4c^8 - 20a^3b^2c^9 + a^4c^{10})x^{60} + 143/3*(b^9c^5 - 12ab^7c^6 + 36a^2b^5c^7 - 30a^3b^3c^8 + 5a^4b^2c^9)x^{57} + 143/6*(b^{10}c^4 - 18ab^8c^5 + 84a^2b^6c^6 - 120a^3b^4c^7 + 45a^4b^2c^8 - 2a^5c^9)x^{54} + 13/3*(2b^{11}c^3 - 55ab^9c^4 + 396a^2b^7c^5 - 924a^3b^5c^6 + 660a^4b^3c^7 - 99a^5b^2c^8)x^{51} + 13/6*(b^{12}c^2 - 44ab^{10}c^3 + 495a^2b^8c^4 - 1848a^3b^6c^5 + 2310a^4b^4c^6 - 792a^5b^2c^7 + 33a^6c^8)x^{48} + 1/3*(b^{13}c - 78ab^{11}c^2 + 1430a^2b^9c^3 - 8580a^3b^7c^4 + 18018a^4b^5c^5 - 12012a^5b^3c^6 + 1716a^6b^2c^7)x^{45} + 1/42*(b^{14} - 182ab^{12}c + 6006a^2b^{10}c^2 - 60060a^3b^8c^3 + 210210a^4b^6c^4 - 252252a^5b^4c^5 + 84084a^6b^2c^6 - 3432a^7c^7)x^{42} - 1/3*(a^2b^{13} - 78a^2b^{11}c + 1430a^3b^9c^2 - 8580a^4b^7c^3 + 18018a^5b^5c^4 - 12012a^6b^3c^5 + 1716a^7b^2c^6)x^{39} + 13/6*(a^2b^{12} - 44a^3b^{10}c + 495a^4b^8c^2 - 1848a^5b^6c^3 + 2310a^6b^4c^4 - 792a^7b^2c^5 + 33a^8c^6)x^{36} - 13/3*(2a^3b^{11} - 55a^4b^9c + 396a^5b^7c^2 - 924a^6b^5c^3 + 660a^7b^3c^4 - 99a^8b^2c^5)x^{33} + 143/6*(a^4b^{10} - 18a^5b^8c + 84a^6b^6c^2 - 120a^7b^4c^3 + 45a^8b^2c^4 - 2a^9c^5)x^{30} - 143/3*(a^5b^9 - 12a^6b^7c + 36a^7b^5c^2 - 30a^8b^3c^3 + 5a^9b^2c^4)x^{27} + 143/6*(3a^6b^8 - 24a^7b^6c + 45a^8b^4c^2 - 20a^9b^2c^3 + a^{10}c^4)x^{24} - 143/21*(12a^7b^7 - 63a^8b^5c + 70a^9b^3c^2 - 14a^{10}b^2c^3)x^{21} + 13/6*(33a^8b^6 - 110a^9b^4c + 66a^{10}b^2c^2 - 4a^{11}c^3)x^{18} - 1/3a^{13}bx^3 - 13/3*(11a^9b^5 - 22a^{10}b^3c + 6a^{11}b^2c^2)x^{15} + 13/6*(11a^{10}b^4 - 12a^{11}b^2c + a^{12}c^2)x^{12} - 13/3*(2a^{11}b^3 - a^{12}b^2c)x^9 + 1/6*(13a^{12}b^2 - 2a^{13}c)x^6
\end{aligned}$$

mupad [B] time = 1.28, size = 1214, normalized size = 60.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(b + 2cx^3)*(bx^3 - a + cx^6)^{13}, x)$

[Out] $x^{36}*((13a^2b^{12})/6 + (143a^8c^6)/2 - (286a^3b^{10}c)/3 + (2145a^4b^8c^2)/2 - 4004a^5b^6c^3 + 5005a^6b^4c^4 - 1716a^7b^2c^5) + x^{48}*((143a^6c^8)/2 + (13b^{12}c^2)/6 - (286ab^{10}c^3)/3 + (2145a^2b^8c^4)/2 - 4004a^3b^6c^5 + 5005a^4b^4c^6 - 1716a^5b^2c^7) - x^{39}*((ab^{13})/3 - 26a^2b^{11}c + 572a^7b^2c^6 + (1430a^3b^9c^2)/3 - 2860a^4b^7c^3 + 6006a^5b^5c^4 - 4004a^6b^3c^5) + x^{45}*((b^{13}c)/3 - 26ab^{11}c^2 + 572a^6b^2c^7 + (1430a^2b^9c^3)/3 - 2860a^3b^7c^4 + 6006a^4b^5c^5 - 4004a^5b^3c^6) + x^{18}*((143a^8b^6)/2 - (26a^{11}c^3)/3 - (715a^9b^4c)/3 + 143a^{10}b^2c^2) - x^{66}*((26a^3c^{11})/3 - (143b^6c^8)/2 + (715ab^4c^9)/3 - 143a^2b^2c^{10}) + x^{30}*((143a^4b^{10})/6 - (143a^9c^5)/3 - 429a^5b^8c + 2002a^6b^6c^2 - 2860a^7b^4c^3 + (2145a^8b^$

$$\begin{aligned}
& 2*c^4)/2) - x^{54}*((143*a^5*c^9)/3 - (143*b^{10}*c^4)/6 + 429*a*b^8*c^5 - 2002 \\
& *a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^{42}*(b^{14}/42 - (\\
& 572*a^7*c^7)/7 + 143*a^2*b^{10}*c^2 - 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 - 6 \\
& 006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^{12}*c)/3) + x^{24}*((143*a^6*b^8) \\
& /2 + (143*a^{10}*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 - (1430*a^9*b^ \\
& 2*c^3)/3) + x^{60}*((143*a^4*c^{10})/6 + (143*b^8*c^6)/2 - 572*a*b^6*c^7 + (214 \\
& 5*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^{14}*x^{84})/42 - x^6*((a^{13}*c)/3 \\
& - (13*a^{12}*b^2)/6) + (13*a^{10}*x^{12}*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 + (1 \\
& 3*c^{10}*x^{72}*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^{13}*b*x^3)/3 + (b*c^{13}*x \\
& ^{81})/3 - (c^{12}*x^{78}*(2*a*c - 13*b^2))/6 - (143*a^7*b*x^{21}*(12*b^6 - 14*a^3* \\
& c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 + (143*b*c^7*x^{63}*(12*b^6 - 14*a^3*c \\
& ^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 - (143*a^5*b*x^{27}*(b^8 + 5*a^4*c^4 + \\
& 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 + (143*b*c^5*x^{57}*(b^8 + 5 \\
& *a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 - (13*a^3*b*x^3 \\
& 3*(2*b^{10} - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^ \\
& 4 - 55*a*b^8*c))/3 + (13*b*c^3*x^{51}*(2*b^{10} - 99*a^5*c^5 + 396*a^2*b^6*c^2 \\
& - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/3 - (13*a^9*b*x^{15}*(11*b \\
& ^4 + 6*a^2*c^2 - 22*a*b^2*c))/3 + (13*b*c^9*x^{69}*(11*b^4 + 6*a^2*c^2 - 22*a \\
& *b^2*c))/3 + (13*a^{11}*b*x^9*(a*c - 2*b^2))/3 - (13*b*c^{11}*x^{75}*(a*c - 2*b^2 \\
&))/3
\end{aligned}$$

sympy [B] time = 0.36, size = 1394, normalized size = 69.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)

[Out] $-a^{13}b^3x^3/3 + b^3c^{13}x^{81}/3 + c^{14}x^{84}/42 + x^{78}(-a^{13}c/3 + 13$
 $*b^2c^{12}/6) + x^{75}(-13ab^3c^{12}/3 + 26b^3c^{11}/3) + x^{72}(13a^2$
 $*c^{12}/6 - 26ab^2c^{11} + 143b^4c^{10}/6) + x^{69}(26a^2b^3c^{11} - 2$
 $86ab^3c^{10}/3 + 143b^5c^9/3) + x^{66}(-26a^3c^{11}/3 + 143a^2b$
 $**2c^{10} - 715ab^4c^9/3 + 143b^6c^8/2) + x^{63}(-286a^3b^3c^{10}$
 $/3 + 1430a^2b^3c^9/3 - 429ab^5c^8 + 572b^7c^7/7) + x^{60}(14$
 $3a^4c^{10}/6 - 1430a^3b^2c^9/3 + 2145a^2b^4c^8/2 - 572ab^6$
 $*c^7 + 143b^8c^6/2) + x^{57}(715a^4b^3c^9/3 - 1430a^3b^3c^8 +$
 $1716a^2b^5c^7 - 572ab^7c^6 + 143b^9c^5/3) + x^{54}(-143a^5$
 $c^9/3 + 2145a^4b^2c^8/2 - 2860a^3b^4c^7 + 2002a^2b^6c^6$
 $- 429ab^8c^5 + 143b^{10}c^4/6) + x^{51}(-429a^5b^3c^8 + 2860a$
 $*4b^3c^7 - 4004a^3b^5c^6 + 1716a^2b^7c^5 - 715ab^9c^4/$
 $3 + 26b^{11}c^3/3) + x^{48}(143a^6c^8/2 - 1716a^5b^2c^7 + 5005$
 $a^4b^4c^6 - 4004a^3b^6c^5 + 2145a^2b^8c^4/2 - 286ab^{10}$
 $c^3/3 + 13b^{12}c^2/6) + x^{45}(572a^6b^3c^7 - 4004a^5b^3c^6 +$
 $6006a^4b^5c^5 - 2860a^3b^7c^4 + 1430a^2b^9c^3/3 - 26ab^{11}$
 $*c^2 + b^{13}c/3) + x^{42}(-572a^7c^7/7 + 2002a^6b^2c^6 - 600$

$$\begin{aligned}
& 6a^{55}b^{44}c^{55} + 5005a^{44}b^{66}c^{44} - 1430a^{33}b^{88}c^{33} + 143a^{22}b^{10}c^{22} - 13ab^{12}c/3 + b^{14}/42) + x^{39}(-572a^{77}b^{c^{66}} + 4004a^{66}b^{33}c^{55} - 6006a^{55}b^{55}c^{44} + 2860a^{44}b^{77}c^{33} - 1430a^{33}b^{99}c^{22}/3 + 26a^{22}b^{11}c - ab^{13}/3) + x^{36}(143a^{88}c^{66}/2 - 1716a^{77}b^{22}c^{55} + 5005a^{66}b^{44}c^{44} - 4004a^{55}b^{66}c^{33} + 2145a^{44}b^{88}c^{22}/2 - 286a^{33}b^{10}c/3 + 13a^{22}b^{12}/6) + x^{33}(429a^{88}b^{c^{55}} - 2860a^{77}b^{33}c^{44} + 4004a^{66}b^{55}c^{33} - 1716a^{55}b^{77}c^{22} + 715a^{44}b^{99}c/3 - 26a^{33}b^{11}/3) + x^{30}(-143a^{99}c^{55}/3 + 2145a^{88}b^{22}c^{44}/2 - 2860a^{77}b^{44}c^{33} + 2002a^{66}b^{66}c^{22} - 429a^{55}b^{88}c + 143a^{44}b^{10}/6) + x^{27}(-715a^{99}b^{c^{44}}/3 + 1430a^{88}b^{33}c^{33} - 1716a^{77}b^{55}c^{22} + 572a^{66}b^{77}c - 143a^{55}b^{99}/3) + x^{24}(143a^{10}c^{44}/6 - 1430a^{99}b^{22}c^{33}/3 + 2145a^{88}b^{44}c^{22}/2 - 572a^{77}b^{66}c + 143a^{66}b^{88}/2) + x^{21}(286a^{10}b^{c^{33}}/3 - 1430a^{99}b^{33}c^{22}/3 + 429a^{88}b^{55}c - 572a^{77}b^{77}/7) + x^{18}(-26a^{11}c^{33}/3 + 143a^{10}b^{22}c^{22} - 715a^{99}b^{44}c/3 + 143a^{88}b^{66}/2) + x^{15}(-26a^{11}b^{c^{22}} + 286a^{10}b^{33}c/3 - 143a^{99}b^{55}/3) + x^{12}(13a^{12}c^{22}/6 - 26a^{11}b^{22}c + 143a^{10}b^{44}/6) + x^9(13a^{12}b^{c/3} - 26a^{11}b^{33}/3) + x^6(-a^{13}c/3 + 13a^{12}b^{2/6})
\end{aligned}$$

$$3.100 \quad \int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^{13} dx$$

Optimal. Leaf size=25

$$\frac{(a - bx^n - cx^{2n})^{14}}{14n}$$

[Out] 1/14*(a-b*x^n-c*x^(2*n))^14/n

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1468, 629}

$$\frac{(a - bx^n - cx^{2n})^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]

[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^{13} dx &= \frac{\text{Subst}\left(\int (b + 2cx) (-a + bx + cx^2)^{13} dx, x, x^n\right)}{n} \\ &= \frac{(a - bx^n - cx^{2n})^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{(x^n (b + cx^n) - a)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]

[Out] (-a + x^n*(b + c*x^n))^14/(14*n)

fricas [B] time = 0.76, size = 1299, normalized size = 51.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 - 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(15*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(14*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^6

$$\frac{6 - 110a^9b^4c + 66a^{10}b^2c^2 - 4a^{11}c^3}{x^{6n}} - 182(11a^9b^5 - 22a^{10}b^3c + 6a^{11}b^2c^2)x^{5n} + 91(11a^{10}b^4 - 12a^{11}b^2c + a^{12}c^2)x^{4n} - 182(2a^{11}b^3 - a^{12}bc)x^{3n} + 7(13a^{12}b^2 - 2a^{13}c)x^{2n} / n$$

giac [B] time = 1.09, size = 1693, normalized size = 67.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 84084*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 4004*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n) - 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(11*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) + 1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
```

$$5*c^2*x^{(9*n)} + 60060*a^8*b^3*c^3*x^{(9*n)} - 10010*a^9*b*c^4*x^{(9*n)} + 3003*a^6*b^8*x^{(8*n)} - 24024*a^7*b^6*c*x^{(8*n)} + 45045*a^8*b^4*c^2*x^{(8*n)} - 20020*a^9*b^2*c^3*x^{(8*n)} + 1001*a^10*c^4*x^{(8*n)} - 3432*a^7*b^7*x^{(7*n)} + 18018*a^8*b^5*c*x^{(7*n)} - 20020*a^9*b^3*c^2*x^{(7*n)} + 4004*a^10*b*c^3*x^{(7*n)} + 3003*a^8*b^6*x^{(6*n)} - 10010*a^9*b^4*c*x^{(6*n)} + 6006*a^10*b^2*c^2*x^{(6*n)} - 364*a^11*c^3*x^{(6*n)} - 2002*a^9*b^5*x^{(5*n)} + 4004*a^10*b^3*c*x^{(5*n)} - 1092*a^11*b*c^2*x^{(5*n)} + 1001*a^10*b^4*x^{(4*n)} - 1092*a^11*b^2*c*x^{(4*n)} + 91*a^12*c^2*x^{(4*n)} - 364*a^11*b^3*x^{(3*n)} + 182*a^12*b*c*x^{(3*n)} + 91*a^12*b^2*x^{(2*n)} - 14*a^13*c*x^{(2*n)} - 14*a^13*b*x^{(n)}/n$$

maple [B] time = 0.06, size = 2046, normalized size = 81.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(n-1)}*(b+2*c*x^n)*(-a+b*x^n+c*x^{(2*n)})^{13},x)$

[Out] $26*b^{11}*c^3/n*(x^n)^{17}-1716/7/n*(x^n)^{14}*a^7*c^7-1716/7*b^7*a^7/n*(x^n)^{7+1}43*b^9*c^5/n*(x^n)^{19}+26*b^3*c^{11}/n*(x^n)^{25}-a*b^{13}/n*(x^n)^{13}-143*a^5*b^9/n*(x^n)^9+1716/7*b^7*c^7/n*(x^n)^{21}+143*b^5*c^9/n*(x^n)^{23}+143/2*a^{10}/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8-143*b^5*a^9/n*(x^n)^5-26*b^{11}*a^3/n*(x^n)^{11}+b^{13}*c/n*(x^n)^{15}+13/2*a^{12}/n*(x^n)^4*c^2+143/2*a^{10}/n*(x^n)^4*b^4-26*a^{11}/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6-143*a^9/n*(x^n)^{10}*c^5+143/2*a^4/n*(x^n)^{10}*b^{10}+429/2*c^8/n*(x^n)^{22}*b^6-c^{13}/n*(x^n)^{26}*a+13/2*c^{12}/n*(x^n)^{26}*b^2+429/2*c^8/n*(x^n)^{16}*a^6+13/2*c^2/n*(x^n)^{16}*b^{12}-143*c^9/n*(x^n)^{18}*a^5+143/2*c^4/n*(x^n)^{18}*b^{10}+143/2*c^{10}/n*(x^n)^{20}*a^4+429/2*c^6/n*(x^n)^{20}*b^8-26*c^{11}/n*(x^n)^{22}*a^3+429/2*a^8/n*(x^n)^{12}*c^6+13/2*a^2/n*(x^n)^{12}*b^{12}-26*a^{11}*b^3/n*(x^n)^3+13/2*c^{12}/n*(x^n)^{24}*a^2+143/2*c^{10}/n*(x^n)^{24}*b^4-a^{13}/n*(x^n)^2*c+13/2*a^{12}/n*(x^n)^2*b^2-a^{13}*b/n*x^n+b*c^{13}/n*(x^n)^{27}-1287*b^5*c^8/n*(x^n)^{21}*a+78*b*c^{11}/n*(x^n)^{23}*a^2-286*b^3*c^{10}/n*(x^n)^{23}*a+286*b*a^{10}/n*(x^n)^7*c^3-1430*b^3*a^9/n*(x^n)^7*c^2+1287*b^5*a^8/n*(x^n)^7*c+715*b*c^9/n*(x^n)^{19}*a^4-4290*b^3*c^8/n*(x^n)^{19}*a^3+5148*b^5*c^7/n*(x^n)^{19}*a^2-1716*b^7*c^6/n*(x^n)^{19}*a-5148*a^7/n*(x^n)^{12}*b^2*c^5+15015*a^6/n*(x^n)^{12}*b^4*c^4-12012*a^5/n*(x^n)^{12}*b^6*c^3+6435/2*a^4/n*(x^n)^{12}*b^8*c^2+1/14*c^{14}/n*(x^n)^{28}-715*a^9/n*(x^n)^6*b^4*c+1/14/n*(x^n)^{14}*b^{14}+6435/2*a^8/n*(x^n)^{10}*b^2*c^4-8580*a^7/n*(x^n)^{10}*b^4*c^3+6006*a^6/n*(x^n)^{10}*b^6*c^2-1287*a^5/n*(x^n)^{10}*b^8*c-1430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c-78*b*a^{11}/n*(x^n)^5*c^2+286*b^3*a^{10}/n*(x^n)^5*c+1287*b*a^8/n*(x^n)^{11}*c^5-1716*a^7*b/n*(x^n)^{13}*c^6+12012*a^6*b^3/n*(x^n)^{13}*c^5-18018*a^5*b^5/n*(x^n)^{13}*c^4+8580*a^4*b^7/n*(x^n)^{13}*c^3-1430*a^3*b^9/n*(x^n)^{13}*c^2+78*a^2*b^{11}/n*(x^n)^{13}*c-715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3-5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c-286*b*c^{10}/n*(x^n)^{21}*a^3+1430*b^3*c^9/n*(x^n)^{21}*a^2-8580*b^3*a^7/n*(x^n)^{11}*c^4+12012*b^5*a^6/n*(x^n)^{11}*c^3-5148*b^7*a^5/n*(x^n)^{11}*c^2+715*b^9*a^4/n*(x^n)^{11}*c+1716*b*c^7/n*(x^n)^{15}*a^6-12012*b^3*c^6/n*(x^n)^{15}*a^5+1$

$$\begin{aligned}
& 8018*b^5*c^5/n*(x^n)^{15*a^4-8580*b^7*c^4/n*(x^n)^{15*a^3+1430*b^9*c^3/n*(x^n)} \\
&)^{15*a^2-78*b^{11}*c^2/n*(x^n)^{15*a-13*b*c^{12}/n*(x^n)^{25*a-1430*c^9/n*(x^n)^2} \\
& 0*a^3*b^2+6435/2*c^8/n*(x^n)^{20*a^2*b^4-1716*c^7/n*(x^n)^{20*a*b^6+429*c^{10}/} \\
& n*(x^n)^{22*a^2*b^2-715*c^9/n*(x^n)^{22*a*b^4+13*a^{12}*b/n*(x^n)^3*c-78*c^{11}/n} \\
& *(x^n)^{24*a*b^2-78*a^{11}/n*(x^n)^4*b^2*c+429*a^{10}/n*(x^n)^6*b^2*c^2-1287*b*c \\
& ^8/n*(x^n)^{17*a^5+8580*b^3*c^7/n*(x^n)^{17*a^4-12012*b^5*c^6/n*(x^n)^{17*a^3+} \\
& 5148*b^7*c^5/n*(x^n)^{17*a^2-715*b^9*c^4/n*(x^n)^{17*a+6006/n*(x^n)^{14*a^6*b^} \\
& 2*c^6-18018/n*(x^n)^{14*a^5*b^4*c^5+15015/n*(x^n)^{14*a^4*b^6*c^4-4290/n*(x^n)} \\
&)^{14*a^3*b^8*c^3+429/n*(x^n)^{14*a^2*b^{10}*c^2-13/n*(x^n)^{14*a*b^{12}*c-286*a^3} \\
& /n*(x^n)^{12*b^{10}*c-5148*c^7/n*(x^n)^{16*a^5*b^2+15015*c^6/n*(x^n)^{16*a^4*b^4} \\
& -12012*c^5/n*(x^n)^{16*a^3*b^6+6435/2*c^4/n*(x^n)^{16*a^2*b^8-286*c^3/n*(x^n)} \\
& ^{16*a*b^{10}+6435/2*c^8/n*(x^n)^{18*a^4*b^2-8580*c^7/n*(x^n)^{18*a^3*b^4+6006*c} \\
& ^6/n*(x^n)^{18*a^2*b^6-1287*c^5/n*(x^n)^{18*a*b^8}
\end{aligned}$$

maxima [B] time = 0.83, size = 2045, normalized size = 81.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima")

[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^13*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n + 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*a*b^5*c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n - 8580*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*a^5*c^9*x^(18*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n - 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286*a*b^10*c^3*x^(16*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5*x^(16*n)/n + 15015*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n - 12012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)/n - 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n - 4290*a^3*b^8*c^3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n - 18018*a^5*b^4*c^5*x^(14*n)/n

$$\begin{aligned}
& + 6006*a^6*b^2*c^6*x^{(14*n)}/n - 1716/7*a^7*c^7*x^{(14*n)}/n - a*b^{13}*x^{(13*n)} \\
& /n + 78*a^2*b^{11}*c*x^{(13*n)}/n - 1430*a^3*b^9*c^2*x^{(13*n)}/n + 8580*a^4*b^7* \\
& c^3*x^{(13*n)}/n - 18018*a^5*b^5*c^4*x^{(13*n)}/n + 12012*a^6*b^3*c^5*x^{(13*n)}/ \\
& n - 1716*a^7*b*c^6*x^{(13*n)}/n + 13/2*a^2*b^{12}*x^{(12*n)}/n - 286*a^3*b^{10}*c*x \\
& ^{(12*n)}/n + 6435/2*a^4*b^8*c^2*x^{(12*n)}/n - 12012*a^5*b^6*c^3*x^{(12*n)}/n + \\
& 15015*a^6*b^4*c^4*x^{(12*n)}/n - 5148*a^7*b^2*c^5*x^{(12*n)}/n + 429/2*a^8*c^6* \\
& x^{(12*n)}/n - 26*a^3*b^{11}*x^{(11*n)}/n + 715*a^4*b^9*c*x^{(11*n)}/n - 5148*a^5*b \\
& ^7*c^2*x^{(11*n)}/n + 12012*a^6*b^5*c^3*x^{(11*n)}/n - 8580*a^7*b^3*c^4*x^{(11*n)} \\
&)/n + 1287*a^8*b*c^5*x^{(11*n)}/n + 143/2*a^4*b^{10}*x^{(10*n)}/n - 1287*a^5*b^8* \\
& c*x^{(10*n)}/n + 6006*a^6*b^6*c^2*x^{(10*n)}/n - 8580*a^7*b^4*c^3*x^{(10*n)}/n + \\
& 6435/2*a^8*b^2*c^4*x^{(10*n)}/n - 143*a^9*c^5*x^{(10*n)}/n - 143*a^5*b^9*x^{(9*n)} \\
&)/n + 1716*a^6*b^7*c*x^{(9*n)}/n - 5148*a^7*b^5*c^2*x^{(9*n)}/n + 4290*a^8*b^3* \\
& c^3*x^{(9*n)}/n - 715*a^9*b*c^4*x^{(9*n)}/n + 429/2*a^6*b^8*x^{(8*n)}/n - 1716*a^ \\
& 7*b^6*c*x^{(8*n)}/n + 6435/2*a^8*b^4*c^2*x^{(8*n)}/n - 1430*a^9*b^2*c^3*x^{(8*n)} \\
& /n + 143/2*a^{10}*c^4*x^{(8*n)}/n - 1716/7*a^7*b^7*x^{(7*n)}/n + 1287*a^8*b^5*c*x \\
& ^{(7*n)}/n - 1430*a^9*b^3*c^2*x^{(7*n)}/n + 286*a^{10}*b*c^3*x^{(7*n)}/n + 429/2*a^ \\
& 8*b^6*x^{(6*n)}/n - 715*a^9*b^4*c*x^{(6*n)}/n + 429*a^{10}*b^2*c^2*x^{(6*n)}/n - 26 \\
& *a^{11}*c^3*x^{(6*n)}/n - 143*a^9*b^5*x^{(5*n)}/n + 286*a^{10}*b^3*c*x^{(5*n)}/n - 78 \\
& *a^{11}*b*c^2*x^{(5*n)}/n + 143/2*a^{10}*b^4*x^{(4*n)}/n - 78*a^{11}*b^2*c*x^{(4*n)}/n \\
& + 13/2*a^{12}*c^2*x^{(4*n)}/n - 26*a^{11}*b^3*x^{(3*n)}/n + 13*a^{12}*b*c*x^{(3*n)}/n + \\
& 13/2*a^{12}*b^2*x^{(2*n)}/n - a^{13}*c*x^{(2*n)}/n - a^{13}*b*x^n/n
\end{aligned}$$

mupad [B] time = 5.78, size = 1401, normalized size = 56.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{(n-1)}*(b+2*c*x^n)*(b*x^n-a+c*x^{(2*n)})^{13},x)$

[Out] $x^{(n-1)}*((x^{(11*n+1)}*((13*a^2*b^{12})/2+(429*a^8*c^6)/2-286*a^3*b^{10}*c+(6435*a^4*b^8*c^2)/2-12012*a^5*b^6*c^3+15015*a^6*b^4*c^4-5148*a^7*b^2*c^5))/n+(x^{(15*n+1)}*((429*a^6*c^8)/2+(13*b^{12}*c^2)/2-286*a*b^{10}*c^3+(6435*a^2*b^8*c^4)/2-12012*a^3*b^6*c^5+15015*a^4*b^4*c^6-5148*a^5*b^2*c^7))/n-(x^{(12*n+1)}*(a*b^{13}-78*a^2*b^{11}*c+1716*a^7*b*c^6+1430*a^3*b^9*c^2-8580*a^4*b^7*c^3+18018*a^5*b^5*c^4-12012*a^6*b^3*c^5))/n+(x^{(14*n+1)}*(b^{13}*c-78*a*b^{11}*c^2+1716*a^6*b*c^7+1430*a^2*b^9*c^3-8580*a^3*b^7*c^4+18018*a^4*b^5*c^5-12012*a^5*b^3*c^6))/n+(x^{(5*n+1)}*((429*a^8*b^6)/2-26*a^{11}*c^3-715*a^9*b^4*c+429*a^{10}*b^2*c^2))/n-(x^{(21*n+1)}*(26*a^3*c^{11}-(429*b^6*c^8)/2+715*a*b^4*c^9-429*a^2*b^2*c^{10}))/n+(x^{(9*n+1)}*((143*a^4*b^{10})/2-143*a^9*c^5-1287*a^5*b^8*c+6006*a^6*b^6*c^2-8580*a^7*b^4*c^3+(6435*a^8*b^2*c^4)/2))/n-(x^{(17*n+1)}*(143*a^5*c^9-(143*b^{10}*c^4)/2+1287*a*b^8*c^5-6006*a^2*b^6*c^6+8580*a^3*b^4*c^7-(6435*a^4*b^2*c^8)/2))/n+(x^{(13*n+1)}*(b^{14}/14-(1716*a^7*c^7)/7+429*a^2*b^{10}*c^2-4290*a^3*b^8*c^3+15015*a^4*b^6*c^4-18018*a^5*b^4*c^5+6006*a^6*b^2*c^6-13*a*b^{12}*c))/n+(x^{(7*n+1)}*($

$$\begin{aligned}
& (429*a^6*b^8)/2 + (143*a^{10}*c^4)/2 - 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2 \\
& - 1430*a^9*b^2*c^3)/n + (x^{(19*n + 1)}*((143*a^4*c^{10})/2 + (429*b^8*c^6)/2 \\
& - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9))/n + (c^{14}*x^{(2 \\
& 7*n + 1)})/(14*n) - (a^{12}*x^{(n + 1)}*(a*c - (13*b^2)/2))/n + (a^{10}*x^{(3*n + 1 \\
&)}*((143*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (c^{10}*x^{(23*n + 1)}*((143 \\
& *b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (b*c^{13}*x^{(26*n + 1)})/n - (c^{12 \\
& *x^{(25*n + 1)}*(a*c - (13*b^2)/2))/n - (a^{13}*b*x)/n - (143*a^7*b*x^{(6*n + 1 \\
&)}*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) + (143*b*c^7*x^{(\\
& 20*n + 1)}*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) - (14 \\
& 3*a^5*b*x^{(8*n + 1)}*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12 \\
& *a*b^6*c))/n + (143*b*c^5*x^{(18*n + 1)}*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - \\
& 30*a^3*b^2*c^3 - 12*a*b^6*c))/n - (13*a^3*b*x^{(10*n + 1)}*(2*b^{10} - 99*a^5*c \\
& ^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n + \\
& (13*b*c^3*x^{(16*n + 1)}*(2*b^{10} - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^ \\
& 4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n - (13*a^9*b*x^{(4*n + 1)}*(11*b^4 + \\
& 6*a^2*c^2 - 22*a*b^2*c))/n + (13*b*c^9*x^{(22*n + 1)}*(11*b^4 + 6*a^2*c^2 - 2 \\
& 2*a*b^2*c))/n + (13*a^{11}*b*x^{(2*n + 1)}*(a*c - 2*b^2))/n - (13*b*c^{11}*x^{(24* \\
& n + 1)}*(a*c - 2*b^2))/n
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)

[Out] Timed out

$$3.101 \quad \int (b + 2cx) (bx + cx^2)^{13} dx$$

Optimal. Leaf size=15

$$\frac{1}{14} (bx + cx^2)^{14}$$

[Out] 1/14*(c*x^2+b*x)^14

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{1}{14} (bx + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b*x + c*x^2)^14/14

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^{13} dx = \frac{1}{14} (bx + cx^2)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13}{2}b^{12}c^2x^{16} + 26b^{11}c^3x^{17} + \frac{143}{2}b^{10}c^4x^{18} + 143b^9c^5x^{19} + \frac{429}{2}b^8c^6x^{20} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^6c^8x^{22} + 143b^5c^9x^{23} + \frac{143}{2}b^4c^{10}x^{24} + 26b^3c^{11}x^{25} + \frac{13}{2}b^2c^{12}x^{26} + b^1c^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]

[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14

fricas [B] time = 0.48, size = 154, normalized size = 10.27

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")

[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14

giac [A] time = 0.40, size = 13, normalized size = 0.87

$$\frac{1}{14}(cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")

[Out] 1/14*(c*x^2 + b*x)^14

maple [B] time = 0.00, size = 155, normalized size = 10.33

$$\frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)

[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2*x^16+b^13*c*x^15+1/14*b^14*x^14

maxima [A] time = 0.44, size = 13, normalized size = 0.87

$$\frac{1}{14}(cx^2 + bx)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")

[Out] 1/14*(c*x^2 + b*x)^14

mupad [B] time = 2.09, size = 154, normalized size = 10.27

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + c^{14}x^{28}/14$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x + c*x^2)^13*(b + 2*c*x), x)

[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + bc^13*x^27 + c^14*x^28/14

sympy [B] time = 0.13, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + c^{14}x^{28}/14$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)

[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 + 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*x**27 + c**14*x**28/14

$$3.102 \quad \int x (b + 2cx^2) (bx^2 + cx^4)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

[Out] 1/28*x^28*(c*x^2+b)^14

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28}(b + cx^2)^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x(b+2cx^2)(bx^2+cx^4)^{13} dx &= \int x^{27}(b+cx^2)^{13}(b+2cx^2) dx \\ &= \frac{1}{2} \text{Subst} \left(\int x^{13}(b+cx)^{13}(b+2cx) dx, x, x^2 \right) \\ &= \frac{1}{28} x^{28} (b+cx^2)^{14} \end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{1}{2}b^{13}cx^{30} + \frac{13}{4}b^{12}c^2x^{32} + 13b^{11}c^3x^{34} + \frac{143}{4}b^{10}c^4x^{36} + \frac{143}{2}b^9c^5x^{38} + \frac{429}{4}b^8c^6x^{40} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^6c^8x^{44} + \frac{143}{2}b^5c^9x^{46} + \frac{143}{4}b^4c^{10}x^{48} + 13b^3c^{11}x^{50} + \frac{13}{4}b^2c^{12}x^{52} + \frac{1}{2}bc^{13}x^{54} + \frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]

[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 + (c^14*x^56)/28

fricas [B] time = 0.77, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8 + \frac{143}{2}x^{38}c^5b^9 + \frac{143}{4}x^{36}c^4b^{10} + 13x^{34}c^3b^{11} + \frac{13}{4}x^{32}c^2b^{12} + \frac{1}{2}x^{30}cb^{13} + \frac{1}{28}x^{28}b^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")

[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 + 143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 13*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14

giac [A] time = 0.38, size = 15, normalized size = 0.94

$$\frac{1}{28} (cx^4 + bx^2)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")

[Out] $\frac{1}{28}(c*x^4 + b*x^2)^{14}$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x)`

[Out] $\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$

maxima [B] time = 0.43, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")`

[Out] $\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)`

[Out] $\frac{(b^{14}x^{28})}{28} + \frac{(c^{14}x^{56})}{28} + \frac{(b^{13}cx^{30})}{2} + \frac{(b^{12}c^2x^{32})}{4} + \frac{(13b^{11}c^3x^{34})}{4} + \frac{(143b^{10}c^4x^{36})}{4} + \frac{(143b^9c^5x^{38})}{2} + \frac{(429b^8c^6x^{40})}{4} + \frac{(858b^7c^7x^{42})}{7} + \frac{(429b^6c^8x^{44})}{4} + \frac{(143b^5c^9x^{46})}{2} + \frac{(143b^4c^{10}x^{48})}{4} + 13b^3c^{11}x^{50} + \frac{(13b^2c^{12}x^{52})}{4}$

sympy [B] time = 0.13, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)`

[Out] $b^{14}x^{28}/28 + b^{13}c x^{30}/2 + 13b^{12}c^2 x^{32}/4 + 13b^{11}c^3 x^{34} + 143b^{10}c^4 x^{36}/4 + 143b^9 c^5 x^{38}/2 + 429b^8 c^6 x^{40}/4 + 858b^7 c^7 x^{42}/7 + 429b^6 c^8 x^{44}/4 + 143b^5 c^9 x^{46}/2 + 143b^4 c^{10} x^{48}/4 + 13b^3 c^{11} x^{50} + 13b^2 c^{12} x^{52}/4 + b c^{13} x^{54}/2 + c^{14} x^{56}/28$

$$3.103 \quad \int x^2 (b + 2cx^3) (bx^3 + cx^6)^{13} dx$$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}(b + cx^3)^{14}$$

[Out] 1/42*x^42*(c*x^3+b)^14

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42}(b + cx^3)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]

[Out] (x^42*(b + c*x^3)^14)/42

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
 \int x^2 (b + 2cx^3) (bx^3 + cx^6)^{13} dx &= \int x^{41} (b + cx^3)^{13} (b + 2cx^3) dx \\
 &= \frac{1}{3} \text{Subst} \left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^3 \right) \\
 &= \frac{1}{42} x^{42} (b + cx^3)^{14}
 \end{aligned}$$

Mathematica [B] time = 0.01, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42} + \frac{1}{3}b^{13}cx^{45} + \frac{13}{6}b^{12}c^2x^{48} + \frac{26}{3}b^{11}c^3x^{51} + \frac{143}{6}b^{10}c^4x^{54} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{2}b^8c^6x^{60} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^6c^8x^{66} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]

[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^81)/3 + (c^14*x^84)/42

fricas [B] time = 0.70, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^2 + \frac{26}{3}x^{75}c^{11}b^3 + \frac{143}{6}x^{72}c^{10}b^4 + \frac{143}{3}x^{69}c^9b^5 + \frac{143}{2}x^{66}c^8b^6 + \frac{572}{7}x^{63}c^7b^7 + \frac{143}{2}x^{60}c^6b^8 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")

[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3 + 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^63*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 + 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14

giac [A] time = 0.51, size = 15, normalized size = 0.94

$$\frac{1}{42} (cx^6 + bx^3)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")

[Out] $1/42*(c*x^6 + b*x^3)^{14}$

maple [B] time = 0.00, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^{13},x)$

[Out] $1/42*c^{14}*x^{84}+1/3*b*c^{13}*x^{81}+13/6*b^2*c^{12}*x^{78}+26/3*b^3*c^{11}*x^{75}+143/6*b^4*c^{10}*x^{72}+143/3*b^5*c^9*x^{69}+143/2*b^6*c^8*x^{66}+572/7*b^7*c^7*x^{63}+143/2*b^8*c^6*x^{60}+143/3*b^9*c^5*x^{57}+143/6*b^{10}*c^4*x^{54}+26/3*b^{11}*c^3*x^{51}+13/6*b^{12}*c^2*x^{48}+1/3*b^{13}*c*x^{45}+1/42*b^{14}*x^{42}$

maxima [B] time = 0.44, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^{13},x, \text{algorithm}="maxima")$

[Out] $1/42*c^{14}*x^{84} + 1/3*b*c^{13}*x^{81} + 13/6*b^2*c^{12}*x^{78} + 26/3*b^3*c^{11}*x^{75} + 143/6*b^4*c^{10}*x^{72} + 143/3*b^5*c^9*x^{69} + 143/2*b^6*c^8*x^{66} + 572/7*b^7*c^7*x^{63} + 143/2*b^8*c^6*x^{60} + 143/3*b^9*c^5*x^{57} + 143/6*b^{10}*c^4*x^{54} + 26/3*b^{11}*c^3*x^{51} + 13/6*b^{12}*c^2*x^{48} + 1/3*b^{13}*c*x^{45} + 1/42*b^{14}*x^{42}$

mupad [B] time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^{13},x)$

[Out] $(b^{14}*x^{42})/42 + (c^{14}*x^{84})/42 + (b^{13}*c*x^{45})/3 + (b*c^{13}*x^{81})/3 + (13*b^{12}*c^2*x^{48})/6 + (26*b^{11}*c^3*x^{51})/3 + (143*b^{10}*c^4*x^{54})/6 + (143*b^9*c^5*x^{57})/3 + (143*b^8*c^6*x^{60})/2 + (572*b^7*c^7*x^{63})/7 + (143*b^6*c^8*x^{66})/2 + (143*b^5*c^9*x^{69})/3 + (143*b^4*c^{10}*x^{72})/6 + (26*b^3*c^{11}*x^{75})/3 + (13*b^2*c^{12}*x^{78})/6$

sympy [B] time = 0.13, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^2x^{48}}{6} + \frac{26b^{11}c^3x^{51}}{3} + \frac{143b^{10}c^4x^{54}}{6} + \frac{143b^9c^5x^{57}}{3} + \frac{143b^8c^6x^{60}}{2} + \frac{572b^7c^7x^{63}}{7} + \frac{143b^6c^8x^{66}}{2} + \frac{143b^5c^9x^{69}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)`

[Out] $b^{14}x^{42}/42 + b^{13}c x^{45}/3 + 13b^{12}c^2 x^{48}/6 + 26b^{11}c^3 x^{51}/3 + 143b^{10}c^4 x^{54}/6 + 143b^9c^5 x^{57}/3 + 143b^8c^6 x^{60}/2 + 572b^7c^7 x^{63}/7 + 143b^6c^8 x^{66}/2 + 143b^5c^9 x^{69}/3 + 143b^4c^{10} x^{72}/6 + 26b^3c^{11} x^{75}/3 + 13b^2c^{12} x^{78}/6 + b c^{13} x^{81}/3 + c^{14} x^{84}/42$

$$3.104 \quad \int x^{-1+n} (b + 2cx^n) (bx^n + cx^{2n})^{13} dx$$

Optimal. Leaf size=21

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int x^{-1+n} (b + 2cx^n) (bx^n + cx^{2n})^{13} dx &= \int x^{-1+14n} (b + cx^n)^{13} (b + 2cx^n) dx \\ &= \frac{\text{Subst}\left(\int x^{13} (b + cx)^{13} (b + 2cx) dx, x, x^n\right)}{n} \\ &= \frac{x^{14n} (b + cx^n)^{14}}{14n} \end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{x^{14n} (b + cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

fricas [B] time = 0.68, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n

giac [B] time = 0.43, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n} + 14bc^{13}x^{27n} + 91b^2c^{12}x^{26n} + 364b^3c^{11}x^{25n} + 1001b^4c^{10}x^{24n} + 2002b^5c^9x^{23n} + 3003b^6c^8x^{22n} + 3432b^7c^7x^{21n} + 3003b^8c^6x^{20n} + 2002b^9c^5x^{19n} + 1001b^{10}c^4x^{18n} + 364b^{11}c^3x^{17n} + 91b^{12}c^2x^{16n} + 14b^{13}cx^{15n} + b^{14}x^{14n}}{14n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")

[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n

$$8*x^{(22*n)} + 3432*b^7*c^7*x^{(21*n)} + 3003*b^8*c^6*x^{(20*n)} + 2002*b^9*c^5*x^{(19*n)} + 1001*b^{10}*c^4*x^{(18*n)} + 364*b^{11}*c^3*x^{(17*n)} + 91*b^{12}*c^2*x^{(16*n)} + 14*b^{13}*c*x^{(15*n)} + b^{14}*x^{(14*n)})/n$$

maple [B] time = 0.04, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n} + \frac{b^{13}cx^{15n}}{n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)

[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*b^3*c^11/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^n)^23+429/2*c^8/n*(x^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14

maxima [B] time = 0.48, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28n}}{14n} + \frac{bc^{13}x^{27n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^8c^6x^{20n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")

[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n

mupad [B] time = 2.63, size = 229, normalized size = 10.90

$$\frac{b^{14}x^{14n}}{14n} + \frac{c^{14}x^{28n}}{14n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)

[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^10*x^(24*n))/(2*n) + (143*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n) + (26*b*c^13*x^(27*n))/n + (c^14*x^(28*n))/(14*n)

$$0*x^{(24*n)} / (2*n) + (26*b^3*c^{11}*x^{(25*n)}) / n + (13*b^2*c^{12}*x^{(26*n)}) / (2*n) + (b^{13}*c*x^{(15*n)}) / n + (b*c^{13}*x^{(27*n)}) / n$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)

[Out] Timed out

$$3.105 \quad \int \frac{b+2cx}{a+bx+cx^2} dx$$

Optimal. Leaf size=11

$$\log(a + bx + cx^2)$$

[Out] ln(c*x^2+b*x+a)

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {628}

$$\log(a + bx + cx^2)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + b*x + c*x^2]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\int \frac{b + 2cx}{a + bx + cx^2} dx = \log(a + bx + cx^2)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.91

$$\log(a + x(b + cx))$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + x*(b + c*x)]

fricas [A] time = 0.75, size = 11, normalized size = 1.00

$$\log(cx^2 + bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")

[Out] log(c*x^2 + b*x + a)

giac [A] time = 0.35, size = 12, normalized size = 1.09

$$\log(|cx^2 + bx + a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")

[Out] log(abs(c*x^2 + b*x + a))

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\ln(cx^2 + bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x+a),x)

[Out] ln(c*x^2+b*x+a)

maxima [A] time = 0.45, size = 11, normalized size = 1.00

$$\log(cx^2 + bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")

[Out] log(c*x^2 + b*x + a)

mupad [B] time = 1.96, size = 11, normalized size = 1.00

$$\ln(cx^2 + bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(a + b*x + c*x^2),x)

[Out] log(a + b*x + c*x^2)

sympy [A] time = 0.16, size = 10, normalized size = 0.91

$$\log(a + bx + cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
```

```
[Out] log(a + b*x + c*x**2)
```

$$3.106 \quad \int \frac{x(b+2cx^2)}{a+bx^2+cx^4} dx$$

Optimal. Leaf size=17

$$\frac{1}{2} \log(a + bx^2 + cx^4)$$

[Out] 1/2*ln(c*x^4+b*x^2+a)

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 628}

$$\frac{1}{2} \log(a + bx^2 + cx^4)$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]

[Out] Log[a + b*x^2 + c*x^4]/2

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(b+2cx^2)}{a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \log(a + bx^2 + cx^4) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{2} \log(a + bx^2 + cx^4)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4), x]

[Out] Log[a + b*x^2 + c*x^4]/2

fricas [A] time = 0.71, size = 15, normalized size = 0.88

$$\frac{1}{2} \log(cx^4 + bx^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a), x, algorithm="fricas")

[Out] 1/2*log(c*x^4 + b*x^2 + a)

giac [A] time = 1.73, size = 16, normalized size = 0.94

$$\frac{1}{2} \log(|cx^4 + bx^2 + a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a), x, algorithm="giac")

[Out] 1/2*log(abs(c*x^4 + b*x^2 + a))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln(cx^4 + bx^2 + a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a), x)

[Out] 1/2*ln(c*x^4+b*x^2+a)

maxima [A] time = 0.43, size = 15, normalized size = 0.88

$$\frac{1}{2} \log(cx^4 + bx^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a), x, algorithm="maxima")

[Out] 1/2*log(c*x^4 + b*x^2 + a)

mupad [B] time = 1.96, size = 15, normalized size = 0.88

$$\frac{\ln(cx^4 + bx^2 + a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4), x)`

[Out] `log(a + b*x^2 + c*x^4)/2`

sympy [A] time = 0.28, size = 14, normalized size = 0.82

$$\frac{\log(a + bx^2 + cx^4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a), x)`

[Out] `log(a + b*x**2 + c*x**4)/2`

$$3.107 \quad \int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6} dx$$

Optimal. Leaf size=17

$$\frac{1}{3} \log(a + bx^3 + cx^6)$$

[Out] 1/3*ln(c*x^6+b*x^3+a)

Rubi [A] time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 628}

$$\frac{1}{3} \log(a + bx^3 + cx^6)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6), x]

[Out] Log[a + b*x^3 + c*x^6]/3

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \log(a + bx^3 + cx^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{3} \log(a + bx^3 + cx^6)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6), x]

[Out] Log[a + b*x^3 + c*x^6]/3

fricas [A] time = 0.63, size = 15, normalized size = 0.88

$$\frac{1}{3} \log(cx^6 + bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a), x, algorithm="fricas")

[Out] 1/3*log(c*x^6 + b*x^3 + a)

giac [A] time = 1.09, size = 16, normalized size = 0.94

$$\frac{1}{3} \log(|cx^6 + bx^3 + a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a), x, algorithm="giac")

[Out] 1/3*log(abs(c*x^6 + b*x^3 + a))

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{\ln(cx^6 + bx^3 + a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a), x)

[Out] 1/3*ln(c*x^6+b*x^3+a)

maxima [A] time = 0.44, size = 15, normalized size = 0.88

$$\frac{1}{3} \log(cx^6 + bx^3 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")`

[Out] `1/3*log(c*x^6 + b*x^3 + a)`

mupad [B] time = 0.05, size = 15, normalized size = 0.88

$$\frac{\ln(cx^6 + bx^3 + a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)`

[Out] `log(a + b*x^3 + c*x^6)/3`

sympy [A] time = 0.41, size = 14, normalized size = 0.82

$$\frac{\log(a + bx^3 + cx^6)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)`

[Out] `log(a + b*x**3 + c*x**6)/3`

$$3.108 \quad \int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=19

$$\frac{\log(a + bx^n + cx^{2n})}{n}$$

[Out] $\ln(a+b*x^n+c*x^{(2*n)})/n$

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 628}

$$\frac{\log(a + bx^n + cx^{2n})}{n}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^{(-1+n)}*(b+2*c*x^n))/(a+b*x^n+c*x^{(2*n)}),x]$

[Out] $\text{Log}[a+b*x^n+c*x^{(2*n)}]/n$

Rule 628

$\text{Int}[(d + (e_*)*(x_))/((a_*) + (b_*)*(x_) + (c_*)*(x_)^2), x_Symbol] :> \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1468

$\text{Int}[(x_)^{(m_*)}*((a_*) + (c_*)*(x_)^{(n2_*)} + (b_*)*(x_)^{(n_*)})^{(p_*)}*((d_*) + (e_*)*(x_)^{(n_*)})^{(q_*)}, x_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{EqQ}[\text{Simplify}[m - n + 1], 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= \frac{\log(a + bx^n + cx^{2n})}{n} \end{aligned}$$

Mathematica [A] time = 0.10, size = 19, normalized size = 1.00

$$\frac{\log(a + bx^n + cx^{2n})}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]

[Out] Log[a + b*x^n + c*x^(2*n)]/n

fricas [A] time = 0.70, size = 19, normalized size = 1.00

$$\frac{\log(cx^{2n} + bx^n + a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] log(c*x^(2*n) + b*x^n + a)/n

giac [A] time = 0.46, size = 19, normalized size = 1.00

$$\frac{\log(cx^{2n} + bx^n + a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] log(c*x^(2*n) + b*x^n + a)/n

maple [A] time = 0.02, size = 24, normalized size = 1.26

$$\frac{\ln\left(b e^{n \ln(x)} + c e^{2n \ln(x)} + a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)+a),x)

[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)

maxima [A] time = 0.60, size = 23, normalized size = 1.21

$$\frac{\log\left(\frac{cx^{2n}+bx^n+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(a+b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] log((c*x^(2*n) + b*xⁿ + a)/c)/n

mupad [B] time = 2.32, size = 121, normalized size = 6.37

$$-\frac{2b \operatorname{atan}\left(\frac{b}{\sqrt{4ac-b^2}} + \frac{2cx^n}{\sqrt{4ac-b^2}}\right) - \ln\left(a + bx^n + cx^{2n}\right) \sqrt{4ac-b^2}}{n\sqrt{4ac-b^2}} - \frac{2b \operatorname{atanh}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ))/(a+b*xⁿ+c*x^(2*n)),x)

[Out] - (2*b*atan(b/(4*a*c - b²)^(1/2) + (2*c*xⁿ)/(4*a*c - b²)^(1/2)) - log(a + b*xⁿ + c*x^(2*n))*(4*a*c - b²)^(1/2))/(n*(4*a*c - b²)^(1/2)) - (2*b*atanh((b + 2*c*xⁿ)/(b² - 4*a*c)^(1/2)))/(n*(b² - 4*a*c)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+n)}*(b+2*c*x^{**n})/(a+b*x^{**n}+c*x^{** (2*n)}),x)

[Out] Timed out

$$3.109 \quad \int \frac{b+2cx}{(a+bx+cx^2)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{7(a+bx+cx^2)^7}$$

[Out] -1/7/(c*x^2+b*x+a)^7

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {629}

$$-\frac{1}{7(a+bx+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]

[Out] -1/(7*(a + b*x + c*x^2)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+bx+cx^2)^7}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]

[Out] -1/7*1/(a + x*(b + c*x))^7

fricas [B] time = 0.64, size = 350, normalized size = 21.88

$$\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 7(3b^2c^5 + ac^6)x^{12} + 7(5b^3c^4 + 6abc^5)x^{11} + 7(5b^4c^3 + 15ab^2c^4 + 3a^2c^5)x^{10} + 7(3b^5c^2 + 6a^2b^3c^3 + 15a^3b^2c^4 + 3a^4b^2c^5)x^9 + 7(b^6c + 15a^2b^4c^2 + 30a^2b^2c^3 + 5a^3c^4)x^8 + 7a^6bx + (b^7 + 42a^5b^2c + 210a^4b^3c^2 + 140a^3b^4c^3)x^7 + a^7 + 7(a^2b^6 + 15a^2b^4c + 30a^3b^2c^2 + 5a^4c^3)x^6 + 7(3a^2b^5 + 20a^3b^3c + 15a^4b^2c^2)x^5 + 7(5a^3b^4 + 15a^4b^2c + 3a^5c^2)x^4 + 7(5a^4b^3 + 6a^5bc)x^3 + 7(3a^5b^2 + a^6c)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b^2*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^5 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^5*b^2 + a^6*c)*x^2)

giac [A] time = 0.40, size = 14, normalized size = 0.88

$$-\frac{1}{7(cx^2 + bx + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x + a)^7

maple [A] time = 0.00, size = 15, normalized size = 0.94

$$-\frac{1}{7(cx^2 + bx + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x)

[Out] -1/7/(c*x^2+b*x+a)^7

maxima [A] time = 0.44, size = 14, normalized size = 0.88

$$-\frac{1}{7(cx^2 + bx + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")

[Out] -1/7/(c*x^2 + b*x + a)^7

mupad [B] time = 3.62, size = 358, normalized size = 22.38

$$7 \left(x^5 \left(105 a^4 b c^2 + 140 a^3 b^3 c + 21 a^2 b^5 \right) + x^9 \left(105 a^2 b c^4 + 140 a b^3 c^3 + 21 b^5 c^2 \right) + x^7 \left(140 a^3 b c^3 + 210 a^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)

[Out] -1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*a*b*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 + 21*b^2*c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))

sympy [B] time = 4.79, size = 359, normalized size = 22.44

$$7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12} \left(49ac^6 + 147b^2c^5 \right) + x^{11} \left(294abc^5 + 245b^3c^4 \right) + x^{10} \left(147a^2c^5 + 735ab \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)

[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b**3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))

$$3.110 \quad \int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx$$

Optimal. Leaf size=18

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

[Out] -1/14/(c*x^4+b*x^2+a)^7

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 629}

$$-\frac{1}{14(a+bx^2+cx^4)^7}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]

[Out] -1/(14*(a + b*x^2 + c*x^4)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{(a+bx+cx^2)^8} dx, x, x^2 \right) \\ &= -\frac{1}{14(a+bx^2+cx^4)^7} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{14(a + bx^2 + cx^4)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]

[Out] -1/14*1/(a + b*x^2 + c*x^4)^7

fricas [B] time = 0.73, size = 352, normalized size = 19.56

$$14(c^7x^{28} + 7bc^6x^{26} + 7(3b^2c^5 + ac^6)x^{24} + 7(5b^3c^4 + 6abc^5)x^{22} + 7(5b^4c^3 + 15ab^2c^4 + 3a^2c^5)x^{20} + 7(3b^5c^2 + 20a^2b^3c^3 + 15a^2b^2c^4)x^{18} + 7(b^6c + 15a^2b^4c^2 + 30a^2b^2c^3 + 5a^3c^4)x^{16} + (b^7 + 42a^2b^5c + 210a^2b^3c^2 + 140a^3b^2c^3)x^{14} + 7(a^2b^6 + 15a^2b^4c + 30a^3b^2c^2 + 5a^4c^3)x^{12} + 7(3a^2b^5 + 20a^3b^3c + 15a^4b^2c^2)x^{10} + 7a^6b^2x^8 + 7(5a^3b^4 + 15a^4b^2c + 3a^5c^2)x^8 + a^7 + 7(5a^4b^3 + 6a^5b^2c)x^6 + 7(3a^5b^2 + a^6c)x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b^2*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^10 + 7*a^6*b*x^8 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b^2*c)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4)

giac [A] time = 6.78, size = 16, normalized size = 0.89

$$-\frac{1}{14(cx^4 + bx^2 + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2 + a)^7

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{14(cx^4 + bx^2 + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x)`

[Out] $-1/14/(c*x^4+b*x^2+a)^7$

maxima [B] time = 0.95, size = 352, normalized size = 19.56

$$14 \left(c^7 x^{28} + 7 b c^6 x^{26} + 7 (3 b^2 c^5 + a c^6) x^{24} + 7 (5 b^3 c^4 + 6 a b c^5) x^{22} + 7 (5 b^4 c^3 + 15 a b^2 c^4 + 3 a^2 c^5) x^{20} + 7 (3 b^5 c^2 + 20 a b^3 c^3 + 15 a^2 b^2 c^4) x^{18} + 7 (b^6 c + 15 a b^4 c^2 + 30 a^2 b^2 c^3 + 5 a^3 c^4) x^{16} + (b^7 + 42 a^2 b^5 c + 210 a^2 b^3 c^2 + 140 a^3 b^2 c^3) x^{14} + 7 (a b^6 + 15 a^2 b^4 c + 30 a^3 b^2 c^2 + 5 a^4 c^3) x^{12} + 7 (3 a^2 b^5 + 20 a^3 b^3 c + 15 a^4 b^2 c^2) x^{10} + 7 a^6 b x^2 + 7 (5 a^3 b^4 + 15 a^4 b^2 c + 3 a^5 c^2) x^8 + a^7 + 7 (5 a^4 b^3 + 6 a^5 b^2 c) x^6 + 7 (3 a^5 b^2 + a^6 c) x^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")`

[Out] $-1/14/(c^7*x^{28} + 7*b*c^6*x^{26} + 7*(3*b^2*c^5 + a*c^6)*x^{24} + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^{22} + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^{20} + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*x^{18} + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^{16} + (b^7 + 42*a^2*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b^2*c^3)*x^{14} + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^{12} + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^{10} + 7*a^6*b*x^2 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b^2*c)*x^6 + 7*(3*a^5*b^2 + a^6*c)*x^4$

mupad [B] time = 12.16, size = 360, normalized size = 20.00

$$14 \left(x^{10} (105 a^4 b c^2 + 140 a^3 b^3 c + 21 a^2 b^5) + x^{18} (105 a^2 b c^4 + 140 a b^3 c^3 + 21 b^5 c^2) + x^{14} (140 a^3 b c^3 + 210 a^2 b^2 c^2 + 42 a^2 b^5 c) + x^{16} (35 a^4 b^3 + 42 a^5 b^2 c) + x^{22} (35 b^3 c^4 + 42 a b^2 c^5) + x^8 (35 a^3 b^4 + 21 a^5 c^2 + 105 a^4 b^2 c) + x^{20} (21 a^2 c^5 + 35 b^4 c^3 + 105 a b^2 c^4) + a^7 + x^{12} (7 a b^6 + 35 a^4 c^3 + 105 a^2 b^4 c + 210 a^3 b^2 c^2) + x^{16} (7 b^6 c + 35 a^3 c^4 + 105 a b^4 c^2 + 210 a^2 b^2 c^3) + c^7 x^{28} + x^4 (7 a^6 c + 21 a^5 b^2) + x^{24} (7 a c^6 + 21 b^2 c^5) + 7 a^6 b x^2 + 7 b c^6 x^{26} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)`

[Out] $-1/(14*(x^{10}*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^{18}*(21*b^5*c^2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^{14}*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b^2*c) + x^{22}*(35*b^3*c^4 + 42*a*b^2*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^{20}*(21*a^2*c^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^{12}*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^{16}*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2*b^2*c^3) + c^7*x^{28} + x^4*(7*a^6*c + 21*a^5*b^2) + x^{24}*(7*a*c^6 + 21*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^{26}))$

sympy [B] time = 7.66, size = 360, normalized size = 20.00

$$14 a^7 + 98 a^6 b x^2 + 98 b c^6 x^{26} + 14 c^7 x^{28} + x^{24} (98 a c^6 + 294 b^2 c^5) + x^{22} (588 a b c^5 + 490 b^3 c^4) + x^{20} (294 a^2 c^5 + 147 a^3 b^2 c^4) + x^{18} (140 a^3 b c^3 + 210 a^2 b^2 c^2 + 42 a^2 b^5 c) + x^{16} (35 a^4 b^3 + 42 a^5 b^2 c) + x^{14} (b^7 + 140 a^3 b c^3 + 210 a^2 b^3 c^2 + 42 a b^5 c) + x^{12} (7 a b^6 + 35 a^4 c^3 + 105 a^2 b^4 c + 210 a^3 b^2 c^2) + x^{10} (3 a^2 b^5 + 20 a^3 b^3 c + 15 a^4 b^2 c^2) + x^8 (7 a^6 c + 21 a^5 b^2) + x^4 (7 a^6 c + 21 a^5 b^2) + a^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)

[Out]
$$-1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98*a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(294*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 + 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b**2*c**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a**2*b**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3*b**2*c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 1960*a**3*b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c + 490*a**3*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 294*a**5*b**2))$$

$$3.111 \quad \int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8} dx$$

Optimal. Leaf size=18

$$-\frac{1}{21(a+bx^3+cx^6)^7}$$

[Out] -1/21/(c*x^6+b*x^3+a)^7

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 629}

$$-\frac{1}{21(a+bx^3+cx^6)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]

[Out] -1/(21*(a + b*x^3 + c*x^6)^7)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8} dx = \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{(a+bx+cx^2)^8} dx, x, x^3 \right)$$

$$= -\frac{1}{21(a+bx^3+cx^6)^7}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{21(a+bx^3+cx^6)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]

[Out] -1/21*1/(a + b*x^3 + c*x^6)^7

fricas [B] time = 0.65, size = 352, normalized size = 19.56

$$21(c^7x^{42} + 7bc^6x^{39} + 7(3b^2c^5 + ac^6)x^{36} + 7(5b^3c^4 + 6abc^5)x^{33} + 7(5b^4c^3 + 15ab^2c^4 + 3a^2c^5)x^{30} + 7(3b^5c^2 + 20a^2b^3c^3 + 15a^2b^2c^4)x^{27} + 7(b^6c + 15a^2b^4c^2 + 30a^2b^2c^3 + 5a^3c^4)x^{24} + (b^7 + 42a^2b^5c + 210a^2b^3c^2 + 140a^3b^2c^3)x^{21} + 7(a^2b^6 + 15a^2b^4c + 30a^3b^2c^2 + 5a^4c^3)x^{18} + 7(3a^2b^5 + 20a^3b^3c + 15a^4b^2c^2)x^{15} + 7(5a^3b^4 + 15a^4b^2c + 3a^5c^2)x^{12} + 7a^6b^2x^9 + 7(5a^4b^3 + 6a^5b^2c)x^6 + a^7 + 7(3a^5b^2 + a^6c)x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4 + 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b^2*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b^2*x^9 + 7*(5*a^4*b^3 + 6*a^5*b^2*c)*x^6 + a^7 + 7*(3*a^5*b^2 + a^6*c)*x^3)

giac [A] time = 22.37, size = 16, normalized size = 0.89

$$-\frac{1}{21(cx^6+bx^3+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

$$^2*b^4*c + 210*a^3*b^2*c^2) + x^{24}*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2*b^2*c^3) + c^7*x^{42} + x^6*(7*a^6*c + 21*a^5*b^2) + x^{36}*(7*a*c^6 + 21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^{39}))$$

sympy [B] time = 11.76, size = 360, normalized size = 20.00

$$21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}(147ac^6 + 441b^2c^5) + x^{33}(882abc^5 + 735b^3c^4) + x^{30}(441a^2c^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)

[Out]
$$-1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(147*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30*(441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2*b**2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 4410*a**2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*a**3*b**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2 + 2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**2*c + 735*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c + 441*a**5*b**2))$$

$$3.112 \quad \int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8} dx$$

Optimal. Leaf size=23

$$-\frac{1}{7n(a+bx^n+cx^{2n})^7}$$

[Out] -1/7/n/(a+b*x^n+c*x^(2*n))^7

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 629}

$$-\frac{1}{7n(a+bx^n+cx^{2n})^7}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]

[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\int \frac{x^{-1+n} (b + 2cx^n)}{(a + bx^n + cx^{2n})^8} dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8} dx, x, x^n\right)}{n}$$

$$= -\frac{1}{7n(a + bx^n + cx^{2n})^7}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7n(a + x^n(b + cx^n))^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]

[Out] -1/7*1/(n*(a + x^n*(b + c*x^n))^7)

fricas [B] time = 0.75, size = 394, normalized size = 17.13

$$7(c^7nx^{14n} + 7bc^6nx^{13n} + 7a^6bnx^n + a^7n + 7(3b^2c^5 + ac^6)nx^{12n} + 7(5b^3c^4 + 6abc^5)nx^{11n} + 7(5b^4c^3 + 15$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")

[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3*a^5*b^2 + a^6*c)*n*x^(2*n))

giac [A] time = 0.63, size = 21, normalized size = 0.91

$$-\frac{1}{7(cx^{2n} + bx^n + a)^7 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(a+b*xⁿ+c*x^(2*n))⁸,x, algorithm="giac")

[Out] -1/7/((c*x^(2*n) + b*xⁿ + a)^{7*n})

maple [A] time = 0.06, size = 22, normalized size = 0.96

$$-\frac{1}{7(bx^n + cx^{2n} + a)^7 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ)/(b*xⁿ+c*x^(2*n)+a)⁸,x)

[Out] -1/7/n/(a+b*xⁿ+c*(xⁿ)²)⁷

maxima [B] time = 2.39, size = 416, normalized size = 18.09

$$7(c^7 n x^{14n} + 7bc^6 n x^{13n} + 7a^6 b n x^n + a^7 n + 7(3b^2 c^5 n + ac^6 n)x^{12n} + 7(5b^3 c^4 n + 6abc^5 n)x^{11n} + 7(5b^4 c^3 n + 4a^2 b^2 c^5 n + a^2 c^6 n)x^{10n} + 7(5b^4 c^3 n + 15a^2 b^2 c^4 n + 3a^2 c^5 n)x^{9n} + 7(3b^5 c^2 n + 20a^2 b^3 c^3 n + 15a^2 b^2 c^4 n)x^{8n} + 7(b^6 c n + 15a^2 b^4 c^2 n + 30a^2 b^2 c^3 n + 5a^3 c^4 n)x^{7n} + (b^7 n + 42a^2 b^5 c n + 210a^2 b^3 c^2 n + 140a^3 b^2 c^3 n)x^{6n} + 7(a^2 b^6 n + 15a^2 b^4 c n + 30a^3 b^2 c^2 n + 5a^4 c^3 n)x^{5n} + 7(3a^2 b^5 n + 20a^3 b^3 c n + 15a^4 b^2 c^2 n)x^{4n} + 7(5a^4 b^3 n + 6a^5 b^2 c n)x^{3n} + 7(3a^5 b^2 n + a^6 c n)x^{2n})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(a+b*xⁿ+c*x^(2*n))⁸,x, algorithm="maxima")

[Out] -1/7/(c^{7*n}*x^(14*n) + 7*b*c^{6*n}*x^(13*n) + 7*a⁶*b*n*xⁿ + a^{7*n} + 7*(3*b²*c^{5*n} + a*c^{6*n})*x^(12*n) + 7*(5*b³*c^{4*n} + 6*a*b*c^{5*n})*x^(11*n) + 7*(5*b⁴*c^{3*n} + 15*a*b²*c^{4*n} + 3*a²*c^{5*n})*x^(10*n) + 7*(3*b⁵*c^{2*n} + 20*a*b³*c^{3*n} + 15*a²*b*c^{4*n})*x^(9*n) + 7*(b⁶*c*n + 15*a*b⁴*c^{2*n} + 30*a²*b²*c^{3*n} + 5*a³*c^{4*n})*x^(8*n) + (b⁷*n + 42*a*b⁵*c*n + 210*a²*b³*c^{2*n} + 140*a³*b²*c^{3*n})*x^(7*n) + 7*(a*b⁶*n + 15*a²*b⁴*c*n + 30*a³*b²*c^{2*n} + 5*a⁴*c^{3*n})*x^(6*n) + 7*(3*a²*b⁵*n + 20*a³*b³*c*n + 15*a⁴*b*c^{2*n})*x^(5*n) + 7*(5*a³*b⁴*n + 15*a⁴*b²*c*n + 3*a⁵*c^{2*n})*x^(4*n) + 7*(5*a⁴*b³*n + 6*a⁵*b*c*n)*x^(3*n) + 7*(3*a⁵*b²*n + a⁶*c*n)*x^(2*n))

mupad [B] time = 23.01, size = 496, normalized size = 21.57

$$7a^7 n + 7b^7 n x^{7n} + 7c^7 n x^{14n} + 49a^6 b n x^n + 49a^6 b^6 n x^{6n} + 49a^6 c n x^{2n} + 49a^6 c^6 n x^{12n} + 49b^6 c n x^{8n} + 49a^2 b^2 c^5 n x^{10n} + 49a^2 b^2 c^6 n x^{10n} + 49a^2 b^3 c^4 n x^{9n} + 49a^2 b^4 c^3 n x^{9n} + 49a^2 b^5 c^2 n x^{9n} + 49a^2 b^6 c n x^{9n} + 49a^3 b^2 c^3 n x^{8n} + 49a^3 b^3 c^2 n x^{8n} + 49a^3 b^4 c n x^{8n} + 49a^3 b^5 c^2 n x^{8n} + 49a^3 b^6 c^3 n x^{8n} + 49a^4 b^2 c^4 n x^{7n} + 49a^4 b^3 c^3 n x^{7n} + 49a^4 b^4 c^2 n x^{7n} + 49a^4 b^5 c n x^{7n} + 49a^4 b^6 c^2 n x^{7n} + 49a^5 b^2 c^3 n x^{6n} + 49a^5 b^3 c^2 n x^{6n} + 49a^5 b^4 c n x^{6n} + 49a^5 b^5 c^2 n x^{6n} + 49a^5 b^6 c^3 n x^{6n} + 49a^6 b^2 c^4 n x^{5n} + 49a^6 b^3 c^3 n x^{5n} + 49a^6 b^4 c^2 n x^{5n} + 49a^6 b^5 c n x^{5n} + 49a^6 b^6 c^2 n x^{5n} + 49a^7 b^2 c^4 n x^{4n} + 49a^7 b^3 c^3 n x^{4n} + 49a^7 b^4 c^2 n x^{4n} + 49a^7 b^5 c n x^{4n} + 49a^7 b^6 c^2 n x^{4n} + 49a^8 b^2 c^4 n x^{3n} + 49a^8 b^3 c^3 n x^{3n} + 49a^8 b^4 c^2 n x^{3n} + 49a^8 b^5 c n x^{3n} + 49a^8 b^6 c^2 n x^{3n} + 49a^9 b^2 c^4 n x^{2n} + 49a^9 b^3 c^3 n x^{2n} + 49a^9 b^4 c^2 n x^{2n} + 49a^9 b^5 c n x^{2n} + 49a^9 b^6 c^2 n x^{2n} + 49a^{10} b^2 c^4 n x^{1n} + 49a^{10} b^3 c^3 n x^{1n} + 49a^{10} b^4 c^2 n x^{1n} + 49a^{10} b^5 c n x^{1n} + 49a^{10} b^6 c^2 n x^{1n} + 49a^{11} b^2 c^4 n x^{0n} + 49a^{11} b^3 c^3 n x^{0n} + 49a^{11} b^4 c^2 n x^{0n} + 49a^{11} b^5 c n x^{0n} + 49a^{11} b^6 c^2 n x^{0n})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(n - 1)*(b + 2*c*xⁿ))/(a + b*xⁿ + c*x^(2*n))⁸,x)

[Out] -1/(7*a^{7*n} + 7*b⁷*n*x^(7*n) + 7*c⁷*n*x^(14*n) + 49*a⁶*b*n*xⁿ + 49*a*b⁶*n*x^(6*n) + 49*a⁶*c*n*x^(2*n) + 49*a*c⁶*n*x^(12*n) + 49*b⁶*c*n*x^(8*n) + 49*a²*b²*c⁵*n*x^(10*n) + 49*a²*b²*c⁶*n*x^(10*n) + 49*a²*b³*c⁴*n*x^(9*n) + 49*a²*b⁴*c³*n*x^(9*n) + 49*a²*b⁵*c²*n*x^(9*n) + 49*a²*b⁶*c*n*x^(9*n) + 49*a³*b²*c³*n*x^(8*n) + 49*a³*b³*c²*n*x^(8*n) + 49*a³*b⁴*c*n*x^(8*n) + 49*a³*b⁵*c²*n*x^(8*n) + 49*a³*b⁶*c³*n*x^(8*n) + 49*a⁴*b²*c⁴*n*x^(7*n) + 49*a⁴*b³*c³*n*x^(7*n) + 49*a⁴*b⁴*c²*n*x^(7*n) + 49*a⁴*b⁵*c*n*x^(7*n) + 49*a⁴*b⁶*c²*n*x^(7*n) + 49*a⁵*b²*c³*n*x^(6*n) + 49*a⁵*b³*c²*n*x^(6*n) + 49*a⁵*b⁴*c*n*x^(6*n) + 49*a⁵*b⁵*c²*n*x^(6*n) + 49*a⁵*b⁶*c³*n*x^(6*n) + 49*a⁶*b²*c⁴*n*x^(5*n) + 49*a⁶*b³*c³*n*x^(5*n) + 49*a⁶*b⁴*c²*n*x^(5*n) + 49*a⁶*b⁵*c*n*x^(5*n) + 49*a⁶*b⁶*c²*n*x^(5*n) + 49*a⁷*b²*c⁴*n*x^(4*n) + 49*a⁷*b³*c³*n*x^(4*n) + 49*a⁷*b⁴*c²*n*x^(4*n) + 49*a⁷*b⁵*c*n*x^(4*n) + 49*a⁷*b⁶*c²*n*x^(4*n) + 49*a⁸*b²*c⁴*n*x^(3*n) + 49*a⁸*b³*c³*n*x^(3*n) + 49*a⁸*b⁴*c²*n*x^(3*n) + 49*a⁸*b⁵*c*n*x^(3*n) + 49*a⁸*b⁶*c²*n*x^(3*n) + 49*a⁹*b²*c⁴*n*x^(2*n) + 49*a⁹*b³*c³*n*x^(2*n) + 49*a⁹*b⁴*c²*n*x^(2*n) + 49*a⁹*b⁵*c*n*x^(2*n) + 49*a⁹*b⁶*c²*n*x^(2*n) + 49*a¹⁰*b²*c⁴*n*x^(1*n) + 49*a¹⁰*b³*c³*n*x^(1*n) + 49*a¹⁰*b⁴*c²*n*x^(1*n) + 49*a¹⁰*b⁵*c*n*x^(1*n) + 49*a¹⁰*b⁶*c²*n*x^(1*n) + 49*a¹¹*b²*c⁴*n*x^(0*n) + 49*a¹¹*b³*c³*n*x^(0*n) + 49*a¹¹*b⁴*c²*n*x^(0*n) + 49*a¹¹*b⁵*c*n*x^(0*n) + 49*a¹¹*b⁶*c²*n*x^(0*n))

$$\begin{aligned}
& + 49*b*c^6*n*x^{(13*n)} + 147*a^5*b^2*n*x^{(2*n)} + 245*a^4*b^3*n*x^{(3*n)} + 24 \\
& 5*a^3*b^4*n*x^{(4*n)} + 147*a^2*b^5*n*x^{(5*n)} + 147*a^5*c^2*n*x^{(4*n)} + 245*a \\
& ^4*c^3*n*x^{(6*n)} + 245*a^3*c^4*n*x^{(8*n)} + 147*a^2*c^5*n*x^{(10*n)} + 147*b^5 \\
& *c^2*n*x^{(9*n)} + 245*b^4*c^3*n*x^{(10*n)} + 245*b^3*c^4*n*x^{(11*n)} + 147*b^2* \\
& c^5*n*x^{(12*n)} + 735*a^4*b^2*c*n*x^{(4*n)} + 980*a^3*b^3*c*n*x^{(5*n)} + 735*a^ \\
& 4*b*c^2*n*x^{(5*n)} + 735*a^2*b^4*c*n*x^{(6*n)} + 980*a^3*b*c^3*n*x^{(7*n)} + 735 \\
& *a*b^4*c^2*n*x^{(8*n)} + 980*a*b^3*c^3*n*x^{(9*n)} + 735*a^2*b*c^4*n*x^{(9*n)} + \\
& 735*a*b^2*c^4*n*x^{(10*n)} + 1470*a^3*b^2*c^2*n*x^{(6*n)} + 1470*a^2*b^3*c^2*n* \\
& x^{(7*n)} + 1470*a^2*b^2*c^3*n*x^{(8*n)} + 294*a^5*b*c*n*x^{(3*n)} + 294*a*b^5*c* \\
& n*x^{(7*n)} + 294*a*b*c^5*n*x^{(11*n)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)

[Out] Timed out

$$3.113 \quad \int \frac{b+2cx}{-a+bx+cx^2} dx$$

Optimal. Leaf size=13

$$\log(a - bx - cx^2)$$

[Out] $\ln(-c*x^2-b*x+a)$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {628}

$$\log(a - bx - cx^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b + 2*c*x)/(-a + b*x + c*x^2), x]$

[Out] $\text{Log}[a - b*x - c*x^2]$

Rule 628

$\text{Int}[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\int \frac{b + 2cx}{-a + bx + cx^2} dx = \log(a - bx - cx^2)$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.92

$$\log(x(b + cx) - a)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(b + 2*c*x)/(-a + b*x + c*x^2), x]$

[Out] $\text{Log}[-a + x*(b + c*x)]$

fricas [A] time = 0.49, size = 13, normalized size = 1.00

$$\log(cx^2 + bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")

[Out] log(c*x^2 + b*x - a)

giac [A] time = 0.39, size = 14, normalized size = 1.08

$$\log(|cx^2 + bx - a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")

[Out] log(abs(c*x^2 + b*x - a))

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\ln(cx^2 + bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x-a),x)

[Out] ln(c*x^2+b*x-a)

maxima [A] time = 0.44, size = 13, normalized size = 1.00

$$\log(cx^2 + bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")

[Out] log(c*x^2 + b*x - a)

mupad [B] time = 0.05, size = 13, normalized size = 1.00

$$\ln(cx^2 + bx - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(b*x - a + c*x^2),x)

[Out] log(b*x - a + c*x^2)

sympy [A] time = 0.15, size = 10, normalized size = 0.77

$$\log(-a + bx + cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
```

```
[Out] log(-a + b*x + c*x**2)
```

$$3.114 \quad \int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx$$

Optimal. Leaf size=19

$$\frac{1}{2} \log(a - bx^2 - cx^4)$$

[Out] 1/2*ln(-c*x^4-b*x^2+a)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1247, 628}

$$\frac{1}{2} \log(a - bx^2 - cx^4)$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4), x]

[Out] Log[a - b*x^2 - c*x^4]/2

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{-a+bx+cx^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \log(a - bx^2 - cx^4) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2} \log(-a + bx^2 + cx^4)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4), x]

[Out] Log[-a + b*x^2 + c*x^4]/2

fricas [A] time = 0.56, size = 17, normalized size = 0.89

$$\frac{1}{2} \log(cx^4 + bx^2 - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a), x, algorithm="fricas")

[Out] 1/2*log(c*x^4 + b*x^2 - a)

giac [A] time = 1.62, size = 18, normalized size = 0.95

$$\frac{1}{2} \log(|cx^4 + bx^2 - a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a), x, algorithm="giac")

[Out] 1/2*log(abs(c*x^4 + b*x^2 - a))

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln(cx^4 + bx^2 - a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a), x)

[Out] 1/2*ln(c*x^4+b*x^2-a)

maxima [A] time = 0.44, size = 17, normalized size = 0.89

$$\frac{1}{2} \log(cx^4 + bx^2 - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a), x, algorithm="maxima")

[Out] 1/2*log(c*x^4 + b*x^2 - a)

mupad [B] time = 0.05, size = 17, normalized size = 0.89

$$\frac{\ln(cx^4 + bx^2 - a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4), x)`

[Out] `log(b*x^2 - a + c*x^4)/2`

sympy [A] time = 0.28, size = 14, normalized size = 0.74

$$\frac{\log(-a + bx^2 + cx^4)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a), x)`

[Out] `log(-a + b*x**2 + c*x**4)/2`

$$3.115 \quad \int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx$$

Optimal. Leaf size=19

$$\frac{1}{3} \log(a - bx^3 - cx^6)$$

[Out] 1/3*ln(-c*x^6-b*x^3+a)

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 628}

$$\frac{1}{3} \log(a - bx^3 - cx^6)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6), x]

[Out] Log[a - b*x^3 - c*x^6]/3

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{b+2cx}{-a+bx+cx^2} dx, x, x^3 \right) \\ &= \frac{1}{3} \log(a - bx^3 - cx^6) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3} \log(-a + bx^3 + cx^6)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]

[Out] Log[-a + b*x^3 + c*x^6]/3

fricas [A] time = 0.62, size = 17, normalized size = 0.89

$$\frac{1}{3} \log(cx^6 + bx^3 - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")

[Out] 1/3*log(c*x^6 + b*x^3 - a)

giac [A] time = 1.04, size = 18, normalized size = 0.95

$$\frac{1}{3} \log(|cx^6 + bx^3 - a|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")

[Out] 1/3*log(abs(c*x^6 + b*x^3 - a))

maple [A] time = 0.00, size = 18, normalized size = 0.95

$$\frac{\ln(cx^6 + bx^3 - a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x)

[Out] 1/3*ln(c*x^6+b*x^3-a)

maxima [A] time = 0.43, size = 17, normalized size = 0.89

$$\frac{1}{3} \log(cx^6 + bx^3 - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")

[Out] 1/3*log(c*x^6 + b*x^3 - a)

mupad [B] time = 0.06, size = 17, normalized size = 0.89

$$\frac{\ln(cx^6 + bx^3 - a)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)

[Out] log(b*x^3 - a + c*x^6)/3

sympy [A] time = 0.39, size = 14, normalized size = 0.74

$$\frac{\log(-a + bx^3 + cx^6)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)

[Out] log(-a + b*x**3 + c*x**6)/3

$$3.116 \quad \int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=21

$$\frac{\log(a - bx^n - cx^{2n})}{n}$$

[Out] $\ln(a-b*x^n-c*x^{(2*n)})/n$

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1468, 628}

$$\frac{\log(a - bx^n - cx^{2n})}{n}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^{(-1+n)}*(b+2*c*x^n))/(-a+b*x^n+c*x^{(2*n)}),x]$

[Out] $\text{Log}[a - b*x^n - c*x^{(2*n)}]/n$

Rule 628

$\text{Int}[(d + (e_*)*(x_))/((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rule 1468

$\text{Int}[(x_)^{(m_*)}*((a_*) + (c_*)*(x_)^{(n2_*)} + (b_*)*(x_)^{(n_*)})^{(p_*)}*((d_*) + (e_*)*(x_)^{(n_*)})^{(q_*)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{EqQ}[\text{Simplify}[m - n + 1], 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2} dx, x, x^n\right)}{n} \\ &= \frac{\log(a - bx^n - cx^{2n})}{n} \end{aligned}$$

Mathematica [A] time = 0.12, size = 21, normalized size = 1.00

$$\frac{\log(a - bx^n - cx^{2n})}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]

[Out] Log[a - b*x^n - c*x^(2*n)]/n

fricas [A] time = 0.68, size = 21, normalized size = 1.00

$$\frac{\log(cx^{2n} + bx^n - a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] log(c*x^(2*n) + b*x^n - a)/n

giac [A] time = 0.37, size = 21, normalized size = 1.00

$$\frac{\log(cx^{2n} + bx^n - a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] log(c*x^(2*n) + b*x^n - a)/n

maple [A] time = 0.02, size = 26, normalized size = 1.24

$$\frac{\ln(-be^{n \ln(x)} - ce^{2n \ln(x)} + a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x)

[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)

maxima [A] time = 0.60, size = 25, normalized size = 1.19

$$\frac{\log\left(\frac{cx^{2n}+bx^n-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(-a+b*xⁿ+c*x^(2*n)),x, algorithm="maxima")

[Out] log((c*x^(2*n) + b*xⁿ - a)/c)/n

mupad [B] time = 2.68, size = 199, normalized size = 9.48

$$\ln\left(\frac{2cx^n}{n} - \left(\frac{1}{n} + \frac{b\sqrt{b^2+4ac}}{nb^2+4acn}\right)(b+2cx^n)\right) + \ln\left(\frac{2cx^n}{n} - \left(\frac{1}{n} - \frac{b\sqrt{b^2+4ac}}{nb^2+4acn}\right)(b+2cx^n)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ))/(b*xⁿ-a+c*x^(2*n)),x)

[Out] log((2*c*xⁿ)/n - (1/n + (b*(4*a*c + b²)^(1/2))/(b²*n + 4*a*c*n))*(b + 2*c*xⁿ))*(1/n + (b*(4*a*c + b²)^(1/2))/(b²*n + 4*a*c*n)) + log((2*c*xⁿ)/n - (1/n - (b*(4*a*c + b²)^(1/2))/(b²*n + 4*a*c*n))*(b + 2*c*xⁿ))*(1/n - (b*(4*a*c + b²)^(1/2))/(b²*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*xⁿ)/(4*a*c + b²)^(1/2)))/(n*(4*a*c + b²)^(1/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+n)}*(b+2*c*x^{**n})/(-a+b*x^{**n}+c*x^{**2}*(2*n)),x)

[Out] Timed out

$$3.117 \quad \int \frac{b+2cx}{(-a+bx+cx^2)^8} dx$$

Optimal. Leaf size=18

$$\frac{1}{7(a-bx-cx^2)^7}$$

[Out] 1/7/(-c*x^2-b*x+a)^7

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {629}

$$\frac{1}{7(a-bx-cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]

[Out] 1/(7*(a - b*x - c*x^2)^7)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{b+2cx}{(-a+bx+cx^2)^8} dx = \frac{1}{7(a-bx-cx^2)^7}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]

[Out] 1/(7*(a - x*(b + c*x))^7)

fricas [B] time = 0.67, size = 354, normalized size = 19.67

$$\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 7(3b^2c^5 - ac^6)x^{12} + 7(5b^3c^4 - 6abc^5)x^{11} + 7(5b^4c^3 - 15ab^2c^4 + 3a^2c^5)x^{10} + 7(3b^5c^2 - 20a^2b^3c^3 + 15a^2b^2c^4)x^9 + 7(b^6c - 15a^2b^4c^2 + 30a^2b^2c^3 - 5a^3c^4)x^8 + 7a^6bx + (b^7 - 42a^2b^5c + 210a^2b^3c^2 - 140a^3b^2c^3)x^7 - a^7 - 7(a^2b^6 - 15a^2b^4c + 30a^3b^2c^2 - 5a^4c^3)x^6 + 7(3a^2b^5 - 20a^3b^3c + 15a^4b^2c^2)x^5 - 7(5a^3b^4 - 15a^4b^2c + 3a^5c^2)x^4 + 7(5a^4b^3 - 6a^5bc)x^3 - 7(3a^5b^2 - a^6c)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")

[Out]
$$-1/7/(c^7x^{14} + 7b^6c^6x^{13} + 7(3b^2c^5 - a^6c)x^{12} + 7(5b^3c^4 - 6a^2b^3c^5)x^{11} + 7(5b^4c^3 - 15a^2b^2c^4 + 3a^2c^5)x^{10} + 7(3b^5c^2 - 20a^2b^3c^3 + 15a^2b^2c^4)x^9 + 7(b^6c - 15a^2b^4c^2 + 30a^2b^2c^3 - 5a^3c^4)x^8 + 7a^6bx + (b^7 - 42a^2b^5c + 210a^2b^3c^2 - 140a^3b^2c^3)x^7 - a^7 - 7(a^2b^6 - 15a^2b^4c + 30a^3b^2c^2 - 5a^4c^3)x^6 + 7(3a^2b^5 - 20a^3b^3c + 15a^4b^2c^2)x^5 - 7(5a^3b^4 - 15a^4b^2c + 3a^5c^2)x^4 + 7(5a^4b^3 - 6a^5bc)x^3 - 7(3a^5b^2 - a^6c)x^2)$$

giac [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7(cx^2 + bx - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")

[Out]
$$-1/7/(c*x^2 + b*x - a)^7$$

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{7(cx^2 + bx - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x)

[Out]
$$-1/7/(c*x^2+b*x-a)^7$$

maxima [A] time = 0.43, size = 16, normalized size = 0.89

$$-\frac{1}{7(cx^2 + bx - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")

[Out] -1/7/(c*x^2 + b*x - a)^7

mupad [B] time = 5.22, size = 358, normalized size = 19.89

$$7 \left(x^5 \left(105 a^4 b c^2 - 140 a^3 b^3 c + 21 a^2 b^5 \right) + x^9 \left(105 a^2 b c^4 - 140 a b^3 c^3 + 21 b^5 c^2 \right) + x^7 \left(-140 a^3 b c^3 + 210 a^2 b^3 c \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)

[Out] -1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*a*b*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 - 21*b^2*c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))

sympy [B] time = 5.06, size = 359, normalized size = 19.94

$$-7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}(-49ac^6 + 147b^2c^5) + x^{11}(-294abc^5 + 245b^3c^4) + x^{10}(147a^2c^5 - 735ab^3c^4 + 147b^5c^2) + x^9(735a^2b^3c^4 - 980ab^3c^3 + 147b^5c^2) + x^8(-245a^3c^4 + 1470a^2b^2c^3 - 735ab^4c^2 + 49b^6c) + x^7(-980a^3b^3c^3 + 1470a^2b^3c^2 - 294a^5b^5c + 7b^7) + x^6(245a^4c^3 - 1470a^3b^2c^2 + 735a^2b^4c - 49ab^6) + x^5(735a^4b^3c^2 - 980a^3b^3c + 147a^2b^5) + x^4(-147a^5c^2 + 735a^4b^2c - 245a^3b^4) + x^3(-294a^5b^3c + 245a^4b^3) + x^2(49a^6c - 147a^5b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)

[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 - 735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))

$$3.118 \quad \int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx$$

Optimal. Leaf size=20

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

[Out] 1/14/(-c*x^4-b*x^2+a)^7

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1247, 629}

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]

[Out] 1/(14*(a - b*x^2 - c*x^4)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{(-a+bx+cx^2)^8} dx, x, x^2 \right) \\ &= \frac{1}{14(a-bx^2-cx^4)^7} \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{1}{14(-a + bx^2 + cx^4)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]

[Out] -1/14*1/(-a + b*x^2 + c*x^4)^7

fricas [B] time = 0.76, size = 356, normalized size = 17.80

$$14(c^7x^{28} + 7bc^6x^{26} + 7(3b^2c^5 - ac^6)x^{24} + 7(5b^3c^4 - 6abc^5)x^{22} + 7(5b^4c^3 - 15ab^2c^4 + 3a^2c^5)x^{20} + 7(3b^5c^2 - 20a^2b^3c^3 + 15a^2b^2c^4)x^{18} + 7(b^6c - 15a^2b^4c^2 + 30a^2b^2c^3 - 5a^3c^4)x^{16} + (b^7 - 42a^2b^5c + 210a^2b^3c^2 - 140a^3b^2c^3)x^{14} - 7(a^2b^6 - 15a^2b^4c + 30a^3b^2c^2 - 5a^4c^3)x^{12} + 7(3a^2b^5 - 20a^3b^3c + 15a^4b^2c^2)x^{10} + 7a^6b^2x^2 - 7(5a^3b^4 - 15a^4b^2c + 3a^5c^2)x^8 - a^7 + 7(5a^4b^3 - 6a^5b^2c)x^6 - 7(3a^5b^2 - a^6c)x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4 - 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b^2*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b^2*c)*x^6 - 7*(3*a^5*b^2 - a^6*c)*x^4)

giac [A] time = 7.19, size = 18, normalized size = 0.90

$$-\frac{1}{14(cx^4 + bx^2 - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2 - a)^7

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{14(cx^4 + bx^2 - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x)`

[Out] $-1/14/(c*x^4+b*x^2-a)^7$

maxima [B] time = 0.97, size = 356, normalized size = 17.80

$$14 \left(c^7 x^{28} + 7 b c^6 x^{26} + 7 (3 b^2 c^5 - a c^6) x^{24} + 7 (5 b^3 c^4 - 6 a b c^5) x^{22} + 7 (5 b^4 c^3 - 15 a b^2 c^4 + 3 a^2 c^5) x^{20} + 7 (3 b^5 c^2 - 15 a^2 b^3 c^3 + 15 a^3 b^2 c^4) x^{18} + 7 (b^6 c - 15 a b^4 c^2 + 30 a^2 b^2 c^3 - 5 a^3 c^4) x^{16} + (b^7 - 42 a b^5 c + 210 a^2 b^3 c^2 - 140 a^3 b^2 c^3) x^{14} - 7 (a b^6 - 15 a^2 b^4 c + 30 a^3 b^2 c^2 - 5 a^4 c^3) x^{12} + 7 (3 a^2 b^5 - 20 a^3 b^3 c + 15 a^4 b^2 c^2) x^{10} + 7 a^6 b^2 x^8 - 7 (5 a^4 b^3 - 6 a^5 b^2 c) x^6 - 7 (3 a^5 b^2 - a^6 c) x^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")`

[Out] $-1/14/(c^7 x^{28} + 7 b c^6 x^{26} + 7 (3 b^2 c^5 - a c^6) x^{24} + 7 (5 b^3 c^4 - 6 a b c^5) x^{22} + 7 (5 b^4 c^3 - 15 a b^2 c^4 + 3 a^2 c^5) x^{20} + 7 (3 b^5 c^2 - 15 a^2 b^3 c^3 + 15 a^3 b^2 c^4) x^{18} + 7 (b^6 c - 15 a b^4 c^2 + 30 a^2 b^2 c^3 - 5 a^3 c^4) x^{16} + (b^7 - 42 a b^5 c + 210 a^2 b^3 c^2 - 140 a^3 b^2 c^3) x^{14} - 7 (a b^6 - 15 a^2 b^4 c + 30 a^3 b^2 c^2 - 5 a^4 c^3) x^{12} + 7 (3 a^2 b^5 - 20 a^3 b^3 c + 15 a^4 b^2 c^2) x^{10} + 7 a^6 b^2 x^8 - 7 (5 a^4 b^3 - 6 a^5 b^2 c) x^6 - 7 (3 a^5 b^2 - a^6 c) x^4$

mupad [B] time = 11.04, size = 360, normalized size = 18.00

$$14 \left(x^{10} (105 a^4 b c^2 - 140 a^3 b^3 c + 21 a^2 b^5) + x^{18} (105 a^2 b c^4 - 140 a b^3 c^3 + 21 b^5 c^2) + x^{14} (-140 a^3 b c^3 + 210 a^2 b^3 c^2 - 42 a b^5 c) + x^{12} (35 a^4 b^3 - 42 a^5 b^2 c) + x^{22} (35 b^3 c^4 - 42 a b^5 c) - x^8 (35 a^3 b^4 + 21 a^5 c^2 - 105 a^4 b^2 c) + x^{20} (21 a^2 c^5 + 35 b^4 c^3 - 105 a b^2 c^4) - a^7 - x^{12} (7 a b^6 - 35 a^4 c^3 - 105 a^2 b^4 c + 210 a^3 b^2 c^2) + x^{16} (7 b^6 c - 35 a^3 c^4 - 105 a b^4 c^2 + 210 a^2 b^2 c^3) + c^7 x^{28} + x^4 (7 a^6 c - 21 a^5 b^2) - x^{24} (7 a^6 c - 21 a^5 b^2) + 7 a^6 b^2 x^2 + 7 b^6 c x^{26} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)`

[Out] $-1/(14*(x^{10}(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^{18}(21*b^5*c^2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^{14}(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^2 - 42*a*b^5*c) + x^6(35*a^4*b^3 - 42*a^5*b*c) + x^{22}(35*b^3*c^4 - 42*a*b^5*c) - x^8(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^{20}(21*a^2*c^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^{12}(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4*c + 210*a^3*b^2*c^2) + x^{16}(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2*b^2*c^3) + c^7*x^{28} + x^4*(7*a^6*c - 21*a^5*b^2) - x^{24}*(7*a*c^6 - 21*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^{26}))$

sympy [B] time = 8.09, size = 360, normalized size = 18.00

$$-14 a^7 + 98 a^6 b x^2 + 98 b c^6 x^{26} + 14 c^7 x^{28} + x^{24} (-98 a c^6 + 294 b^2 c^5) + x^{22} (-588 a b c^5 + 490 b^3 c^4) + x^{20} (294 a^2 c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)

[Out]
$$\begin{aligned} & -1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-9 \\ & 8*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20*(\\ & 294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 \\ & - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**2*b* \\ & *2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2940*a \\ & **2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940*a**3* \\ & b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2 - 1960* \\ & a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**2*c - 49 \\ & 0*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c - 294 \\ & *a**5*b**2)) \end{aligned}$$

$$3.119 \quad \int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8} dx$$

Optimal. Leaf size=20

$$\frac{1}{21(a-bx^3-cx^6)^7}$$

[Out] 1/21/(-c*x^6-b*x^3+a)^7

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 629}

$$\frac{1}{21(a-bx^3-cx^6)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]

[Out] 1/(21*(a - b*x^3 - c*x^6)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\int \frac{x^2 (b + 2cx^3)}{(-a + bx^3 + cx^6)^8} dx = \frac{1}{3} \text{Subst} \left(\int \frac{b + 2cx}{(-a + bx + cx^2)^8} dx, x, x^3 \right)$$

$$= \frac{1}{21 (a - bx^3 - cx^6)^7}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{1}{21 (-a + bx^3 + cx^6)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]

[Out] -1/21*1/(-a + b*x^3 + c*x^6)^7

fricas [B] time = 0.53, size = 356, normalized size = 17.80

$$21 (c^7 x^{42} + 7bc^6 x^{39} + 7(3b^2c^5 - ac^6)x^{36} + 7(5b^3c^4 - 6abc^5)x^{33} + 7(5b^4c^3 - 15ab^2c^4 + 3a^2c^5)x^{30} + 7(3b^5c^2 - 20a^2b^3c^3 + 15a^2b^2c^4)x^{27} + 7(b^6c - 15a^2b^4c^2 + 30a^2b^2c^3 - 5a^3c^4)x^{24} + (b^7 - 42a^2b^5c + 210a^2b^3c^2 - 140a^3b^2c^3)x^{21} - 7(a^2b^6 - 15a^2b^4c + 30a^3b^2c^2 - 5a^4c^3)x^{18} + 7(3a^2b^5 - 20a^3b^3c + 15a^4b^2c^2)x^{15} - 7(5a^3b^4 - 15a^4b^2c + 3a^5c^2)x^{12} + 7a^6b^2x^9 + 7(5a^4b^3 - 6a^5b^2c)x^6 - a^7 - 7(3a^5b^2 - a^6c)x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b^2*c^3)*x^21 - 7*(a^2*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b^2*x^9 + 7*(5*a^4*b^3 - 6*a^5*b^2*c)*x^6 - a^7 - 7*(3*a^5*b^2 - a^6*c)*x^3)

giac [A] time = 22.35, size = 18, normalized size = 0.90

$$\frac{1}{21 (cx^6 + bx^3 - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")

[Out] -1/21/(c*x^6 + b*x^3 - a)^7

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$-\frac{1}{21(c x^6 + b x^3 - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x)

[Out] -1/21/(c*x^6+b*x^3-a)^7

maxima [B] time = 0.94, size = 356, normalized size = 17.80

$$21(c^7 x^{42} + 7bc^6 x^{39} + 7(3b^2 c^5 - ac^6)x^{36} + 7(5b^3 c^4 - 6abc^5)x^{33} + 7(5b^4 c^3 - 15ab^2 c^4 + 3a^2 c^5)x^{30} + 7(3b^5 c^2 - 20a^2 b^3 c^3 + 15a^2 b^2 c^4)x^{27} + 7(b^6 c - 15a^2 b^4 c^2 + 30a^2 b^2 c^3 - 5a^3 c^4)x^{24} + (b^7 - 42a^2 b^5 c + 210a^2 b^3 c^2 - 140a^3 b^2 c^3)x^{21} - 7(a^2 b^6 - 15a^2 b^4 c + 30a^3 b^2 c^2 - 5a^4 c^3)x^{18} + 7(3a^2 b^5 - 20a^3 b^3 c + 15a^4 b^2 c^2)x^{15} - 7(5a^3 b^4 - 15a^4 b^2 c + 3a^5 c^2)x^{12} + 7a^6 b^2 x^9 + 7(5a^4 b^3 - 6a^5 b^2 c)x^9 - a^7 - 7(3a^5 b^2 - a^6 c)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4 - 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b^2*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b^2*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^9 + 7*(5*a^4*b^3 - 6*a^5*b^2*c)*x^9 - a^7 - 7*(3*a^5*b^2 - a^6*c)*x^6

mupad [B] time = 16.60, size = 360, normalized size = 18.00

$$21(x^{15}(105a^4bc^2 - 140a^3b^3c + 21a^2b^5) + x^{27}(105a^2bc^4 - 140ab^3c^3 + 21b^5c^2) + x^{21}(-140a^3bc^3 + 21a^4b^2c^2 - 140a^2b^3c^3 + 105a^2b^2c^4) + x^{21}(b^7 - 140a^3b^2c^3 + 210a^2b^3c^2 - 42a^2b^5c) + x^9(35a^4b^3 - 42a^5b^2c) + x^{33}(35b^3c^4 - 42a^2b^3c^5) - x^{12}(35a^3b^4 + 21a^5c^2 - 105a^4b^2c) + x^{30}(21a^2c^5 + 35b^4c^3 - 105a^2b^2c^4) - a^7 - x^{18}(7a^2b^6 - 35a^4c^3 - 105a^3b^2c^2) + 7a^6b^2x^9 + 7(5a^4b^3 - 6a^5b^2c)x^9 - a^7 - 7(3a^5b^2 - a^6c)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)

[Out] -1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b^2*c^2) + x^27*(21*b^5*c^4 - 140*a*b^3*c^3 + 105*a^2*b^2*c^4) + x^21*(b^7 - 140*a^3*b^2*c^3 + 210*a^2*b^3*c^2 - 42*a^2*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b^2*c) + x^33*(35*b^3*c^4 - 42*a^2*b^3*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a^2*c^5 + 35*b^4*c^3 - 105*a^2*b^2*c^4) - a^7 - x^18*(7*a^2*b^6 - 35*a^4*c^3 - 105*a^3*b^2*c^2) + 7a^6b^2x^9 + 7(5a^4b^3 - 6a^5b^2c)x^9 - a^7 - 7(3a^5b^2 - a^6c)x^6

$$\begin{aligned} & ^2*b^4*c + 210*a^3*b^2*c^2) + x^{24}*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + \\ & 210*a^2*b^2*c^3) + c^7*x^{42} + x^6*(7*a^6*c - 21*a^5*b^2) - x^{36}*(7*a*c^6 - \\ & 21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^{39}) \end{aligned}$$

sympy [B] time = 11.79, size = 360, normalized size = 18.00

$$-21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}(-147ac^6 + 441b^2c^5) + x^{33}(-882abc^5 + 735b^3c^4) + x^{30}(441a^2c^5 - 2205a^2b^2c^4 + 735ab^3c^3 - 2940a^3b^2c^2 + 441a^4b^2c) + x^{27}(2205a^2b^3c^3 - 2940a^3b^2c^2 + 441a^4b^2c) + x^{24}(-735a^3c^4 + 4410a^2b^2c^3 - 2205a^3b^2c^2 + 147a^4b^2c) + x^{21}(-2940a^3b^3c^3 + 4410a^2b^3c^2 - 882a^3b^3c + 21a^4b^3) + x^{18}(735a^4c^3 - 4410a^3b^2c^2 + 2205a^4b^2c - 147a^5b^2) + x^{15}(2205a^4b^3c^2 - 2940a^5b^3c + 441a^6b^3) + x^{12}(-441a^5c^2 + 2205a^6b^2c - 735a^7b^2) + x^9(-882a^5b^3c + 735a^6b^3) + x^6(147a^6c - 441a^7b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)

[Out] -1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x**30*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*a**2*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3 + 4410*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 - 4410*a**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*c**2 - 2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a**4*b**2*c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c - 441*a**5*b**2))

$$3.120 \quad \int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8} dx$$

Optimal. Leaf size=25

$$\frac{1}{7n(a-bx^n-cx^{2n})^7}$$

[Out] 1/7/n/(a-b*x^n-c*x^(2*n))^7

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1468, 629}

$$\frac{1}{7n(a-bx^n-cx^{2n})^7}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]

[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\int \frac{x^{-1+n} (b + 2cx^n)}{(-a + bx^n + cx^{2n})^8} dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8} dx, x, x^n\right)}{n}$$

$$= \frac{1}{7n(a - bx^n - cx^{2n})^7}$$

Mathematica [A] time = 0.07, size = 23, normalized size = 0.92

$$\frac{1}{7n(a - x^n(b + cx^n))^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]

[Out] 1/(7*n*(a - x^n*(b + c*x^n))^7)

fricas [B] time = 0.70, size = 397, normalized size = 15.88

$$7(c^7nx^{14n} + 7bc^6nx^{13n} + 7a^6bnx^n - a^7n + 7(3b^2c^5 - ac^6)nx^{12n} + 7(5b^3c^4 - 6abc^5)nx^{11n} + 7(5b^4c^3 - 15abc^4 - 15a^2b^2c^4 + 3a^2c^5)nx^{10n} + 7(3b^5c^2 - 20a^2b^3c^3 + 15a^2b^2c^4)nx^{9n} + 7(b^6c - 15a^2b^4c^2 + 30a^2b^2c^3 - 5a^3c^4)nx^{8n} + (b^7 - 42a^2b^5c + 210a^2b^3c^2 - 140a^3b^2c^3)nx^{7n} - 7(a^2b^6 - 15a^2b^4c + 30a^3b^2c^2 - 5a^4c^3)nx^{6n} + 7(3a^2b^5 - 20a^3b^3c + 15a^4b^2c^2)nx^{5n} - 7(5a^3b^4 - 15a^4b^2c + 3a^5c^2)nx^{4n} + 7(5a^4b^3 - 6a^5b^2c)nx^{3n} - 7(3a^5b^2 - a^6c)nx^{2n})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")

[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 + 15*a^2*b^2*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b^2*c^3)*n*x^(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b^2*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b^2*c)*n*x^(3*n) - 7*(3*a^5*b^2 - a^6*c)*n*x^(2*n))

giac [A] time = 0.79, size = 23, normalized size = 0.92

$$\frac{1}{7n(cx^{2n} + bx^n - a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(-a+b*xⁿ+c*x^(2*n))⁸,x, algorithm="giac")

[Out] -1/7/((c*x^(2*n) + b*xⁿ - a)^{7*n})

maple [A] time = 0.07, size = 24, normalized size = 0.96

$$\frac{1}{7(-bx^n - cx^{2n} + a)^7 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ)/(-a+b*xⁿ+c*x^(2*n))⁸,x)

[Out] 1/7/n/(-c*(xⁿ)²-b*xⁿ+a)⁷

maxima [B] time = 2.42, size = 419, normalized size = 16.76

$$7(c^7 n x^{14n} + 7bc^6 n x^{13n} + 7a^6 b n x^n - a^7 n + 7(3b^2 c^5 n - ac^6 n)x^{12n} + 7(5b^3 c^4 n - 6abc^5 n)x^{11n} + 7(5b^4 c^3 n -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)/(-a+b*xⁿ+c*x^(2*n))⁸,x, algorithm="maxima")

[Out] -1/7/(c^{7*n}*x^(14*n) + 7*b*c^{6*n}*x^(13*n) + 7*a⁶*b*n*xⁿ - a^{7*n} + 7*(3*b²*c^{5*n} - a*c^{6*n})*x^(12*n) + 7*(5*b³*c^{4*n} - 6*a*b*c^{5*n})*x^(11*n) + 7*(5*b⁴*c^{3*n} - 15*a*b²*c^{4*n} + 3*a²*c^{5*n})*x^(10*n) + 7*(3*b⁵*c^{2*n} - 20*a*b³*c^{3*n} + 15*a²*b*c^{4*n})*x^(9*n) + 7*(b⁶*c*n - 15*a*b⁴*c^{2*n} + 30*a²*b²*c^{3*n} - 5*a³*c^{4*n})*x^(8*n) + (b⁷*n - 42*a*b⁵*c*n + 210*a²*b³*c²*n - 140*a³*b*c³*n)*x^(7*n) - 7*(a*b⁶*n - 15*a²*b⁴*c*n + 30*a³*b²*c²*n - 5*a⁴*c³*n)*x^(6*n) + 7*(3*a²*b⁵*n - 20*a³*b³*c*n + 15*a⁴*b*c²*n)*x^(5*n) - 7*(5*a³*b⁴*n - 15*a⁴*b²*c*n + 3*a⁵*c²*n)*x^(4*n) + 7*(5*a⁴*b³*n - 6*a⁵*b*c*n)*x^(3*n) - 7*(3*a⁵*b²*n - a⁶*c*n)*x^(2*n))

mupad [B] time = 22.40, size = 496, normalized size = 19.84

$$7b^7 n x^{7n} - 7a^7 n + 7c^7 n x^{14n} + 49a^6 b n x^n - 49a^6 b^6 n x^{6n} + 49a^6 c n x^{2n} - 49a^6 c^6 n x^{12n} + 49b^6 c n x^{8n} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ))/(b*xⁿ-a+c*x^(2*n))⁸,x)

[Out] -1/(7*b⁷*n*x^(7*n) - 7*a⁷*n + 7*c⁷*n*x^(14*n) + 49*a⁶*b*n*xⁿ - 49*a*b⁶*n*x^(6*n) + 49*a⁶*c*n*x^(2*n) - 49*a*c⁶*n*x^(12*n) + 49*b⁶*c*n*x^(8*n))

$$\begin{aligned}
& + 49*b*c^6*n*x^{(13*n)} - 147*a^5*b^2*n*x^{(2*n)} + 245*a^4*b^3*n*x^{(3*n)} - 24 \\
& 5*a^3*b^4*n*x^{(4*n)} + 147*a^2*b^5*n*x^{(5*n)} - 147*a^5*c^2*n*x^{(4*n)} + 245*a \\
& ^4*c^3*n*x^{(6*n)} - 245*a^3*c^4*n*x^{(8*n)} + 147*a^2*c^5*n*x^{(10*n)} + 147*b^5 \\
& *c^2*n*x^{(9*n)} + 245*b^4*c^3*n*x^{(10*n)} + 245*b^3*c^4*n*x^{(11*n)} + 147*b^2* \\
& c^5*n*x^{(12*n)} + 735*a^4*b^2*c*n*x^{(4*n)} - 980*a^3*b^3*c*n*x^{(5*n)} + 735*a^ \\
& 4*b*c^2*n*x^{(5*n)} + 735*a^2*b^4*c*n*x^{(6*n)} - 980*a^3*b*c^3*n*x^{(7*n)} - 735 \\
& *a*b^4*c^2*n*x^{(8*n)} - 980*a*b^3*c^3*n*x^{(9*n)} + 735*a^2*b*c^4*n*x^{(9*n)} - \\
& 735*a*b^2*c^4*n*x^{(10*n)} - 1470*a^3*b^2*c^2*n*x^{(6*n)} + 1470*a^2*b^3*c^2*n* \\
& x^{(7*n)} + 1470*a^2*b^2*c^3*n*x^{(8*n)} - 294*a^5*b*c*n*x^{(3*n)} - 294*a*b^5*c* \\
& n*x^{(7*n)} - 294*a*b*c^5*n*x^{(11*n)}
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)

[Out] Timed out

$$3.121 \quad \int \frac{b+2cx}{bx+cx^2} dx$$

Optimal. Leaf size=10

$$\log(bx + cx^2)$$

[Out] $\ln(c*x^2+b*x)$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {628}

$$\log(bx + cx^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b + 2*c*x)/(b*x + c*x^2), x]$

[Out] $\text{Log}[b*x + c*x^2]$

Rule 628

$\text{Int}[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] \text{ :> S imp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] \text{ /; FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\int \frac{b + 2cx}{bx + cx^2} dx = \log(bx + cx^2)$$

Mathematica [A] time = 0.00, size = 9, normalized size = 0.90

$$\log(b + cx) + \log(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(b + 2*c*x)/(b*x + c*x^2), x]$

[Out] $\text{Log}[x] + \text{Log}[b + c*x]$

fricas [A] time = 0.81, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")

[Out] log(c*x^2 + b*x)

giac [A] time = 0.46, size = 11, normalized size = 1.10

$$\log(|cx^2 + bx|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")

[Out] log(abs(c*x^2 + b*x))

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$\ln((cx + b)x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x),x)

[Out] ln(x*(c*x+b))

maxima [A] time = 0.44, size = 10, normalized size = 1.00

$$\log(cx^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")

[Out] log(c*x^2 + b*x)

mupad [B] time = 0.05, size = 8, normalized size = 0.80

$$\ln(x(b + cx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b + 2*c*x)/(b*x + c*x^2),x)

[Out] log(x*(b + c*x))

sympy [A] time = 0.12, size = 8, normalized size = 0.80

$$\log(bx + cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x**2+b*x),x)

[Out] log(b*x + c*x**2)

$$3.122 \quad \int \frac{x(b+2cx^2)}{bx^2+cx^4} dx$$

Optimal. Leaf size=16

$$\frac{1}{2} \log(bx^2 + cx^4)$$

[Out] 1/2*ln(c*x^4+b*x^2)

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 72}

$$\frac{1}{2} \log(b + cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx &= \int \frac{b+2cx^2}{x(b+cx^2)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b+cx} \right) dx, x, x^2 \right) \\
&= \log(x) + \frac{1}{2} \log(b+cx^2)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{2} \log(b+cx^2) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4), x]

[Out] Log[x] + Log[b + c*x^2]/2

fricas [A] time = 0.84, size = 13, normalized size = 0.81

$$\frac{1}{2} \log(cx^2 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2), x, algorithm="fricas")

[Out] 1/2*log(c*x^2 + b) + log(x)

giac [A] time = 0.49, size = 15, normalized size = 0.94

$$\frac{1}{2} \log(|cx^4 + bx^2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2), x, algorithm="giac")

[Out] 1/2*log(abs(c*x^4 + b*x^2))

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x) + \frac{\ln(cx^2 + b)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x)`

[Out] `ln(x)+1/2*ln(c*x^2+b)`

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{2} \log(cx^2 + b) + \frac{1}{2} \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")`

[Out] `1/2*log(c*x^2 + b) + 1/2*log(x^2)`

mupad [B] time = 0.06, size = 13, normalized size = 0.81

$$\frac{\ln(cx^2 + b)}{2} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)`

[Out] `log(b + c*x^2)/2 + log(x)`

sympy [A] time = 0.19, size = 12, normalized size = 0.75

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)`

[Out] `log(x) + log(b/c + x**2)/2`

$$3.123 \quad \int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx$$

Optimal. Leaf size=16

$$\frac{1}{3} \log(bx^3 + cx^6)$$

[Out] 1/3*ln(c*x^6+b*x^3)

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1584, 446, 72}

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]

[Out] Log[x] + Log[b + c*x^3]/3

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^2(b + 2cx^3)}{bx^3 + cx^6} dx &= \int \frac{b + 2cx^3}{x(b + cx^3)} dx \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{b + 2cx}{x(b + cx)} dx, x, x^3 \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(\frac{1}{x} + \frac{c}{b + cx} \right) dx, x, x^3 \right) \\
&= \log(x) + \frac{1}{3} \log(b + cx^3)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.94

$$\frac{1}{3} \log(b + cx^3) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]

[Out] Log[x] + Log[b + c*x^3]/3

fricas [A] time = 0.84, size = 13, normalized size = 0.81

$$\frac{1}{3} \log(cx^3 + b) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")

[Out] 1/3*log(c*x^3 + b) + log(x)

giac [A] time = 0.34, size = 15, normalized size = 0.94

$$\frac{1}{3} \log(|cx^6 + bx^3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")

[Out] 1/3*log(abs(c*x^6 + b*x^3))

maple [A] time = 0.01, size = 14, normalized size = 0.88

$$\ln(x) + \frac{\ln(cx^3 + b)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x)`

[Out] `ln(x)+1/3*ln(c*x^3+b)`

maxima [A] time = 0.43, size = 17, normalized size = 1.06

$$\frac{1}{3} \log(cx^3 + b) + \frac{1}{3} \log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")`

[Out] `1/3*log(c*x^3 + b) + 1/3*log(x^3)`

mupad [B] time = 1.99, size = 13, normalized size = 0.81

$$\frac{\ln(cx^3 + b)}{3} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)`

[Out] `log(b + c*x^3)/3 + log(x)`

sympy [A] time = 0.20, size = 12, normalized size = 0.75

$$\log(x) + \frac{\log\left(\frac{b}{c} + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)`

[Out] `log(x) + log(b/c + x**3)/3`

$$3.124 \quad \int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx$$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

[Out] ln(x)+ln(b+c*x^n)/n

Rubi [A] time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 72}

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}} dx &= \int \frac{b+2cx^n}{x(b+cx^n)} dx \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x} + \frac{c}{b+cx}\right) dx, x, x^n\right)}{n} \\
&= \log(x) + \frac{\log(b+cx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1+n)*(b+2*c*x^n))/(b*x^n+c*x^(2*n)),x]

[Out] Log[x] + Log[b + c*x^n]/n

fricas [A] time = 0.87, size = 17, normalized size = 1.13

$$\frac{n \log(x) + \log(cx^n + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] (n*log(x) + log(c*x^n + b))/n

giac [A] time = 0.37, size = 17, normalized size = 1.13

$$\frac{\log(|cx^n + b|)}{n} + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] log(abs(c*x^n + b))/n + log(abs(x))

maple [A] time = 0.02, size = 18, normalized size = 1.20

$$\ln(x) + \frac{\ln(c e^{n \ln(x)} + b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x)`

[Out] `ln(x)+1/n*ln(c*exp(n*ln(x))+b)`

maxima [B] time = 0.44, size = 47, normalized size = 3.13

$$b \left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn} \right) + \frac{2 \log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")`

[Out] `b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n`

mupad [B] time = 2.23, size = 28, normalized size = 1.87

$$\frac{2 \left(\ln(b + c x^n) - \operatorname{atanh}\left(\frac{2 c x^n}{b} + 1\right) \right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(n-1)*(b+2*c*x^n))/(b*x^n+c*x^(2*n)),x)`

[Out] `(2*(log(b+c*x^n)-atanh((2*c*x^n)/b+1)))/n`

sympy [A] time = 31.23, size = 48, normalized size = 3.20

$$\begin{cases} \log(x) & \text{for } c = 0 \wedge n = 0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n = 0 \\ \frac{n^2 \log(x)}{n^2-n} - \frac{n \log(x)}{n^2-n} & \text{for } c = 0 \\ \log(x) + \frac{\log\left(\frac{b}{c} + x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0)), (n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

$$3.125 \quad \int \frac{b+2cx}{(bx+cx^2)^8} dx$$

Optimal. Leaf size=15

$$-\frac{1}{7(bx+cx^2)^7}$$

[Out] -1/7/(c*x^2+b*x)^7

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$-\frac{1}{7(bx+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/(7*(b*x + c*x^2)^7)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{b+2cx}{(bx+cx^2)^8} dx = -\frac{1}{7(bx+cx^2)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]

[Out] -1/7*1/(x^7*(b + c*x)^7)

fricas [B] time = 0.82, size = 81, normalized size = 5.40

$$\frac{1}{7(c^7x^{14} + 7bc^6x^{13} + 21b^2c^5x^{12} + 35b^3c^4x^{11} + 35b^4c^3x^{10} + 21b^5c^2x^9 + 7b^6cx^8 + b^7x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")

[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)

giac [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")

[Out] -1/7/(c*x^2 + b*x)^7

maple [B] time = 0.02, size = 177, normalized size = 11.80

$$\frac{c^7}{7(cx+b)^7b^7} + \frac{c^7}{(cx+b)^6b^8} + \frac{4c^7}{(cx+b)^5b^9} + \frac{12c^7}{(cx+b)^4b^{10}} + \frac{30c^7}{(cx+b)^3b^{11}} + \frac{66c^7}{(cx+b)^2b^{12}} + \frac{132c^7}{(cx+b)b^{13}} - \frac{132c^6}{b^{13}x} + \frac{66c^5}{b^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)

[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b^9*c^2/x^5+1/b^8*c/x^6+132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7

maxima [A] time = 0.42, size = 13, normalized size = 0.87

$$\frac{1}{7(cx^2 + bx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")

[Out] $-1/7/(c*x^2 + b*x)^7$

mupad [B] time = 4.30, size = 12, normalized size = 0.80

$$-\frac{1}{7x^7(b+cx)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x)/(b*x + c*x^2)^8, x)`

[Out] $-1/(7*x^7*(b + c*x)^7)$

sympy [B] time = 0.88, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)/(c*x**2+b*x)**8, x)`

[Out] $-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**14)$

$$3.126 \quad \int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

[Out] -1/14/x^14/(c*x^2+b)^7

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]

[Out] -1/(14*x^14*(b + c*x^2)^7)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol]
:> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx &= \int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^2 \right) \\ &= -\frac{1}{14x^{14}(b+cx^2)^7} \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]

[Out] -1/14*1/(x^14*(b + c*x^2)^7)

fricas [B] time = 0.88, size = 81, normalized size = 5.06

$$-\frac{1}{14(c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")

[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)

giac [A] time = 0.45, size = 15, normalized size = 0.94

$$-\frac{1}{14(cx^4 + bx^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")

[Out] -1/14/(c*x^4 + b*x^2)^7

maple [B] time = 0.02, size = 197, normalized size = 12.31

$$\frac{\left(-\frac{b^6}{7(c x^2+b)^7 c} - \frac{b^5}{(c x^2+b)^6 c} - \frac{4b^4}{(c x^2+b)^5 c} - \frac{12b^3}{(c x^2+b)^4 c} - \frac{30b^2}{(c x^2+b)^3 c} - \frac{66b}{(c x^2+b)^2 c} - \frac{132}{(c x^2+b)c} \right) c^8}{2b^{13}} - \frac{66c^6}{b^{13}x^2} + \frac{33c^5}{b^{12}x^4} - \frac{15c^4}{b^{11}x^6} + \frac{6c^3}{b^{10}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x)

[Out] $-1/14/b^7/x^{14}-66/b^{13}*c^6/x^2+33/b^{12}*c^5/x^4-15/b^{11}*c^4/x^6+6/b^{10}*c^3/x^8-2/b^9*c^2/x^{10}+1/2/b^8*c/x^{12}-1/2*c^8/b^{13}*(-12*b^3/c/(c*x^2+b)^4-30*b^2/c/(c*x^2+b)^3-132/c/(c*x^2+b)-b^5/c/(c*x^2+b)^6-4*b^4/c/(c*x^2+b)^5-66*b/c/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^7)$

maxima [B] time = 0.52, size = 81, normalized size = 5.06

$$\frac{1}{14(c^7x^{28} + 7bc^6x^{26} + 21b^2c^5x^{24} + 35b^3c^4x^{22} + 35b^4c^3x^{20} + 21b^5c^2x^{18} + 7b^6cx^{16} + b^7x^{14})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")

[Out] $-1/14/(c^7*x^{28} + 7*b*c^6*x^{26} + 21*b^2*c^5*x^{24} + 35*b^3*c^4*x^{22} + 35*b^4*c^3*x^{20} + 21*b^5*c^2*x^{18} + 7*b^6*c*x^{16} + b^7*x^{14})$

mupad [B] time = 2.33, size = 14, normalized size = 0.88

$$-\frac{1}{14x^{14}(cx^2+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)

[Out] $-1/(14*x^{14}*(b + c*x^2)^7)$

sympy [B] time = 1.38, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)

[Out] $-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**7*x**28)$

$$3.127 \quad \int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8} dx$$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

[Out] -1/21/x^21/(c*x^3+b)^7

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]

[Out] -1/(21*x^21*(b + c*x^3)^7)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2 (b + 2cx^3)}{(bx^3 + cx^6)^8} dx &= \int \frac{b + 2cx^3}{x^{22} (b + cx^3)^8} dx \\ &= \frac{1}{3} \text{Subst} \left(\int \frac{b + 2cx}{x^8 (b + cx)^8} dx, x, x^3 \right) \\ &= -\frac{1}{21x^{21} (b + cx^3)^7} \end{aligned}$$

Mathematica [A] time = 0.04, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21} (b + cx^3)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]

[Out] -1/21*1/(x^21*(b + c*x^3)^7)

fricas [B] time = 0.81, size = 81, normalized size = 5.06

$$-\frac{1}{21 (c^7 x^{42} + 7 b c^6 x^{39} + 21 b^2 c^5 x^{36} + 35 b^3 c^4 x^{33} + 35 b^4 c^3 x^{30} + 21 b^5 c^2 x^{27} + 7 b^6 c x^{24} + b^7 x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")

[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)

giac [A] time = 0.61, size = 15, normalized size = 0.94

$$-\frac{1}{21 (cx^6 + bx^3)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")

[Out] -1/21/(c*x^6 + b*x^3)^7

maple [B] time = 0.01, size = 197, normalized size = 12.31

$$\frac{\left(\frac{b^6}{7(c^3x^3+b)^7c} - \frac{b^5}{(c^3x^3+b)^6c} - \frac{4b^4}{(c^3x^3+b)^5c} - \frac{12b^3}{(c^3x^3+b)^4c} - \frac{30b^2}{(c^3x^3+b)^3c} - \frac{66b}{(c^3x^3+b)^2c} - \frac{132}{(c^3x^3+b)c} \right) c^8}{3b^{13}} - \frac{44c^6}{b^{13}x^3} + \frac{22c^5}{b^{12}x^6} - \frac{10c^4}{b^{11}x^9} + \frac{4c^3}{b^{10}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x)

[Out] $-1/21/b^7/x^{21}-44/b^{13}*c^6/x^3+22/b^{12}*c^5/x^6-10/b^{11}*c^4/x^9+4/b^{10}*c^3/x^{12}-4/3/b^9*c^2/x^{15}+1/3/b^8*c/x^{18}-1/3*c^8/b^{13}*(-12*b^3/c/(c*x^3+b)^4-30*b^2/c/(c*x^3+b)^3-132/c/(c*x^3+b)-b^5/c/(c*x^3+b)^6-4*b^4/c/(c*x^3+b)^5-66*b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^7)$

maxima [B] time = 0.51, size = 81, normalized size = 5.06

$$\frac{1}{21(c^7x^{42} + 7bc^6x^{39} + 21b^2c^5x^{36} + 35b^3c^4x^{33} + 35b^4c^3x^{30} + 21b^5c^2x^{27} + 7b^6cx^{24} + b^7x^{21})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")

[Out] $-1/21/(c^7*x^{42} + 7*b*c^6*x^{39} + 21*b^2*c^5*x^{36} + 35*b^3*c^4*x^{33} + 35*b^4*c^3*x^{30} + 21*b^5*c^2*x^{27} + 7*b^6*c*x^{24} + b^7*x^{21})$

mupad [B] time = 5.07, size = 14, normalized size = 0.88

$$-\frac{1}{21x^{21}(cx^3+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)

[Out] $-1/(21*x^{21}*(b + c*x^3)^7)$

sympy [B] time = 1.90, size = 87, normalized size = 5.44

$$\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)

[Out] $-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c**7*x**42)$

$$3.128 \quad \int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx$$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7

Rubi [A] time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol]
:> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+n} (b + 2cx^n)}{(bx^n + cx^{2n})^8} dx = \int \frac{x^{-1-7n} (b + 2cx^n)}{(b + cx^n)^8} dx$$

$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-7n}}{7n (b + cx^n)^7}$$

Mathematica [A] time = 0.18, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n (b + cx^n)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

fricas [B] time = 0.74, size = 105, normalized size = 5.00

$$\frac{1}{7(c^7 n x^{14n} + 7 b c^6 n x^{13n} + 21 b^2 c^5 n x^{12n} + 35 b^3 c^4 n x^{11n} + 35 b^4 c^3 n x^{10n} + 21 b^5 c^2 n x^{9n} + 7 b^6 c n x^{8n} + b^7 n x^{7n})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")

[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x^(8*n) + b^7*n*x^(7*n))

giac [A] time = 0.44, size = 20, normalized size = 0.95

$$-\frac{1}{7(cx^{2n} + bx^n)^7 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")

[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)

maple [B] time = 0.05, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n} + \frac{cx^{-6n}}{b^8n} - \frac{4c^2x^{-5n}}{b^9n} + \frac{12c^3x^{-4n}}{b^{10n}} - \frac{30c^4x^{-3n}}{b^{11n}} + \frac{66c^5x^{-2n}}{b^{12n}} + \frac{(9009b^5cx^n + 20020b^4c^2x^{2n} + 24024b^3c^3x^{3n} + 16380c^4x^{4n})}{7(cx^n + b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x)

[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7

maxima [B] time = 0.66, size = 612, normalized size = 29.14

$$-\frac{1}{105} b \left(\frac{360360 c^{13} x^{13n} + 2342340 bc^{12} x^{12n} + 6426420 b^2 c^{11} x^{11n} + 9579570 b^3 c^{10} x^{10n} + 8270262 b^4 c^9 x^{9n} + 4018014 b^5 c^8 x^{8n} + 934362 b^6 c^7 x^{7n} + 45045 b^7 c^6 x^{6n} - 5005 b^8 c^5 x^{5n} + 1001 b^9 c^4 x^{4n} - 273 b^{10} c^3 x^{3n} + 91 b^{11} c^2 x^{2n} - 35 b^{12} c x^n + 15 b^{13}}{b^{14} c^7 n x^{14n} + 7 b^{15} c^6 n x^{13n} + 21 b^{16} c^5 n x^{12n} + \dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")

[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n + b)/c)/(b^14*n))

mupad [B] time = 2.36, size = 107, normalized size = 5.10

$$\frac{1}{7b^7nx^{7n} + 7c^7nx^{14n} + 49b^6cnx^{8n} + 49bc^6nx^{13n} + 147b^5c^2nx^{9n} + 245b^4c^3nx^{10n} + 245b^3c^4nx^{11n} + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8, x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*c^5*n*x^(12*n))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8, x)
```

```
[Out] Timed out
```

$$3.129 \quad \int (b + 2cx) (a + bx + cx^2)^p dx$$

Optimal. Leaf size=20

$$\frac{(a + bx + cx^2)^{p+1}}{p + 1}$$

[Out] (c*x^2+b*x+a)^(1+p)/(1+p)

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {629}

$$\frac{(a + bx + cx^2)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]

[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (a + bx + cx^2)^p dx = \frac{(a + bx + cx^2)^{1+p}}{1 + p}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{(a + x(b + cx))^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]

[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)

fricas [A] time = 0.97, size = 28, normalized size = 1.40

$$\frac{(cx^2 + bx + a)(cx^2 + bx + a)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)

giac [A] time = 0.41, size = 20, normalized size = 1.00

$$\frac{(cx^2 + bx + a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")

[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 21, normalized size = 1.05

$$\frac{(cx^2 + bx + a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x)

[Out] (c*x^2+b*x+a)^(p+1)/(p+1)

maxima [A] time = 0.42, size = 20, normalized size = 1.00

$$\frac{(cx^2 + bx + a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")

[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)

mupad [B] time = 2.04, size = 39, normalized size = 1.95

$$\left(\frac{a}{p+1} + \frac{bx}{p+1} + \frac{cx^2}{p+1}\right)(cx^2 + bx + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x)*(a + b*x + c*x^2)^p, x)`

[Out] `(a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p`

sympy [B] time = 57.11, size = 104, normalized size = 5.20

$$\begin{cases} \frac{a(a+bx+cx^2)^p}{p+1} + \frac{bx(a+bx+cx^2)^p}{p+1} + \frac{cx^2(a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)*(c*x**2+b*x+a)**p, x)`

[Out] `Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p + 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x - sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)), True))`

$$3.130 \quad \int x (b + 2cx^2) (a + bx^2 + cx^4)^p dx$$

Optimal. Leaf size=25

$$\frac{(a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1247, 629}

$$\frac{(a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]

[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (b + 2cx^2) (a + bx^2 + cx^4)^p dx &= \frac{1}{2} \text{Subst} \left(\int (b + 2cx) (a + bx + cx^2)^p dx, x, x^2 \right) \\ &= \frac{(a + bx^2 + cx^4)^{1+p}}{2(1+p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]

[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))

fricas [A] time = 0.76, size = 33, normalized size = 1.32

$$\frac{(cx^4 + bx^2 + a)(cx^4 + bx^2 + a)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)

giac [A] time = 0.46, size = 23, normalized size = 0.92

$$\frac{(cx^4 + bx^2 + a)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")

[Out] 1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{(cx^4 + bx^2 + a)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x)

[Out] 1/2*(c*x^4+b*x^2+a)^(p+1)/(p+1)

maxima [A] time = 0.60, size = 33, normalized size = 1.32

$$\frac{(cx^4 + bx^2 + a)(cx^4 + bx^2 + a)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)

mupad [B] time = 2.09, size = 49, normalized size = 1.96

$$(cx^4 + bx^2 + a)^p \left(\frac{a}{2p+2} + \frac{bx^2}{2p+2} + \frac{cx^4}{2p+2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)

[Out] (a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)

[Out] Timed out

$$3.131 \quad \int x^2 (b + 2cx^3) (a + bx^3 + cx^6)^p dx$$

Optimal. Leaf size=25

$$\frac{(a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1468, 629}

$$\frac{(a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]

[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (b + 2cx^3) (a + bx^3 + cx^6)^p dx &= \frac{1}{3} \text{Subst} \left(\int (b + 2cx) (a + bx + cx^2)^p dx, x, x^3 \right) \\ &= \frac{(a + bx^3 + cx^6)^{1+p}}{3(1+p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{(a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]

[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

fricas [A] time = 0.79, size = 33, normalized size = 1.32

$$\frac{(cx^6 + bx^3 + a)(cx^6 + bx^3 + a)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")

[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)

giac [A] time = 0.32, size = 23, normalized size = 0.92

$$\frac{(cx^6 + bx^3 + a)^{p+1}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")

[Out] 1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)

maple [A] time = 0.01, size = 24, normalized size = 0.96

$$\frac{(cx^6 + bx^3 + a)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x)

[Out] 1/3*(c*x^6+b*x^3+a)^(p+1)/(p+1)

maxima [A] time = 0.58, size = 33, normalized size = 1.32

$$\frac{(cx^6 + bx^3 + a)(cx^6 + bx^3 + a)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)

mupad [B] time = 2.12, size = 49, normalized size = 1.96

$$(cx^6 + bx^3 + a)^p \left(\frac{a}{3p+3} + \frac{bx^3}{3p+3} + \frac{cx^6}{3p+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)

[Out] (a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)

[Out] Timed out

$$3.132 \quad \int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=27

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

[Out] (a+b*x^n+c*x^(2*n))^(1+p)/n/(1+p)

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1468, 629}

$$\frac{(a + bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)]^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^{-1+n} (b + 2cx^n) (a + bx^n + cx^{2n})^p dx &= \frac{\text{Subst}\left(\int (b + 2cx) (a + bx + cx^2)^p dx, x, x^n\right)}{n} \\ &= \frac{(a + bx^n + cx^{2n})^{1+p}}{n(1+p)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{(a + x^n (b + cx^n))^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(a+b*xⁿ+c*x^(2*n))^p,x]

[Out] (a+xⁿ*(b+c*xⁿ))^(1+p)/(n*(1+p))

fricas [A] time = 0.88, size = 38, normalized size = 1.41

$$\frac{(cx^{2n} + bx^n + a)(cx^{2n} + bx^n + a)^p}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(a+b*xⁿ+c*x^(2*n))^p,x, algorithm="fricas")

[Out] (c*x^(2*n) + b*xⁿ + a)*(c*x^(2*n) + b*xⁿ + a)^p/(n*p + n)

giac [A] time = 0.84, size = 27, normalized size = 1.00

$$\frac{(cx^{2n} + bx^n + a)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(a+b*xⁿ+c*x^(2*n))^p,x, algorithm="giac")

[Out] (c*x^(2*n) + b*xⁿ + a)^(p+1)/(n*(p+1))

maple [A] time = 0.06, size = 40, normalized size = 1.48

$$\frac{(bx^n + cx^{2n} + a)(bx^n + cx^{2n} + a)^p}{(p+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x⁽ⁿ⁻¹⁾*(b+2*c*xⁿ)*(b*xⁿ+c*x^(2*n)+a)^p,x)

[Out] (a+b*xⁿ+c*(xⁿ)²)/(p+1)/n*(a+b*xⁿ+c*(xⁿ)²)^p

maxima [A] time = 0.72, size = 39, normalized size = 1.44

$$\frac{(cx^{2n} + bx^n + a)(cx^{2n} + bx^n + a)^p}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(a+b*xⁿ+c*x^(2*n))^p,x, algorithm="maxima")

[Out] (c*x^(2*n) + b*xⁿ + a)*(c*x^(2*n) + b*xⁿ + a)^p/(n*(p + 1))

mupad [B] time = 2.57, size = 56, normalized size = 2.07

$$(a + bx^n + cx^{2n})^p \left(\frac{a}{n(p+1)} + \frac{bx^n}{n(p+1)} + \frac{cx^{2n}}{n(p+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n - 1)*(b + 2*c*xⁿ)*(a + b*xⁿ + c*x^(2*n))^p,x)

[Out] (a + b*xⁿ + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*xⁿ)/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+n)}*(b+2*c*x^{**n})*(a+b*x^{**n}+c*x^{**2}*(2*n))^{**p},x)

[Out] Timed out

$$3.133 \quad \int (b + 2cx) (-a + bx + cx^2)^p dx$$

Optimal. Leaf size=22

$$\frac{(-a + bx + cx^2)^{p+1}}{p + 1}$$

[Out] (c*x^2+b*x-a)^(1+p)/(1+p)

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {629}

$$\frac{(-a + bx + cx^2)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]

[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (-a + bx + cx^2)^p dx = \frac{(-a + bx + cx^2)^{1+p}}{1 + p}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$\frac{(x(b + cx) - a)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]

[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)

fricas [A] time = 0.86, size = 32, normalized size = 1.45

$$\frac{(cx^2 + bx - a)(cx^2 + bx - a)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)

giac [A] time = 0.40, size = 22, normalized size = 1.00

$$\frac{(cx^2 + bx - a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")

[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(cx^2 + bx - a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x)

[Out] (c*x^2+b*x-a)^(p+1)/(p+1)

maxima [A] time = 0.42, size = 22, normalized size = 1.00

$$\frac{(cx^2 + bx - a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")

[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)

mupad [B] time = 2.05, size = 42, normalized size = 1.91

$$\left(\frac{bx}{p+1} - \frac{a}{p+1} + \frac{cx^2}{p+1}\right)(cx^2 + bx - a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)`

[Out] `((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p`

sympy [B] time = 56.66, size = 104, normalized size = 4.73

$$\begin{cases} -\frac{a(-a+bx+cx^2)^p}{p+1} + \frac{bx(-a+bx+cx^2)^p}{p+1} + \frac{cx^2(-a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)`

[Out] `Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c)), True))`

$$3.134 \quad \int x (b + 2cx^2) (-a + bx^2 + cx^4)^p dx$$

Optimal. Leaf size=27

$$\frac{(-a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1247, 629}

$$\frac{(-a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]

[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1247

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x (b + 2cx^2) (-a + bx^2 + cx^4)^p dx &= \frac{1}{2} \text{Subst} \left(\int (b + 2cx) (-a + bx + cx^2)^p dx, x, x^2 \right) \\ &= \frac{(-a + bx^2 + cx^4)^{1+p}}{2(1+p)} \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{(-a + bx^2 + cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]

[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))

fricas [A] time = 0.91, size = 37, normalized size = 1.37

$$\frac{(cx^4 + bx^2 - a)(cx^4 + bx^2 - a)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)

giac [A] time = 0.45, size = 25, normalized size = 0.93

$$\frac{(cx^4 + bx^2 - a)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")

[Out] 1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{(cx^4 + bx^2 - a)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x)

[Out] 1/2*(c*x^4+b*x^2-a)^(p+1)/(p+1)

maxima [A] time = 0.60, size = 37, normalized size = 1.37

$$\frac{(cx^4 + bx^2 - a)(cx^4 + bx^2 - a)^p}{2(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)

mupad [B] time = 2.05, size = 52, normalized size = 1.93

$$(cx^4 + bx^2 - a)^p \left(\frac{bx^2}{2p+2} - \frac{a}{2p+2} + \frac{cx^4}{2p+2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)

[Out] (b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)

[Out] Timed out

$$3.135 \quad \int x^2 (b + 2cx^3) (-a + bx^3 + cx^6)^p dx$$

Optimal. Leaf size=27

$$\frac{(-a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1468, 629}

$$\frac{(-a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int x^2 (b + 2cx^3) (-a + bx^3 + cx^6)^p dx &= \frac{1}{3} \text{Subst} \left(\int (b + 2cx) (-a + bx + cx^2)^p dx, x, x^3 \right) \\ &= \frac{(-a + bx^3 + cx^6)^{1+p}}{3(1+p)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{(-a + bx^3 + cx^6)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]

[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

fricas [A] time = 0.60, size = 37, normalized size = 1.37

$$\frac{(cx^6 + bx^3 - a)(cx^6 + bx^3 - a)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")

[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)

giac [A] time = 0.39, size = 25, normalized size = 0.93

$$\frac{(cx^6 + bx^3 - a)^{p+1}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")

[Out] 1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)

maple [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{(cx^6 + bx^3 - a)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x)

[Out] 1/3*(c*x^6+b*x^3-a)^(p+1)/(p+1)

maxima [A] time = 0.59, size = 37, normalized size = 1.37

$$\frac{(cx^6 + bx^3 - a)(cx^6 + bx^3 - a)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)

mupad [B] time = 2.08, size = 52, normalized size = 1.93

$$(cx^6 + bx^3 - a)^p \left(\frac{bx^3}{3p+3} - \frac{a}{3p+3} + \frac{cx^6}{3p+3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)

[Out] (b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)

[Out] Timed out

$$3.136 \quad \int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=29

$$\frac{(-a + bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

[Out] $(-a+b*x^n+c*x^{(2*n)})^{(1+p)}/n/(1+p)$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1468, 629}

$$\frac{(-a + bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(-1+n)}*(b+2*c*x^n)*(-a+b*x^n+c*x^{(2*n)})^p,x]$

[Out] $(-a+b*x^n+c*x^{(2*n)})^{(1+p)}/(n*(1+p))$

Rule 629

$\text{Int}[(d_+ + (e_+)*(x_+))*((a_+ + (b_+)*(x_+) + (c_+)*(x_+)^2)^{(p_+)}), x_Symbol] \rightarrow \text{Simp}[(d*(a + b*x + c*x^2)^{(p+1)})/(b*(p+1)), x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 1468

$\text{Int}[(x_+)^{(m_+)}*((a_+ + (c_+)*(x_+)^{(n2_+)} + (b_+)*(x_+)^{(n_+}))^{(p_+)}*((d_+ + (e_+)*(x_+)^{(n_+}))^{(q_+)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{EqQ}[\text{Simplify}[m - n + 1], 0]$

Rubi steps

$$\begin{aligned} \int x^{-1+n} (b + 2cx^n) (-a + bx^n + cx^{2n})^p dx &= \frac{\text{Subst}\left(\int (b + 2cx) (-a + bx + cx^2)^p dx, x, x^n\right)}{n} \\ &= \frac{(-a + bx^n + cx^{2n})^{1+p}}{n(1+p)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.97

$$\frac{(x^n (b + cx^n) - a)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]

[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))

fricas [A] time = 0.93, size = 42, normalized size = 1.45

$$\frac{(cx^{2n} + bx^n - a)(cx^{2n} + bx^n - a)^p}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)

giac [A] time = 0.82, size = 29, normalized size = 1.00

$$\frac{(cx^{2n} + bx^n - a)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")

[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))

maple [A] time = 0.06, size = 45, normalized size = 1.55

$$\frac{(-bx^n - cx^{2n} + a)(bx^n + cx^{2n} - a)^p}{(p+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x)

[Out] -(-c*(x^n)^2-b*x^n+a)/(p+1)/n*(-a+b*x^n+c*(x^n)^2)^p

maxima [A] time = 0.71, size = 43, normalized size = 1.48

$$\frac{(cx^{2n} + bx^n - a)(cx^{2n} + bx^n - a)^p}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x⁽⁻¹⁺ⁿ⁾*(b+2*c*xⁿ)*(-a+b*xⁿ+c*x^(2*n))^p,x, algorithm="maxima")

[Out] (c*x^(2*n) + b*xⁿ - a)*(c*x^(2*n) + b*xⁿ - a)^p/(n*(p + 1))

mupad [B] time = 2.54, size = 59, normalized size = 2.03

$$\left(\frac{bx^n}{n(p+1)} - \frac{a}{n(p+1)} + \frac{cx^{2n}}{n(p+1)} \right) (bx^n - a + cx^{2n})^p$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n - 1)*(b + 2*c*xⁿ)*(b*xⁿ - a + c*x^(2*n))^p,x)

[Out] ((b*xⁿ)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*xⁿ - a + c*x^(2*n))^p

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{**(-1+n)}*(b+2*c*x^{**n})*(-a+b*x^{**n}+c*x^{** (2*n)})^{**p},x)

[Out] Timed out

$$3.137 \quad \int (b + 2cx) (bx + cx^2)^p dx$$

Optimal. Leaf size=19

$$\frac{(bx + cx^2)^{p+1}}{p + 1}$$

[Out] (c*x^2+b*x)^(1+p)/(1+p)

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {629}

$$\frac{(bx + cx^2)^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (b*x + c*x^2)^(1 + p)/(1 + p)

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int (b + 2cx) (bx + cx^2)^p dx = \frac{(bx + cx^2)^{1+p}}{1 + p}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{(x(b + cx))^{p+1}}{p + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]

[Out] (x*(b + c*x))^(1 + p)/(1 + p)

fricas [A] time = 0.81, size = 26, normalized size = 1.37

$$\frac{(cx^2 + bx)(cx^2 + bx)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")

[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)

giac [A] time = 0.41, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")

[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$\frac{(cx + b)x(c x^2 + bx)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)

[Out] (c*x+b)*x/(p+1)*(c*x^2+b*x)^p

maxima [A] time = 0.43, size = 19, normalized size = 1.00

$$\frac{(cx^2 + bx)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")

[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)

mupad [B] time = 2.03, size = 23, normalized size = 1.21

$$\frac{x(c x^2 + b x)^p (b + c x)}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x + c*x^2)^p*(b + 2*c*x), x)`

[Out] `(x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)`

sympy [A] time = 0.66, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*c*x+b)*(c*x**2+b*x)**p,x)`

[Out] `Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(x) + log(b/c + x), True))`

$$3.138 \quad \int x (b + 2cx^2) (bx^2 + cx^4)^p dx$$

Optimal. Leaf size=24

$$\frac{(bx^2 + cx^4)^{p+1}}{2(p+1)}$$

[Out] 1/2*(c*x^4+b*x^2)^(1+p)/(1+p)

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1588}

$$\frac{(bx^2 + cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]

[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x (b + 2cx^2) (bx^2 + cx^4)^p dx = \frac{(bx^2 + cx^4)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^2 (x^2 (b + cx^2))^p \left(\frac{cx^2}{b} + 1\right)^{-p} \left(2c(p+1)x^2 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^2}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]

[Out] $(x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*\text{Hypergeometric2F1}[-p, 1 + p, 2 + p, -((c*x^2)/b)] + 2*c*(1 + p)*x^2*\text{Hypergeometric2F1}[-p, 2 + p, 3 + p, -((c*x^2)/b)])))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)$

fricas [A] time = 0.76, size = 31, normalized size = 1.29

$$\frac{(cx^4 + bx^2)(cx^4 + bx^2)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")

[Out] $1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)$

giac [A] time = 0.40, size = 22, normalized size = 0.92

$$\frac{(cx^4 + bx^2)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")

[Out] $1/2*(c*x^4 + b*x^2)^{(p + 1)}/(p + 1)$

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{(cx^2 + b)x^2(cx^4 + bx^2)^p}{2p + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x)

[Out] $1/2*(c*x^2+b)*x^2/(p+1)*(c*x^4+b*x^2)^p$

maxima [A] time = 0.59, size = 35, normalized size = 1.46

$$\frac{(cx^4 + bx^2)e^{(p \log(cx^2+b)+2p \log(x))}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")

[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)

mupad [B] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^2 (cx^2 + b) (cx^4 + bx^2)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)

[Out] (x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))

sympy [B] time = 17.12, size = 85, normalized size = 3.54

$$\begin{cases} \frac{bx^2(bx^2+cx^4)^p}{2p+2} + \frac{cx^4(bx^2+cx^4)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x) + \frac{\log(-i\sqrt{b}\sqrt{\frac{1}{c}}+x)}{2} + \frac{\log(i\sqrt{b}\sqrt{\frac{1}{c}}+x)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)

[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/c) + x)/2 + log(I*sqrt(b)*sqrt(1/c) + x)/2, True))

$$3.139 \quad \int x^2 (b + 2cx^3) (bx^3 + cx^6)^p dx$$

Optimal. Leaf size=24

$$\frac{(bx^3 + cx^6)^{p+1}}{3(p+1)}$$

[Out] 1/3*(c*x^6+b*x^3)^(1+p)/(1+p)

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1588}

$$\frac{(bx^3 + cx^6)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]

[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int x^2 (b + 2cx^3) (bx^3 + cx^6)^p dx = \frac{(bx^3 + cx^6)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 4.04

$$\frac{x^3 (x^3 (b + cx^3))^p \left(\frac{cx^3}{b} + 1\right)^{-p} \left(2c(p+1)x^3 {}_2F_1\left(-p, p+2; p+3; -\frac{cx^3}{b}\right) + b(p+2) {}_2F_1\left(-p, p+1; p+2; -\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]

[Out] $(x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*\text{Hypergeometric2F1}[-p, 1 + p, 2 + p, -((c*x^3)/b)] + 2*c*(1 + p)*x^3*\text{Hypergeometric2F1}[-p, 2 + p, 3 + p, -((c*x^3)/b)]))/((3*(1 + p)*(2 + p)*(1 + (c*x^3)/b))^p)$

fricas [A] time = 0.73, size = 31, normalized size = 1.29

$$\frac{(cx^6 + bx^3)(cx^6 + bx^3)^p}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")

[Out] $1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)$

giac [A] time = 0.67, size = 22, normalized size = 0.92

$$\frac{(cx^6 + bx^3)^{p+1}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")

[Out] $1/3*(c*x^6 + b*x^3)^{(p + 1)}/(p + 1)$

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{(cx^3 + b)x^3(cx^6 + bx^3)^p}{3p + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x)

[Out] $1/3*(c*x^3+b)*x^3/(p+1)*(c*x^6+b*x^3)^p$

maxima [A] time = 0.59, size = 35, normalized size = 1.46

$$\frac{(cx^6 + bx^3)e^{(p \log(cx^3+b)+3p \log(x))}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")

[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)

mupad [B] time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^3 (cx^3 + b) (cx^6 + bx^3)^p}{3(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)

[Out] (x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)

[Out] Timed out

$$3.140 \quad \int x^{-1+n} (b + 2cx^n) (bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=26

$$\frac{(bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

[Out] (b*x^n+c*x^(2*n))^(1+p)/n/(1+p)

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2034, 629}

$$\frac{(bx^n + cx^{2n})^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]

[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 2034

Int[(x_)^(m_)*((b_)*(x_)^(k_) + (a_)*(x_)^(j_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\int x^{-1+n} (b + 2cx^n) (bx^n + cx^{2n})^p dx = \frac{\text{Subst}\left(\int (b + 2cx) (bx + cx^2)^p dx, x, x^n\right)}{n}$$

$$= \frac{(bx^n + cx^{2n})^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.13, size = 111, normalized size = 4.27

$$\frac{x^{-np} (x^n (b + cx^n))^p \left(\frac{cx^n}{b} + 1\right)^{-p} \left(b(p+2)x^{n(p+1)} {}_2F_1\left(-p, p+1; p+2; -\frac{cx^n}{b}\right) + 2c(p+1)x^{n(p+2)} {}_2F_1\left(-p, p+2; p+3; -\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^{−1+n}*(b+2*c*xⁿ)*(b*xⁿ+c*x^(2*n))^p,x]

[Out] ((xⁿ*(b+c*xⁿ))^p*(b*(2+p)*x^{(n*(1+p))}*Hypergeometric2F1[−p, 1+p, 2+p, −((c*xⁿ)/b)] + 2*c*(1+p)*x^{(n*(2+p))}*Hypergeometric2F1[−p, 2+p, 3+p, −((c*xⁿ)/b)]))/((n*(1+p)*(2+p)*x^(n*p)*(1+(c*xⁿ)/b)^p)

fricas [A] time = 0.64, size = 36, normalized size = 1.38

$$\frac{(cx^{2n} + bx^n)(cx^{2n} + bx^n)^p}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{−1+n}*(b+2*c*xⁿ)*(b*xⁿ+c*x^(2*n))^p,x, algorithm="fricas")

[Out] (c*x^(2*n) + b*xⁿ)*(c*x^(2*n) + b*xⁿ)^p/(n*p + n)

giac [A] time = 0.83, size = 26, normalized size = 1.00

$$\frac{(cx^{2n} + bx^n)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^{−1+n}*(b+2*c*xⁿ)*(b*xⁿ+c*x^(2*n))^p,x, algorithm="giac")

[Out] (c*x^(2*n) + b*xⁿ)^(p+1)/(n*(p+1))

maple [C] time = 0.11, size = 155, normalized size = 5.96

$$\frac{(cx^n + b)x^n e^{\frac{(-i\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(i(cx^n+b))) \operatorname{csgn}(i(cx^n+b)x^n) + i\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(i(cx^n+b)x^n)^2 + i\pi \operatorname{csgn}(i(cx^n+b)) \operatorname{csgn}(i(cx^n+b)x^n)^2 - i\pi \operatorname{csgn}(i(cx^n+b)x^n)^3 + 2 \ln(x^n))}{2}}}{(p+1)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)`

[Out] `x^n*(c*x^n+b)/(p+1)/n*exp(1/2*p*(-I*Pi*csgn(I*x^n*(c*x^n+b))^3+I*Pi*csgn(I*x^n*(c*x^n+b))^2*csgn(I*x^n)+I*Pi*csgn(I*x^n*(c*x^n+b))^2*csgn(I*(c*x^n+b))-I*Pi*csgn(I*x^n*(c*x^n+b))*csgn(I*x^n)*csgn(I*(c*x^n+b))+2*ln(x^n)+2*ln(c*x^n+b))`

maxima [A] time = 0.77, size = 40, normalized size = 1.54

$$\frac{(cx^{2n} + bx^n)e^{(p \log(cx^n+b) + p \log(x^n))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")`

[Out] `(c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))`

mupad [B] time = 2.13, size = 34, normalized size = 1.31

$$\frac{x^n (b + cx^n) (bx^n + cx^{2n})^p}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(n-1)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)`

[Out] `(x^n*(b+c*x^n)*(b*x^n+c*x^(2*n))^p)/(n*(p+1))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)`

[Out] Timed out

$$3.141 \quad \int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=196

$$\frac{(fx)^{m+1} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{f(m+1)(b-\sqrt{b^2-4ac})} + \frac{(fx)^{m+1} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)(\sqrt{b^2-4ac}+b)}$$

[Out] (f*x)^(1+m)*hypergeom([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(f*x)^(1+m)*hypergeom([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))

Rubi [A] time = 0.29, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1560, 364}

$$\frac{(fx)^{m+1} \left(\frac{2cd-be}{\sqrt{b^2-4ac}} + e \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right)}{f(m+1)(b-\sqrt{b^2-4ac})} + \frac{(fx)^{m+1} \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)(\sqrt{b^2-4ac}+b)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x]

[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(b - Sqrt[b^2 - 4*a*c])*f*(1 + m) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])]/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 1560

Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ

[q, 0])

Rubi steps

$$\begin{aligned}
\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}} dx &= \int \left(\frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (fx)^m}{b - \sqrt{b^2-4ac} + 2cx^n} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (fx)^m}{b + \sqrt{b^2-4ac} + 2cx^n} \right) dx \\
&= \left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \int \frac{(fx)^m}{b + \sqrt{b^2-4ac} + 2cx^n} dx + \left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) \int \frac{(fx)^m}{b - \sqrt{b^2-4ac} + 2cx^n} dx \\
&= \frac{\left(e + \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (fx)^{1+m} {}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{2cx^n}{b - \sqrt{b^2-4ac}}\right)}{\left(b - \sqrt{b^2-4ac}\right) f(1+m)} + \frac{\left(e - \frac{2cd-be}{\sqrt{b^2-4ac}}\right) (fx)^{1+m} {}_2F_1\left(1, \frac{1+m}{n}; \frac{1+m+n}{n}; -\frac{2cx^n}{b + \sqrt{b^2-4ac}}\right)}{\left(b + \sqrt{b^2-4ac}\right) f(1+m)}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 158, normalized size = 0.81

$$\frac{x(fx)^m \left(\left(d\sqrt{b^2-4ac} - 2ae + bd \right) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; \frac{2cx^n}{\sqrt{b^2-4ac}-b}\right) + \left(d\sqrt{b^2-4ac} + 2ae - bd \right) {}_2F_1\left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; \frac{2cx^n}{\sqrt{b^2-4ac}+b}\right) \right)}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]

[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{bx^n + cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a),x)

[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x)

[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)

[Out] Timed out

$$3.142 \quad \int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2} dx$$

Optimal. Leaf size=374

$$\frac{c(fx)^{m+1} \left((m-n+1)(bd-2ae) - \frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) c(fx)^{m+1}}{af(m+1)n(b^2-4ac) \left(b - \sqrt{b^2-4ac} \right)}$$

[Out] $(f*x)^{(1+m)}*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n/(a+b*x^n+c*x^{(2*n)})-c*(f*x)^{(1+m)}*hypergeom([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}))*((-2*a*e+b*d)*(1+m-n)+(-4*a*c*d*(1+m-2*n)+b^2*d*(1+m-n)-2*a*b*e*n)/(-4*a*c+b^2)^{(1/2)})/a/(-4*a*c+b^2)/f/(1+m)/n/(b-(-4*a*c+b^2)^{(1/2)})-c*(f*x)^{(1+m)}*hypergeom([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}))*((-2*a*e+b*d)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*b*e*n)/(-4*a*c+b^2)^{(1/2)})/a/(-4*a*c+b^2)/f/(1+m)/n/(b+(-4*a*c+b^2)^{(1/2)})$

Rubi [A] time = 1.38, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1558, 1560, 364}

$$\frac{c(fx)^{m+1} \left((m-n+1)(bd-2ae) - \frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}} \right) {}_2F_1 \left(1, \frac{m+1}{n}; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}} \right) c(fx)^{m+1}}{af(m+1)n(b^2-4ac) \left(b - \sqrt{b^2-4ac} \right)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]

[Out] $((f*x)^{(1+m)}*(b^2*d-2*a*c*d-a*b*e+c*(b*d-2*a*e)*x^n))/(a*(b^2-4*a*c)*f*n*(a+b*x^n+c*x^{(2*n)}))-c*((b*d-2*a*e)*(1+m-n)-(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*b*e*n)/Sqrt[b^2-4*a*c])*(f*x)^{(1+m)}*Hypergeometric2F1[1, (1+m)/n, (1+m+n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c])]/(a*(b^2-4*a*c)*(b-Sqrt[b^2-4*a*c])*f*(1+m)*n)-c*((b*d-2*a*e)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*b*e*n)/Sqrt[b^2-4*a*c])*(f*x)^{(1+m)}*Hypergeometric2F1[1, (1+m)/n, (1+m+n)/n, (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/(a*(b^2-4*a*c)*(b+Sqrt[b^2-4*a*c])*f*(1+m)*n)$

Rule 364

Int[((c_.)*(x_)^(m_.))*((a_.)+(b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a]]/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILT

$Q[p, 0] \parallel \text{GtQ}[a, 0]$

Rule 1558

$\text{Int}[(f_)(x_)]^{(m_)} * ((d_ + (e_)(x_)^{(n_)})) * ((a_ + (b_)(x_)^{(n_)} + (c_)(x_)^{(n2_)})^{(p_)}, x_Symbol] := -\text{Simp}[(f*x)^{(m+1)} * (a + b*x^n + c*x^{(2*n)})^{(p+1)} * (d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n) / (a*f*n*(p+1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(a*n*(p+1)*(b^2 - 4*a*c)), \text{Int}[(f*x)^m * (a + b*x^n + c*x^{(2*n)})^{(p+1)} * \text{Simp}[d*(b^2*(m+n*(p+1)+1) - 2*a*c*(m+2*n*(p+1)+1) - a*b*e*(m+1) + (m+n*(2*p+3)+1)*(b*d - 2*a*e)*c*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{ILtQ}[p + 1, 0]$

Rule 1560

$\text{Int}[(f_)(x_)]^{(m_)} * ((a_ + (c_)(x_)^{(n2_)} + (b_)(x_)^{(n_)})^{(p_)} * ((d_ + (e_)(x_)^{(n_)})^{(q_)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(f*x)^m * (d + e*x^n)^q * (a + b*x^n + c*x^{(2*n)})^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p, q\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& (\text{IGtQ}[p, 0] \parallel \text{IGtQ}[q, 0])$

Rubi steps

$$\begin{aligned} \int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^2} dx &= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)fn(a + bx^n + cx^{2n})} - \int \frac{(fx)^m (-abe(1+m) - 2acd(1+m-2n) + b^2d(1+m-n))}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} dx \\ &= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)fn(a + bx^n + cx^{2n})} - \int \frac{\left(\frac{c(bd-2ae)(1+m-n) + \frac{c(b^2d-4acd+b^2dm-4acd-m-b^2c)}{\sqrt{b^2-4ac}}}{b - \sqrt{b^2-4ac} + 2cx^n} \right)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} dx \\ &= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)fn(a + bx^n + cx^{2n})} - \frac{c \left((bd - 2ae)(1 + m - n) - \frac{4acd(1+m-2n)}{\sqrt{b^2-4ac}} \right)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} \\ &= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{a(b^2 - 4ac)fn(a + bx^n + cx^{2n})} - \frac{c \left((bd - 2ae)(1 + m - n) - \frac{4acd(1+m-2n)}{\sqrt{b^2-4ac}} \right)}{a(b^2 - 4ac)n(a + bx^n + cx^{2n})} \end{aligned}$$

Mathematica [B] time = 6.52, size = 5363, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]

[Out] Result too large to show

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)(fx)^m}{c^2x^{4n} + b^2x^{2n} + 2abx^n + a^2 + 2(bc x^n + ac)x^{2n}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")

[Out] integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n + a*c)*x^(2*n)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{(cx^{2n} + bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{(bx^n + cx^{2n} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^2,x)

[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(b^2df^m - (2cdf^m + bef^m)a)xx^m + (bcd f^m - 2acef^m)xe^{(m \log(x) + n \log(x))}}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} - \int \frac{(b^2df^m(m-n+1) - (2cdf^m(m-n+1) - 2acef^m))x^{m+n}}{a^2b^2n - 4a^3cn + (ab^2cn - 4a^2c^2n)x^{2n} + (ab^3n - 4a^2bcn)x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")

[Out] ((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c*e*f^m)*x*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n + 1) - (2*c*d*f^m*(m - 2*n + 1) + b*e*f^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m - n + 1) - 2*a*c*e*f^m*(m - n + 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f x)^m (d + e x^n)}{(a + b x^n + c x^{2n})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x)

[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)

[Out] Timed out

$$3.143 \quad \int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3} dx$$

Optimal. Leaf size=816

$$c \left((-d(m-2n+1)b^3 + ae(m+1)b^2 + 2acd(2m-7n+2)b - 4a^2ce(m-3n+1))(m-n+1) + \frac{-d(m^2+(2-3n)m+2n^2)}{2a^2} \right)$$

2a²

[Out] $\frac{1}{2} (f*x)^{(1+m)} * (b^2*d - 2*a*c*d - a*b*e + c*(-2*a*e + b*d) * x^n) / a / (-4*a*c + b^2) / f / n$
 $/ (a + b*x^n + c*x^{(2*n)})^2 + \frac{1}{2} (f*x)^{(1+m)} * ((-2*a*c + b^2) * (a*b*e*(1+m) + 2*a*c*d*(1+m - 4*n) - b^2*d*(1+m - 2*n)) + a*b*c*(-2*a*e + b*d)*(1+m - 3*n) + c*(a*b^2*e*(1+m) + 2*a*b*c*d*(2+2*m - 7*n) - 4*a^2*c*e*(1+m - 3*n) - b^3*d*(1+m - 2*n)) * x^n) / a^2 / (-4*a*c + b^2)^2 / f / n^2 / (a + b*x^n + c*x^{(2*n)}) - \frac{1}{2} * c * (f*x)^{(1+m)} * \text{hypergeom}([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n / (b - (-4*a*c + b^2)^{(1/2)})) * ((a*b^2*e*(1+m) + 2*a*b*c*d*(2+2*m - 7*n) - 4*a^2*c*e*(1+m - 3*n) - b^3*d*(1+m - 2*n)) * (1+m - n) + (a*b^3*e*(1+m) * (1+m - n) - 4*a^2*b*c*e*(1+m^2 + m*(2-n) - n - 3*n^2) - b^4*d*(1+m^2 + m*(2-3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1+m^2 + m*(2-4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1+m^2 + m*(2-6*n) - 6*n + 8*n^2)) / (-4*a*c + b^2)^{(1/2)}) / a^2 / (-4*a*c + b^2)^2 / f / (1+m) / n^2 / (b - (-4*a*c + b^2)^{(1/2)}) - \frac{1}{2} * c * (f*x)^{(1+m)} * \text{hypergeom}([1, (1+m)/n], [(1+m+n)/n], -2*c*x^n / (b + (-4*a*c + b^2)^{(1/2)})) * ((a*b^2*e*(1+m) + 2*a*b*c*d*(2+2*m - 7*n) - 4*a^2*c*e*(1+m - 3*n) - b^3*d*(1+m - 2*n)) * (1+m - n) + (-a*b^3*e*(1+m) * (1+m - n) + 4*a^2*b*c*e*(1+m^2 + m*(2-n) - n - 3*n^2) + b^4*d*(1+m^2 + m*(2-3*n) - 3*n + 2*n^2) - 6*a*b^2*c*d*(1+m^2 + m*(2-4*n) - 4*n + 3*n^2) + 8*a^2*c^2*d*(1+m^2 + m*(2-6*n) - 6*n + 8*n^2)) / (-4*a*c + b^2)^{(1/2)}) / a^2 / (-4*a*c + b^2)^2 / f / (1+m) / n^2 / (b + (-4*a*c + b^2)^{(1/2)})$

Rubi [A] time = 4.55, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1558, 1560, 364}

$$c \left((-d(m-2n+1)b^3 + ae(m+1)b^2 + 2acd(2m-7n+2)b - 4a^2ce(m-3n+1))(m-n+1) + \frac{-d(m^2+(2-3n)m+2n^2)}{2a^2} \right)$$

2a²

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x]

[Out] $((f*x)^{(1+m)} * (b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e) * x^n)) / (2*a*(b^2 - 4*a*c) * f * n * (a + b*x^n + c*x^{(2*n)})^2) + ((f*x)^{(1+m)} * ((b^2 - 2*a*c) * (a*b*e*(1+m) + 2*a*c*d*(1+m - 4*n) - b^2*d*(1+m - 2*n)) + a*b*c*(b*d - 2*a*e)*(1+m - 3*n) + c*(a*b^2*e*(1+m) + 2*a*b*c*d*(2+2*m - 7*n) - 4*a^2*c*e*(1+m - 3*n) - b^3*d*(1+m - 2*n)) * x^n) / (2*a^2*(b^2 - 4*a*c)^2 * f * n^2)$

$$2*(a + b*x^n + c*x^{(2*n)}) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2)))/\text{Sqrt}[b^2 - 4*a*c]]*(f*x)^{(1 + m)}*\text{Hypergeometric2F1}[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c])]/(2*a^2*(b^2 - 4*a*c)^2*(b - \text{Sqrt}[b^2 - 4*a*c]))*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2)))/\text{Sqrt}[b^2 - 4*a*c]]*(f*x)^{(1 + m)}*\text{Hypergeometric2F1}[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c])]/(2*a^2*(b^2 - 4*a*c)^2*(b + \text{Sqrt}[b^2 - 4*a*c]))*f*(1 + m)*n^2)$$

Rule 364

$$\text{Int}[\text{((c_.)*(x_))}^{(m_.)}*\text{((a_.) + (b_.)*(x_)^{(n_)})}^{(p_)}, x_Symbol] \text{ :> } \text{Simp}[(a^p*(c*x)^{(m + 1)}*\text{Hypergeometric2F1}[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/ (c*(m + 1)), x] \text{ /; } \text{FreeQ}\{a, b, c, m, n, p\}, x\} \&\& \text{!IGtQ}[p, 0] \&\& (\text{ILtQ}[p, 0] \text{ || } \text{GtQ}[a, 0])$$

Rule 1558

$$\text{Int}[\text{((f_.)*(x_))}^{(m_.)}*\text{((d_.) + (e_.)*(x_)^{(n_)})}*\text{((a_.) + (b_.)*(x_)^{(n_)})} + (c_.)*(x_)^{(n2_)})}^{(p_)}, x_Symbol] \text{ :> } -\text{Simp}[\text{((f*x)}^{(m + 1)}*(a + b*x^n + c*x^{(2*n)})^{(p + 1)}*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/ (a*f*n*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(a*n*(p + 1)*(b^2 - 4*a*c)), \text{Int}[(f*x)^m*(a + b*x^n + c*x^{(2*n)})^{(p + 1)}*\text{Simp}[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^n, x], x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{ILtQ}[p + 1, 0]$$

Rule 1560

$$\text{Int}[\text{((f_.)*(x_))}^{(m_.)}*\text{((a_.) + (c_.)*(x_)^{(n2_.)})} + (b_.)*(x_)^{(n_)})}^{(p_.)}*\text{((d_.) + (e_.)*(x_)^{(n_)})}^{(q_.)}, x_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^{(2*n)})^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f, m, n, p, q\}, x\} \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& (\text{IGtQ}[p, 0] \text{ || } \text{IGtQ}[q, 0])$$

Rubi steps

$$\begin{aligned}
\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^3} dx &= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} - \int \frac{(fx)^m (-abe(1+m) - 2acd(1+m-4n) + b^2d(1+m-2n))}{(a + bx^n + cx^{2n})^2} dx \\
&= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} + \frac{(fx)^{1+m} ((b^2 - 2ac) (abe(1 + m) + 2acd))}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} \\
&= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} + \frac{(fx)^{1+m} ((b^2 - 2ac) (abe(1 + m) + 2acd))}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} \\
&= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} + \frac{(fx)^{1+m} ((b^2 - 2ac) (abe(1 + m) + 2acd))}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} \\
&= \frac{(fx)^{1+m} (b^2d - 2acd - abe + c(bd - 2ae)x^n)}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2} + \frac{(fx)^{1+m} ((b^2 - 2ac) (abe(1 + m) + 2acd))}{2a (b^2 - 4ac) fn (a + bx^n + cx^{2n})^2}
\end{aligned}$$

Mathematica [B] time = 7.54, size = 20515, normalized size = 25.14

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]

[Out] Result too large to show

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(ex^n + d) (fx)^m}{c^3 x^{6n} + b^3 x^{3n} + 3ab^2 x^{2n} + 3a^2 b x^n + a^3 + 3(bc^2 x^n + ac^2) x^{4n} + 3(b^2 c x^{2n} + 2abc x^n + a^2 c) x^{2n}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")

[Out] integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a^2*c)*x^(2*n)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{(cx^{2n} + bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)(fx)^m}{(bx^n + cx^{2n} + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^3,x)

[Out] int((f*x)^m*(e*x^n+d)/(b*x^n+c*x^(2*n)+a)^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) + 2*c^2*d*f^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) + b^3*e*f^m*(m - n + 1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1) + 4*a^2*c^3*e*f^m*(m - 3*n + 1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2) + b^2*c^2*e*f^m*(m + 1))*a)*x*e^{(m*\log(x) + 3*n*\log(x))} + (2*b^4*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m - 9*n + 4) + 2*c^3*d*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n + 9) + 2*b^3*c*e*f^m*(m + 1))*a)*x*e^{(m*\log(x) + 2*n*\log(x))} + (b^5*d*f^m*(m - 2*n + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m - 4*n + 3) + 2*b*c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) + b^4*e*f^m*(m + 1))*a)*x*e^{(m*\log(x) + n*\log(x))}/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5* \end{aligned}$$

```

c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^
3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 +
16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*((m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*c^2*d*f^m + (2
*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2 - ((5*m^2 - m*(21*n - 10) + 16
*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^3*e*f^m)*a)*x^m
+ ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*(m^2 - 2*m*(2*n -
1) + 3*n^2 - 4*n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n - 4) + 7*n^2 - 9*n
+ 2)*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*e*f^m)*a)*e^(m*log(x) +
n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^
2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*
c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx)^m (d + ex^n)}{(a + bx^n + cx^{2n})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x)

[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)

[Out] Timed out

$$3.144 \quad \int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3} - c^{2/3}d^{2/3}x + \sqrt[3]{c}dx^{4/3}} dx$$

Optimal. Leaf size=47

$$-\frac{3 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x} + d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

[Out] $-3*\ln(c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x^{(1/3)} + d^{(2/3)}*x^{(2/3)})/c^{(1/3)}/d^{(2/3)}$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1594, 1468, 628}

$$-\frac{3 \log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x} + d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x + c^(1/3)*d*x^(4/3)), x]

[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)]/(c^(1/3)*d^(2/3)))

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1468

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3} - c^{2/3}d^{2/3}x + \sqrt[3]{c}dx^{4/3}} dx &= \int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}\sqrt[3]{x}}{(c\sqrt[3]{d} - c^{2/3}d^{2/3}\sqrt[3]{x} + \sqrt[3]{c}dx^{2/3})x^{2/3}} dx \\ &= 3 \text{Subst} \left(\int \frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{c\sqrt[3]{d} - c^{2/3}d^{2/3}x + \sqrt[3]{c}dx^2} dx, x, \sqrt[3]{x} \right) \\ &= -\frac{3 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x} + d^{2/3}x^{2/3})}{\sqrt[3]{c}d^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{3 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x} + d^{2/3}x^{2/3})}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x + c^(1/3)*d*x^(4/3)), x]

[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)]/(c^(1/3)*d^(2/3)))

fricas [A] time = 0.89, size = 33, normalized size = 0.70

$$-\frac{3 \log(dx^{\frac{2}{3}} - c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}} + c^{\frac{2}{3}}d^{\frac{1}{3}})}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3)*d*x^(4/3)), x, algorithm="fricas")

[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2/3))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3)*d*x^(4/3)), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error
 %%%{%%{%%{%%{%%{1, [1]%%}, 0] : [1, 0, 0, %%{-1, [1]%%}, [1]%%}, 0] : [1, 0, 0, %%{-1, [1]%%}, [2]%%}+%%{%%{%%{%%{-1, [1]%%}, 0, 0] : [1, 0, 0, %%{-1, [1]%%}, [1]%%}, [1]%%}+%%{%%{%%{%%{1, [2]%%}, [0]%%}, 0, 0] : [1, 0, 0, %%{-1, [1]%%}, [0]%%} / %%{%%{%%{%%{-1, 0] : [1, 0, 0, %%{-1, [1]%%}, [2]%%}, [2]%%}+%%{%%{%%{%%{1, 0, 0] : [1, 0, 0, %%{-1, [1]%%}, [1]%%}, 0, 0] : [1, 0, 0, %%{-1, [1]%%}, [1]%%}+%%{%%{%%{%%{-1, [1]%%}, [1]%%}, 0] : [1, 0, 0, %%{-1, [1]%%}, [0]%%} Error: Bad Argument Value

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{3 \ln\left(-c^{\frac{1}{3}} d x^{\frac{2}{3}} + c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}} - c d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}} d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3)*d*x^(4/3)),x)

[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*x^(2/3)*d-c*d^(1/3))

maxima [A] time = 0.45, size = 34, normalized size = 0.72

$$\frac{3 \log\left(c^{\frac{1}{3}} d x^{\frac{2}{3}} - c^{\frac{2}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}} + c d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}} d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3)*d*x^(4/3)),x, algorithm="maxima")

[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^(2/3))

mupad [B] time = 2.46, size = 31, normalized size = 0.66

$$\frac{3 \ln\left(x^{2/3} + \frac{c^{2/3}}{d^{2/3}} - \frac{c^{1/3} x^{1/3}}{d^{1/3}}\right)}{c^{1/3} d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x + c^(1/3)*d*x^(4/3)),x)

[Out] $-(3*\log(x^{2/3} + c^{2/3}/d^{2/3} - (c^{1/3}*x^{1/3})/d^{1/3}))/((c^{1/3}*d^{2/3}))$

sympy [C] time = 6.32, size = 126, normalized size = 2.68

$$\frac{3 \log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}} + \sqrt[3]{x} - \frac{\sqrt{3}i\sqrt[4]{c^3}\sqrt[4]{d^3}}{2\sqrt[3]{cd}}\right)}{\sqrt[3]{c}d^{2/3}} - \frac{3 \log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}} + \sqrt[3]{x} + \frac{\sqrt{3}i\sqrt[4]{c^3}\sqrt[4]{d^3}}{2\sqrt[3]{cd}}\right)}{\sqrt[3]{c}d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(2/3)*x+c**(1/3)*d*x**(4/3)),x)`

[Out] $-3*\log(-c^{1/3}/(2*d^{1/3}) + x^{1/3} - \sqrt{3}*I*\sqrt{c^{4/3}}*\sqrt{d^{4/3}}/(2*c^{1/3}*d))/(c^{1/3}*d^{2/3}) - 3*\log(-c^{1/3}/(2*d^{1/3}) + x^{1/3} + \sqrt{3}*I*\sqrt{c^{4/3}}*\sqrt{d^{4/3}}/(2*c^{1/3}*d))/(c^{1/3}*d^{2/3})$

$$3.145 \quad \int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=245

$$\frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{m+1}{n}; 1, -q; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac} \left(b - \sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{m}{n}; 1, -q; \frac{m+n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac} \left(b + \sqrt{b^2-4ac}\right)}$$

[Out] $2*c*(f*x)^{(1+m)}*(d+e*x^n)^q*AppellF1((1+m)/n, 1, -q, (1+m+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}), -e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(b-(-4*a*c+b^2)^{(1/2)})/(-4*a*c+b^2)^{(1/2)}-2*c*(f*x)^{(1+m)}*(d+e*x^n)^q*AppellF1((1+m)/n, 1, -q, (1+m+n)/n, -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}), -e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(-4*a*c+b^2)^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})$

Rubi [A] time = 0.54, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1556, 511, 510}

$$\frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{m+1}{n}; 1, -q; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac} \left(b - \sqrt{b^2-4ac}\right)} - \frac{2c(fx)^{m+1} (d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{m}{n}; 1, -q; \frac{m+n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac} \left(b + \sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

[In] Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

[Out] $(2*c*(f*x)^{(1+m)}*(d+e*x^n)^q*AppellF1[(1+m)/n, 1, -q, (1+m+n)/n, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]), -(e*x^n/d)]/(\text{Sqrt}[b^2 - 4*a*c]*(b - \text{Sqrt}[b^2 - 4*a*c])*f*(1+m)*(1+(e*x^n/d)^q) - (2*c*(f*x)^{(1+m)}*(d+e*x^n)^q*AppellF1[(1+m)/n, 1, -q, (1+m+n)/n, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]), -(e*x^n/d)]/(\text{Sqrt}[b^2 - 4*a*c]*(b + \text{Sqrt}[b^2 - 4*a*c])*f*(1+m)*(1+(e*x^n/d)^q)$

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -(b*x^n)/a, -(d*x^n)/c]]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x^n)^p), x_Symbol]


```

n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rule 1556

```

Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^(n_))^(q_))/((a_) + (c_)*(x_)^(
n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(fx)^m (d + ex^n)^q}{a + bx^n + cx^{2n}} dx &= \frac{(2c) \int \frac{(fx)^m (d + ex^n)^q}{b - \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{(fx)^m (d + ex^n)^q}{b + \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} \\
&= \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{(fx)^m \left(1 + \frac{ex^n}{d}\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{(fx)^m \left(1 + \frac{ex^n}{d}\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} \\
&= \frac{2c (fx)^{1+m} (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1+m}{n}; 1, -q; \frac{1+m+n}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{ex^n}{d}\right)}{\sqrt{b^2 - 4ac} (b - \sqrt{b^2 - 4ac})} - \frac{2c (fx)^{1+m} (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1+m}{n}; 1, -q; \frac{1+m+n}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, -\frac{ex^n}{d}\right)}{\sqrt{b^2 - 4ac} (b + \sqrt{b^2 - 4ac})}
\end{aligned}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (ex^n + d)^q}{bx^n + cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

[Out] int((f*x)^m*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(fx)^m (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)

[Out] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)

[Out] Exception raised: HeuristicGCDFailed

$$3.146 \quad \int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=210

$$\frac{2cx^3(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{n+3}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac} - 4ac + b^2\right)} - \frac{2cx^3(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{n+3}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac} - 4ac + b^2\right)}$$

[Out] $-2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n, 1, -q, (3+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^{(1/2)})-2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n, 1, -q, (3+n)/n, -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^{(1/2)})$

Rubi [A] time = 0.50, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1556, 511, 510}

$$\frac{2cx^3(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{n+3}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac} - 4ac + b^2\right)} - \frac{2cx^3(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{n+3}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac} - 4ac + b^2\right)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^{(2*n)}), x]$

[Out] $(-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]), -(e*x^n/d)])/(3*(b^2 - 4*a*c - b*\text{Sqrt}[b^2 - 4*a*c])*(1 + (e*x^n/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]), -(e*x^n/d)])/(3*(b^2 - 4*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*(1 + (e*x^n/d)^q)$

Rule 510

$\text{Int}[(e*x^m*(a + b*x^n)^p*(c + d*x^n)^q), x_Symbol] \rightarrow \text{Simp}[(a^p*c^q*(e*x)^{m+1}*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b*x^n/a), -(d*x^n/c)])/(e*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{NeQ}[m, n - 1] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0]) \ \&\& \ (\text{IntegerQ}[q] \ || \ \text{GtQ}[c, 0])$

Rule 511

$\text{Int}[(e*x^m*(a + b*x^n)^p*(c + d*x^n)^q), x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n/a)^{\text{FracPart}[p]}), \text{Int}[(e*x)^m*(1 + (b*x^n/a))^p*(c + d*x^n)^q, x], x] /;$

FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 1556

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[((f*x)^(m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[((f*x)^(m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2 (d + ex^n)^q}{a + bx^n + cx^{2n}} dx &= \frac{(2c) \int \frac{x^2 (d + ex^n)^q}{b - \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{(2c) \int \frac{x^2 (d + ex^n)^q}{b + \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} \\ &= \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{x^2 \left(1 + \frac{ex^n}{d}\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} - \frac{\left(2c (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{x^2 \left(1 + \frac{ex^n}{d}\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^n} dx}{\sqrt{b^2 - 4ac}} \\ &= -\frac{2cx^3 (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{3+n}{n}; -\frac{2cx^n}{b - \sqrt{b^2 - 4ac}}, -\frac{ex^n}{d}\right)}{3(b^2 - 4ac - b\sqrt{b^2 - 4ac})} - \frac{2cx^3 (d + ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n}; 1, -q; \frac{3+n}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}}, -\frac{ex^n}{d}\right)}{3(b^2 - 4ac + b\sqrt{b^2 - 4ac})} \end{aligned}$$

Mathematica [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^2 (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x^2 (ex^n + d)^q}{bx^n + cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

[Out] int(x^2*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)

[Out] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)`

[Out] Exception raised: HeuristicGCDFailed

$$3.147 \quad \int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=206

$$\frac{cx^2(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{2}{n}; 1, -q; \frac{n+2}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac} - 4ac + b^2} - \frac{cx^2(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{2}{n}; 1, -q; \frac{n+2}{n}; -\frac{2c}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac} - 4ac + b^2}$$

[Out] $-c*x^2*(d+e*x^n)^q*AppellF1(2/n, 1, -q, (2+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*(d+e*x^n)^q*AppellF1(2/n, 1, -q, (2+n)/n, -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))$

Rubi [A] time = 0.38, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1556, 511, 510}

$$\frac{cx^2(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{2}{n}; 1, -q; \frac{n+2}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac} - 4ac + b^2} - \frac{cx^2(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(\frac{2}{n}; 1, -q; \frac{n+2}{n}; -\frac{2c}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac} - 4ac + b^2}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

[Out] $-((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -(e*x^n)/d])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -(e*x^n)/d])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^n)/d)^q)$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 1556

Int[(((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^(n_))^(q_))/((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_)), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}} dx &= \frac{(2c) \int \frac{x(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{x(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} \\ &= \frac{\left(2c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q}\right) \int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q}\right) \int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} \\ &= -\frac{cx^2(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{2}{n}; 1, -q; \frac{2+n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex^n)^q \left(1+\frac{ex^n}{d}\right)^{-q}}{b^2-4ac+b\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n+d)^q x}{cx^{2n}+bx^n+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{x(e x^n + d)^q}{b x^n + c x^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

[Out] int(x*(e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x(d + e x^n)^q}{a + b x^n + c x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)

[Out] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.148 \quad \int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}} dx$$

Optimal. Leaf size=194

$$\frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1+\frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1+\frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

[Out] $-2*c*x*(d+e*x^n)^q*AppellF1(1/n, 1, -q, 1+1/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(d+e*x^n)^q*AppellF1(1/n, 1, -q, 1+1/n, -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)), -e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))$

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1428, 430, 429}

$$\frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1+\frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2} - \frac{2cx(d+ex^n)^q \left(\frac{ex^n}{d}+1\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1+\frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]

[Out] $(-2*c*x*(d+e*x^n)^q*AppellF1[n^(-1), 1, -q, 1+n^(-1), (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), -(e*x^n/d)])/((b^2-4*a*c-b*Sqrt[b^2-4*a*c])*(1+(e*x^n/d)^q)-(2*c*x*(d+e*x^n)^q*AppellF1[n^(-1), 1, -q, 1+n^(-1), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c]), -(e*x^n/d)])/((b^2-4*a*c+b*Sqrt[b^2-4*a*c])*(1+(e*x^n/d)^q))$

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1+1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
 := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 1428

Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}} dx &= \frac{(2c) \int \frac{(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} \\ &= \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n} dx}{\sqrt{b^2-4ac}} \\ &= -\frac{2cx(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1 + \frac{1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b^2 - 4ac - b\sqrt{b^2 - 4ac}} - \frac{2cx(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(\frac{1}{n}; 1, -q; 1 + \frac{1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{b^2 - 4ac + b\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]

[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{bx^n + cx^{2n} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

[Out] int((e*x^n+d)^q/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d + ex^n)^q}{a + bx^n + cx^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x)

[Out] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)

[Out] Exception raised: HeuristicGCDFailed

$$3.149 \quad \int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})} dx$$

Optimal. Leaf size=263

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1} {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd-(b-\sqrt{b^2-4ac})e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1} {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{an(q+1)\left(2cd-e\left(b+\sqrt{b^2-4ac}\right)\right)}$$

[Out] $-(d+e*x^n)^{(1+q)}*\text{hypergeom}([1, 1+q], [2+q], 1+e*x^n/d)/a/d/n/(1+q)+c*(d+e*x^n)^{(1+q)}*\text{hypergeom}([1, 1+q], [2+q], 2*c*(d+e*x^n)/(2*c*d-e*(b-(-4*a*c+b^2)^{(1/2)})))*(1+b/(-4*a*c+b^2)^{(1/2)})/a/n/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^{(1/2)}))+c*(d+e*x^n)^{(1+q)}*\text{hypergeom}([1, 1+q], [2+q], 2*c*(d+e*x^n)/(2*c*d-e*(b+(-4*a*c+b^2)^{(1/2)})))*(1-b/(-4*a*c+b^2)^{(1/2)})/a/n/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^{(1/2)}))$

Rubi [A] time = 0.73, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1474, 960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1} {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd-(b-\sqrt{b^2-4ac})e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1} {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{an(q+1)\left(2cd-e\left(b+\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x]

[Out] $(c*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*(d + e*x^n)^{(1 + q)}*\text{Hypergeometric2F1}[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e)])/ (a*(2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*(d + e*x^n)^{(1 + q)}*\text{Hypergeometric2F1}[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)])/ (a*(2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e)*n*(1 + q)) - ((d + e*x^n)^{(1 + q)}*\text{Hypergeometric2F1}[1, 1 + q, 2 + q, 1 + (e*x^n)/d])/ (a*d*n*(1 + q))$

Rule 65

Int[((b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/ (d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))]/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
+ (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^n)^q}{x(a + bx^n + cx^{2n})} dx &= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x(a+bx+cx^2)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a(a+bx+cx^2)}\right) dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x} dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2} dx, x, x^n\right)}{an} \\
&= \frac{(d + ex^n)^{1+q} {}_2F_1\left(1, 1 + q; 2 + q; 1 + \frac{ex^n}{d}\right)}{adn(1 + q)} + \frac{\text{Subst}\left(\int \left(\frac{\left(-c - \frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b - \sqrt{b^2-4ac} + 2cx} + \frac{\left(-c + \frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b + \sqrt{b^2-4ac} + 2cx}\right) dx, x, x^n\right)}{an} \\
&= \frac{(d + ex^n)^{1+q} {}_2F_1\left(1, 1 + q; 2 + q; 1 + \frac{ex^n}{d}\right)}{adn(1 + q)} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \text{Subst}\left(\int \frac{(d+ex)^q}{b + \sqrt{b^2-4ac} + 2cx} dx, x, x^n\right)}{an} \\
&= \frac{c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)(d + ex^n)^{1+q} {}_2F_1\left(1, 1 + q; 2 + q; \frac{2c(d+ex^n)}{2cd - (b - \sqrt{b^2-4ac})e}\right)}{a\left(2cd - (b - \sqrt{b^2-4ac})e\right)n(1 + q)} + \frac{c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)(d + ex^n)^{1+q} {}_2F_1\left(1, 1 + q; 2 + q; \frac{2c(d+ex^n)}{2cd - (b + \sqrt{b^2-4ac})e}\right)}{a\left(2cd - (b + \sqrt{b^2-4ac})e\right)n(1 + q)}
\end{aligned}$$

Mathematica [A] time = 0.67, size = 218, normalized size = 0.83

$$\frac{(d + ex^n)^{q+1} \left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd + (\sqrt{b^2-4ac}-b)e}\right)}{e(\sqrt{b^2-4ac}-b) + 2cd} + \frac{c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) {}_2F_1\left(1, q+1; q+2; \frac{2c(ex^n+d)}{2cd - (b + \sqrt{b^2-4ac})e}\right)}{2cd - e(\sqrt{b^2-4ac} + b)} - \frac{{}_2F_1\left(1, q+1; q+2; \frac{ex^n}{d} + 1\right)}{d} \right)}{an(q + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x]

[Out] ((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n)/d]/d))/(a*n*(1 + q))

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q}{cxx^{2n} + bxx^n + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(bx^n + cx^{2n} + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^n+d)^q/x/(b*x^n+c*x^(2*n)+a),x)

[Out] int((e*x^n+d)^q/x/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + ex^n)^q}{x(a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

$$3.150 \quad \int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})} dx$$

Optimal. Leaf size=212

$$\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac} - 4ac + b^2\right)} + \frac{2c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac} - 4ac + b^2\right)}$$

[Out] $2*c*(d+e*x^n)^q*AppellF1(-1/n, 1, -q, (-1+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*(d+e*x^n)^q*AppellF1(-1/n, 1, -q, (-1+n)/n, -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)), -e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))$

Rubi [A] time = 0.49, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1556, 511, 510}

$$\frac{2c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac} - 4ac + b^2\right)} + \frac{2c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac} - 4ac + b^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]

[Out] $(2*c*(d+e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1-n)/n), (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), -(e*x^n/d)]/((b^2-4*a*c-b*Sqrt[b^2-4*a*c])*x*(1+(e*x^n/d)^q) + (2*c*(d+e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1-n)/n), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c]), -(e*x^n/d)]/((b^2-4*a*c+b*Sqrt[b^2-4*a*c])*x*(1+(e*x^n/d)^q))$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n-1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x

$\wedge n)/a)^{\text{FracPart}[p]}$, $\text{Int}[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{NeQ}[m, n - 1] \ \&\& \ \text{!(IntegerQ}[p] \ || \ \text{GtQ}[a, 0])]$

Rule 1556

$\text{Int}[(((f_.)*(x_))^{(m_.)*((d_.) + (e_.)*(x_)^{(n_.))^{(q_.))}/((a_.) + (c_.)*(x_)^{(n2_.) + (b_.)*(x_)^{(n_.))}, x_Symbol] \text{:>} \text{With}[\{r = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/r, \text{Int}[(f*x)^m*(d + e*x^n)^q/(b - r + 2*c*x^n), x], x] - \text{Dist}[(2*c)/r, \text{Int}[(f*x)^m*(d + e*x^n)^q/(b + r + 2*c*x^n), x], x]] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, m, n, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\int \frac{(d + ex^n)^q}{x^2(a + bx^n + cx^{2n})} dx = \frac{(2c) \int \frac{(d+ex^n)^q}{x^2(b-\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{(d+ex^n)^q}{x^2(b+\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{x^2(b-\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{x^2(b+\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{\left(b^2 - 4ac - b\sqrt{b^2 - 4ac}\right)x} + \frac{2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n}; 1, -q; -\frac{1-n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{\left(b^2 - 4ac + b\sqrt{b^2 - 4ac}\right)x}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^n)^q}{x^2(a + bx^n + cx^{2n})} dx$$

Verification is Not applicable to the result.

[In] $\text{Integrate}[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]$

[Out] $\text{Integrate}[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]$

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^2x^{2n} + bx^2x^n + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(bx^n + cx^{2n} + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^n+d)^q/x^2/(b*x^n+c*x^(2*n)+a),x)

[Out] int((e*x^n+d)^q/x^2/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + ex^n)^q}{x^2 (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x)

```
[Out] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.151 \quad \int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})} dx$$

Optimal. Leaf size=210

$$\frac{c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x^2(-b\sqrt{b^2-4ac}-4ac+b^2)} + \frac{c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2c}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x^2(b\sqrt{b^2-4ac}-4ac+b^2)}$$

[Out] $c*(d+e*x^n)^q*AppellF1(-2/n, 1, -q, (-2+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*(d+e*x^n)^q*AppellF1(-2/n, 1, -q, (-2+n)/n, -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)), -e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))$

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1556, 511, 510}

$$\frac{c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x^2(-b\sqrt{b^2-4ac}-4ac+b^2)} + \frac{c(d+ex^n)^q \left(\frac{ex^n}{d} + 1\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2c}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{x^2(b\sqrt{b^2-4ac}-4ac+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]

[Out] $(c*(d+e*x^n)^q*AppellF1[-2/n, 1, -q, -((2-n)/n), (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), -(e*x^n/d)]/((b^2-4*a*c-b*Sqrt[b^2-4*a*c])*x^2*(1+(e*x^n/d)^q) + (c*(d+e*x^n)^q*AppellF1[-2/n, 1, -q, -((2-n)/n), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c]), -(e*x^n/d)]/((b^2-4*a*c+b*Sqrt[b^2-4*a*c])*x^2*(1+(e*x^n/d)^q))$

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x

$\wedge n)/a)^{\text{FracPart}[p]}$, $\text{Int}[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$
 $\text{FreeQ}[\{a, b, c, d, e, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[m, -1] \ \&\& \ \text{NeQ}[m, n - 1] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{GtQ}[a, 0])$

Rule 1556

$\text{Int}[(((f._)*(x._))^m)*((d._) + (e._)*(x._)^n)^q)/((a._) + (c._)*(x._)^n2._) + (b._)*(x._)^n), x_Symbol] := \text{With}[\{r = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/r, \text{Int}[(f*x)^m*(d + e*x^n)^q]/(b - r + 2*c*x^n), x], x] - \text{Dist}[(2*c)/r, \text{Int}[(f*x)^m*(d + e*x^n)^q]/(b + r + 2*c*x^n), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, m, n, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rubi steps

$$\int \frac{(d + ex^n)^q}{x^3(a + bx^n + cx^{2n})} dx = \frac{(2c) \int \frac{(d+ex^n)^q}{x^3(b-\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}} - \frac{(2c) \int \frac{(d+ex^n)^q}{x^3(b+\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{x^3(b-\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^n}{d}\right)^q}{x^3(b+\sqrt{b^2-4ac}+2cx^n)} dx}{\sqrt{b^2-4ac}}$$

$$= \frac{c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{\left(b^2 - 4ac - b\sqrt{b^2 - 4ac}\right) x^2} + \frac{c(d+ex^n)^q \left(1 + \frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n}; 1, -q; -\frac{2-n}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, -\frac{ex^n}{d}\right)}{\left(b^2 - 4ac + b\sqrt{b^2 - 4ac}\right) x^2}$$

Mathematica [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^n)^q}{x^3(a + bx^n + cx^{2n})} dx$$

Verification is Not applicable to the result.

[In] $\text{Integrate}[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]$

[Out] $\text{Integrate}[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]$

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^3x^{2n} + bx^3x^n + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(bx^n + cx^{2n} + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^n+d)^q/x^3/(b*x^n+c*x^(2*n)+a),x)

[Out] int((e*x^n+d)^q/x^3/(b*x^n+c*x^(2*n)+a),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^n + d)^q}{(cx^{2n} + bx^n + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + ex^n)^q}{x^3 (a + bx^n + cx^{2n})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x)

```
[Out] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

$$3.152 \quad \int (fx)^m (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=498

$$\frac{d^2(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

[Out] $d^{2*(f*x)^{(1+m)*(a+b*x^n+c*x^{2n})^p}$ AppellF1((1+m)/n, -p, -p, (1+m+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+2*d*e*x^(1+n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p AppellF1((1+m+n)/n, -p, -p, (1+m+2*n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e^{2*x^{(1+2*n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p} AppellF1((1+m+2*n)/n, -p, -p, (1+m+3*n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^(1/2)), -2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)

Rubi [A] time = 0.61, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1560, 1385, 510, 20}

$$\frac{d^2(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^{2n})^p,x]

[Out] $(d^{2*(f*x)^{(1+m)*(a+b*x^n+c*x^{2n})^p}$ AppellF1[(1+m)/n, -p, -p, (1+m+n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/(f*(1+m)*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)+(2*d*e*x^{(1+n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p} AppellF1[(1+m+n)/n, -p, -p, (1+m+2*n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/((1+m+n)*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)+e^{2*x^{(1+2*n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p} AppellF1[(1+m+2*n)/n, -p, -p, (1+m+3*n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/((1+m+2*n)*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^q_], x_Symbol] := Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m + 1)), x] /; FreeQ[{a,
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx &= \int \left(d^2 (fx)^m (a + bx^n + cx^{2n})^p + 2dex^n (fx)^m (a + bx^n + cx^{2n})^p + e^2 x^{2n} (fx)^m (a + bx^n + cx^{2n})^p \right) dx \\
&= d^2 \int (fx)^m (a + bx^n + cx^{2n})^p dx + (2de) \int x^n (fx)^m (a + bx^n + cx^{2n})^p dx \\
&= (2dex^{-m} (fx)^m) \int x^{m+n} (a + bx^n + cx^{2n})^p dx + (e^2 x^{-m} (fx)^m) \int x^{m+2n} (a + bx^n + cx^{2n})^p dx \\
&= \frac{d^2 (fx)^{1+m} \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{-p} (a + bx^n + cx^{2n})^p F_1\left(\frac{m+1}{n}; \frac{m+1}{n}, \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)}{f(1+m)} \\
&= \frac{d^2 (fx)^{1+m} \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)^{-p} (a + bx^n + cx^{2n})^p F_1\left(\frac{m+1}{n}; \frac{m+1}{n}, \frac{2cx^n}{b + \sqrt{b^2 - 4ac}}\right)}{f(1+m)}
\end{aligned}$$

Mathematica [A] time = 1.14, size = 391, normalized size = 0.79

$$x(fx)^m \left(\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b} \right)^{-p} (a + x^n (b + cx^n))^p \left(d^2 (m^2 + m(3n + 2) + 2n^2 + 3n + 1) F_1\left(\frac{m+1}{n}; \frac{m+1}{n}, \frac{2cx^n}{b - \sqrt{b^2 - 4ac}}\right) + e^2 x^{2n} (fx)^m (a + bx^n + cx^{2n})^p \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + e*(1 + m)*x^n*(2*d*(1 + m + 2*n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + e*(1 + m + n)*x^n*AppellF1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 + m)*(1 + m + n)*(1 + m + 2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(e^2 x^{2n} + 2dex^n + d^2\right)(cx^{2n} + bx^n + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,
x)
```

```
giac [F(-2)]    time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Simplification assuming x near 0Simplificati
on assuming f near 0Simplification assuming x near 0Simplification assuming
f near 0Unable to divide, perhaps due to rounding error%%{128,[1,0,5,3,0,
5,4,1,6,1]%%}+%%{512,[1,0,5,3,0,5,3,1,6,1]%%}+%%{768,[1,0,5,3,0,5,2,1,6
,1]%%}+%%{512,[1,0,5,3,0,5,1,1,6,1]%%}+%%{128,[1,0,5,3,0,5,0,1,6,1]%%}
+%%{-96,[1,0,5,3,0,4,4,3,5,1]%%}+%%{-384,[1,0,5,3,0,4,3,3,5,1]%%}+%%{-
576,[1,0,5,3,0,4,2,3,5,1]%%}+%%{-384,[1,0,5,3,0,4,1,3,5,1]%%}+%%{-96,[1
,0,5,3,0,4,0,3,5,1]%%}+%%{24,[1,0,5,3,0,3,4,5,4,1]%%}+%%{96,[1,0,5,3,0,
3,3,5,4,1]%%}+%%{144,[1,0,5,3,0,3,2,5,4,1]%%}+%%{96,[1,0,5,3,0,3,1,5,4,
1]%%}+%%{24,[1,0,5,3,0,3,0,5,4,1]%%}+%%{-2,[1,0,5,3,0,2,4,7,3,1]%%}+%%
{-8,[1,0,5,3,0,2,3,7,3,1]%%}+%%{-12,[1,0,5,3,0,2,2,7,3,1]%%}+%%{-8,[1,
0,5,3,0,2,1,7,3,1]%%}+%%{-2,[1,0,5,3,0,2,0,7,3,1]%%}+%%{64,[1,0,5,2,1,5
,3,1,6,1]%%}+%%{192,[1,0,5,2,1,5,2,1,6,1]%%}+%%{192,[1,0,5,2,1,5,1,1,6,
1]%%}+%%{64,[1,0,5,2,1,5,0,1,6,1]%%}+%%{-48,[1,0,5,2,1,4,3,3,5,1]%%}+%%
{-144,[1,0,5,2,1,4,2,3,5,1]%%}+%%{-144,[1,0,5,2,1,4,1,3,5,1]%%}+%%{-4
8,[1,0,5,2,1,4,0,3,5,1]%%}+%%{12,[1,0,5,2,1,3,3,5,4,1]%%}+%%{36,[1,0,5,
2,1,3,2,5,4,1]%%}+%%{36,[1,0,5,2,1,3,1,5,4,1]%%}+%%{12,[1,0,5,2,1,3,0,5
,4,1]%%}+%%{-1,[1,0,5,2,1,2,3,7,3,1]%%}+%%{-3,[1,0,5,2,1,2,2,7,3,1]%%}
+%%{-3,[1,0,5,2,1,2,1,7,3,1]%%}+%%{-1,[1,0,5,2,1,2,0,7,3,1]%%}+%%{64,[
1,0,5,2,0,5,3,1,6,1]%%}+%%{192,[1,0,5,2,0,5,2,1,6,1]%%}+%%{192,[1,0,5,2
,0,5,1,1,6,1]%%}+%%{64,[1,0,5,2,0,5,0,1,6,1]%%}+%%{-48,[1,0,5,2,0,4,3,3
,5,1]%%}+%%{-144,[1,0,5,2,0,4,2,3,5,1]%%}+%%{-144,[1,0,5,2,0,4,1,3,5,1]
%%}+%%{-48,[1,0,5,2,0,4,0,3,5,1]%%}+%%{12,[1,0,5,2,0,3,3,5,4,1]%%}+%%
{36,[1,0,5,2,0,3,2,5,4,1]%%}+%%{36,[1,0,5,2,0,3,1,5,4,1]%%}+%%{12,[1,0,
5,2,0,3,0,5,4,1]%%}+%%{-1,[1,0,5,2,0,2,3,7,3,1]%%}+%%{-3,[1,0,5,2,0,2,2
,7,3,1]%%}+%%{-3,[1,0,5,2,0,2,1,7,3,1]%%}+%%{-1,[1,0,5,2,0,2,0,7,3,1]%%
}+%%{128,[0,0,5,2,1,5,3,0,7,1]%%}+%%{384,[0,0,5,2,1,5,2,0,7,1]%%}+%%{
384,[0,0,5,2,1,5,1,0,7,1]%%}+%%{128,[0,0,5,2,1,5,0,0,7,1]%%}+%%{-96,[0,
0,5,2,1,4,3,2,6,1]%%}+%%{-288,[0,0,5,2,1,4,2,2,6,1]%%}+%%{-288,[0,0,5,2
,1,4,1,2,6,1]%%}+%%{-96,[0,0,5,2,1,4,0,2,6,1]%%}+%%{24,[0,0,5,2,1,3,3,4
,5,1]%%}+%%{72,[0,0,5,2,1,3,2,4,5,1]%%}+%%{72,[0,0,5,2,1,3,1,4,5,1]%%}
+%%{24,[0,0,5,2,1,3,0,4,5,1]%%}+%%{-2,[0,0,5,2,1,2,3,6,4,1]%%}+%%{-6,[
0,0,5,2,1,2,2,6,4,1]%%}+%%{-6,[0,0,5,2,1,2,1,6,4,1]%%}+%%{-2,[0,0,5,2,1
,2,0,6,4,1]%%}+%%{128,[0,0,5,2,0,5,3,0,7,1]%%}+%%{384,[0,0,5,2,0,5,2,0,
```

```

7,1]%%}+%%{384,[0,0,5,2,0,5,1,0,7,1]%%}+%%{128,[0,0,5,2,0,5,0,0,7,1]%%}
}+%%{-96,[0,0,5,2,0,4,3,2,6,1]%%}+%%{-288,[0,0,5,2,0,4,2,2,6,1]%%}+%%{-
288,[0,0,5,2,0,4,1,2,6,1]%%}+%%{-96,[0,0,5,2,0,4,0,2,6,1]%%}+%%{24,[0,
0,5,2,0,3,3,4,5,1]%%}+%%{72,[0,0,5,2,0,3,2,4,5,1]%%}+%%{72,[0,0,5,2,0,3
,1,4,5,1]%%}+%%{24,[0,0,5,2,0,3,0,4,5,1]%%}+%%{-2,[0,0,5,2,0,2,3,6,4,1]
%%}+%%{-6,[0,0,5,2,0,2,2,6,4,1]%%}+%%{-6,[0,0,5,2,0,2,1,6,4,1]%%}+%%{-
2,[0,0,5,2,0,2,0,6,4,1]%%} / %%{64,[0,0,5,3,0,5,3,0,6,0]%%}+%%{192,[0,
0,5,3,0,5,2,0,6,0]%%}+%%{192,[0,0,5,3,0,5,1,0,6,0]%%}+%%{64,[0,0,5,3,0,
5,0,0,6,0]%%}+%%{-48,[0,0,5,3,0,4,3,2,5,0]%%}+%%{-144,[0,0,5,3,0,4,2,2,
5,0]%%}+%%{-144,[0,0,5,3,0,4,1,2,5,0]%%}+%%{-48,[0,0,5,3,0,4,0,2,5,0]%%
}+%%{12,[0,0,5,3,0,3,3,4,4,0]%%}+%%{36,[0,0,5,3,0,3,2,4,4,0]%%}+%%{36
,[0,0,5,3,0,3,1,4,4,0]%%}+%%{12,[0,0,5,3,0,3,0,4,4,0]%%}+%%{-1,[0,0,5,3
,0,2,3,6,3,0]%%}+%%{-3,[0,0,5,3,0,2,2,6,3,0]%%}+%%{-3,[0,0,5,3,0,2,1,6,
3,0]%%}+%%{-1,[0,0,5,3,0,2,0,6,3,0]%%} Error: Bad Argument Value

```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ex^n + d)^2 (fx)^m (bx^n + cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^n+d)^2*(b*x^n+c*x^(2*n)+a)^p,x)
```

```
[Out] int((f*x)^m*(e*x^n+d)^2*(b*x^n+c*x^(2*n)+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)^2 (cx^{2n} + bx^n + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (d + ex^n)^2 (a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```


sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)

[Out] Timed out

$$3.153 \quad \int (fx)^m (d + ex^n) (a + bx^n + cx^{2n})^p dx$$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

[Out] $d*(f*x)^{(1+m)*(a+b*x^n+c*x^{2n})^p*AppellF1((1+m)/n, -p, -p, (1+m+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}), -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)})/f/(1+m)/((1+2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}))^p)+e*x^{(1+n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p*AppellF1((1+m+n)/n, -p, -p, (1+m+2*n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}), -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)})/(1+m+n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}))^p)$

Rubi [A] time = 0.37, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1560, 1385, 510, 20}

$$\frac{d(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]

[Out] $(d*(f*x)^{(1+m)*(a+b*x^n+c*x^{2n})^p*AppellF1[(1+m)/n, -p, -p, (1+m+n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/(f*(1+m)*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)+(e*x^{(1+n)*(f*x)^m*(a+b*x^n+c*x^{2n})^p*AppellF1[(1+m+n)/n, -p, -p, (1+m+2*n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])]/((1+m+n)*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)$

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -

$q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[m, -1] \&\& \text{NeQ}[m, n - 1] \&\& (\text{IntegerQ}[p] \mid\mid \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] \mid\mid \text{GtQ}[c, 0])$

Rule 1385

$\text{Int}[(d_*)*(x_*)^{(m_*)}*((a_*) + (c_*)*(x_*)^{(n2_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n + c*x^{(2*n)})^{\text{FracPart}[p]}]/((1 + (2*c*x^n)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}*(1 + (2*c*x^n)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}), \text{Int}[(d*x)^m*(1 + (2*c*x^n)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n, p\}, x] \&\& \text{EqQ}[n2, 2*n]$

Rule 1560

$\text{Int}[(f_*)*(x_*)^{(m_*)}*((a_*) + (c_*)*(x_*)^{(n2_*)} + (b_*)*(x_*)^{(n_*)})^{(p_*)}*(d_*) + (e_*)*(x_*)^{(n_*)})^{(q_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(f*x)^m*(d + e*x^n)^q*(a + b*x^n + c*x^{(2*n)})^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p, q\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& (\text{IGtQ}[p, 0] \mid\mid \text{IGtQ}[q, 0])$

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^n) (a + bx^n + cx^{2n})^p dx &= \int \left(d(fx)^m (a + bx^n + cx^{2n})^p + ex^n (fx)^m (a + bx^n + cx^{2n})^p \right) dx \\ &= d \int (fx)^m (a + bx^n + cx^{2n})^p dx + e \int x^n (fx)^m (a + bx^n + cx^{2n})^p dx \\ &= (ex^{-m} (fx)^m) \int x^{m+n} (a + bx^n + cx^{2n})^p dx + \left(d \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right) \right) \\ &= \frac{d(fx)^{1+m} \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{1}{2} \right)}{f(1+m)} \\ &= \frac{d(fx)^{1+m} \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{1}{2} \right)}{f(1+m)} \end{aligned}$$

Mathematica [A] time = 0.60, size = 273, normalized size = 0.85

$$\frac{x(fx)^m \left(\frac{-\sqrt{b^2 - 4ac} + b + 2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(\frac{\sqrt{b^2 - 4ac} + b + 2cx^n}{\sqrt{b^2 - 4ac} + b} \right)^{-p} (a + x^n (b + cx^n))^p \left(d(m + n + 1) F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right) \right)}{(m+1)(m+n+1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) / ((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((ex^n + d)(cx^{2n} + bx^n + a)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)(cx^{2n} + bx^n + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (ex^n + d)(fx)^m (bx^n + cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^n+d)*(b*x^n+c*x^(2*n)+a)^p,x)

[Out] int((f*x)^m*(e*x^n+d)*(b*x^n+c*x^(2*n)+a)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^n + d)(cx^{2n} + bx^n + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")

[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (f x)^m (d + e x^n) (a + b x^n + c x^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)

[Out] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)

[Out] Timed out

3.154 $\int (fx)^m (a + bx^n + cx^{2n})^p dx$

Optimal. Leaf size=158

$$\frac{(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

[Out] $(f*x)^{(1+m)}*(a+b*x^n+c*x^{(2*n)})^p*AppellF1((1+m)/n, -p, -p, (1+m+n)/n, -2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)}), -2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)}))/f/(1+m)/((1+2*c*x^n/(b-(-4*a*c+b^2)^{(1/2)})))^p/((1+2*c*x^n/(b+(-4*a*c+b^2)^{(1/2)})))^p)$

Rubi [A] time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1385, 510}

$$\frac{(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}} + 1 \right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b} + 1 \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}} \right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]

[Out] $((f*x)^{(1+m)}*(a+b*x^n+c*x^{(2*n)})^p*AppellF1[(1+m)/n, -p, -p, (1+m+n)/n, (-2*c*x^n)/(b-Sqrt[b^2-4*a*c]), (-2*c*x^n)/(b+Sqrt[b^2-4*a*c])])/((f*x)^{(1+m)}*(1+(2*c*x^n)/(b-Sqrt[b^2-4*a*c]))^p*(1+(2*c*x^n)/(b+Sqrt[b^2-4*a*c]))^p)$

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m+1)*AppellF1[(m+1)/n, -p, -q, 1+(m+1)/n, -((b*x^n)/a), -((d*x^n)/c)])/((e*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 1385

Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(d*x)^(m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n]

Rubi steps

$$\int (fx)^m (a + bx^n + cx^{2n})^p dx = \left(\left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p \right) \int (fx)^m dx$$

$$= \frac{(fx)^{1+m} \left(1 + \frac{2cx^n}{b - \sqrt{b^2 - 4ac}} \right)^{-p} \left(1 + \frac{2cx^n}{b + \sqrt{b^2 - 4ac}} \right)^{-p} (a + bx^n + cx^{2n})^p F_1 \left(\frac{1+m}{n}; -p, -p \right)}{f(1+m)}$$

Mathematica [A] time = 0.30, size = 181, normalized size = 1.15

$$\frac{x(fx)^m \left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}} \right)^{-p} \left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b} \right)^{-p} (a + x^n (b + cx^n))^p F_1 \left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b+\sqrt{b^2-4ac}}, \frac{2cx^n}{\sqrt{b^2-4ac}-b} \right)}{m+1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]

[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) / ((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(cx^{2n} + bx^n + a\right)^p (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + bx^n + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (fx)^m (bx^n + cx^{2n} + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)`

[Out] `int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx^{2n} + bx^n + a)^p (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")`

[Out] `integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (fx)^m (a + bx^n + cx^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(a + b*x^n + c*x^(2*n))^p,x)`

[Out] `int((f*x)^m*(a + b*x^n + c*x^(2*n))^p, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)`

[Out] Timed out

$$3.155 \quad \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Optimal. Leaf size=34

$$\text{Int} \left(\frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n}, x \right)$$

[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]

[Out] Defer[Int] [((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]

Rubi steps

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx = \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Mathematica [A] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]

[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")

[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (bx^n + cx^{2n} + a)^p}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d),x)

[Out] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x)

```
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n), x)
```

```
[Out] Timed out
```

$$3.156 \quad \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Optimal. Leaf size=34

$$\text{Int} \left(\frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2}, x \right)$$

[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]

[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]

Rubi steps

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx = \int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Mathematica [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]

[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]

fricas [A] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(cx^{2n} + bx^n + a)^p (fx)^m}{e^2 x^{2n} + 2 dex^n + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")

[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (bx^n + cx^{2n} + a)^p}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d)^2,x)

[Out] int((f*x)^m*(b*x^n+c*x^(2*n)+a)^p/(e*x^n+d)^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x)
```

```
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```
(* ::Text:: *)
```

```
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
```

```
(*1 = rational function*)
```

```
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
```

```
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
```

```
(*6 = appell function*)
```

```
(*7 = rootsum function*)
```

```
(*8 = integrate function*)
```

```
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
```

```
  If[AtomQ[expn],
```

```
    1,
```

```
  If[ListQ[expn],
```

```
    Max[Map[ExpnType,expn]],
```

```
  If[Head[expn]===Power,
```

```
    If[IntegerQ[expn[[2]],
```

```
      ExpnType[expn[[1]],
```

```
    If[Head[expn[[2]]]===Rational,
```

```
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
```

```
        1,
```

```
        Max[ExpnType[expn[[1]],2]],
```

```
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
```

```
    If[Head[expn]===Plus || Head[expn]===Times,
```

```
      Max[ExpnType[First[expn],ExpnType[Rest[expn]]],
```

```
    If[ElementaryFunctionQ[Head[expn]],
```

```
      Max[3,ExpnType[expn[[1]]],
```

```
    If[SpecialFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
```

```
    If[HypergeometricFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
```

```
    If[AppellFunctionQ[Head[expn]],
```

```
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```



```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
            ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc:

```

```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```


4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex

```

```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```